Pop Quiz!

What is the energy source for Enceladus’ geysers?

• Saturn’s rotational energy
• Enceladus’s orbital energy
• Elastic energy stored in Enceladus’s tidal deformation
• Thermal energy in Enceladus’ ocean
Pop Quiz!

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• Thermal energy in Enceladus’ ocean
• All of the above
Orbital Architecture of Saturn System

4:2 Resonance

2:1 Resonance

Moons migrate outward via tides
Orbital Architecture of Jovian System

Moons migrate outward via tides

Callisto, Ganymede, Europa, Io

2:1 Resonance

Click or Touch an Object to Explore

Objects Not to Scale

Jupiter
Tidal Basics

• Tidal Potential is

\[ U_{\text{tide}} \approx \frac{GM M'}{R} \frac{M'}{M} \left( \frac{R}{a} \right)^3 \]

• Tidal Displacement is

\[ \frac{\Delta R}{R} \approx \frac{U_{\text{tide}}}{gR} \approx \frac{M'}{M} \left( \frac{R}{a} \right)^3 \]

• Energy in tidal bulge is

\[ E_{\text{tide}} \approx \frac{U_{\text{tide}} M \Delta R}{R} \approx \frac{GM^2}{R} \left( \frac{M'}{M} \right)^2 \left( \frac{R}{a} \right)^6 \]
Classical Tidal Theory

\[ \theta = \frac{1}{Q} \]

\[ T = \frac{3}{2} \frac{G m^2 R_S^5 k_{2S}}{a^6 Q_S} \]
Tidal Torques
Mind your T’s and Q’s

• Define migration time scale

\[ t_{\text{tide}} = - \frac{E_{\text{orb}}}{\dot{E}_{\text{tide}}} = \frac{a}{\dot{a}_{\text{tide}}} \]

• And effective tidal Q can be defined as

\[ Q \equiv 3k_2 \frac{M_m}{M_p} \left( \frac{R_p}{a_m} \right)^5 \Omega_m t_{\text{tide}} \]
Equilibrium Tides

- Tidal bulge raised by gravity of companion
- Friction acts on shear associated with bulge
  - Turbulent viscosity in convective envelope
  - Viscoelasticity of solid core
- Energy is dissipated, producing tidal torque

\[ \dot{E}_{eq} \sim \frac{\nu_{\text{turb}}}{R^2} E_{\text{tide,eq}} \]
Dynamical Tides

- Dynamical Tide
  - Waves excited by gravitational forcing of companion
  - Friction dissipates waves
  - Energy is dissipated, producing tidal torque

\[ \dot{E}_{\text{dyn}} \approx k^2 \nu_{\text{eff}} E_{\text{tide, dyn}} \]

Mirouh et al. 2015
Dynamical Tides

• Waves and or oscillations in the planet

• Energy dissipation rate varies strongly with forcing frequency

• Tidal dissipation greatly enhanced around resonant peaks where

\[ \omega_\alpha \approx \omega_f = m(\Omega_p - \Omega_m) \]

Ogilvie & Lin 2004
Modeling Saturnian Tides
Modeling Saturnian Tides

![Graph showing tidal Q vs. semi-major axis (R_s)]

- **Tidal Q**
- **Semi-major axis (R_s)**
- **Eq. Tide**
- **Dy. Tide**

Legend:
- **Q_{tide}**
- **Q_{evol}**

Key:
- Mi
- En
- Te
- Di
- Rh
Measurements

- Measured effective $Q$ values are different from one another and smaller than expected
- Inconsistent with equilibrium tides
Resonance Locking

- Frequencies of resonant peaks are dependent on planet’s internal structure
- Planet’s internal structures gradually evolve
  - Cooling
  - Compositional settling, e.g., Helium rain

\[ T_{Sa} = \frac{G M^2}{R_{Sa} L_{Sa}} \approx 100 \text{ Gyr} \]

- Frequencies of resonant peaks evolve on planetary evolution timescale

\[ \dot{\omega}_\alpha = \frac{\omega_\alpha}{t_\alpha} \]
Stable Fixed Point

t_{\text{evol}} \sim t_{\alpha}

\alpha \sim \text{tide}

$\alpha$
Tidal Dynamics in Resonance Lock

• Migration rate is simply

\[
\frac{\dot{a}}{a} = \frac{2}{3} \left[ \frac{\omega_{\alpha}}{m \Omega_m t_{\alpha}} - \frac{\Omega_p}{\Omega_m t_p} \right]
\]

• Tidal timescale is closely related to planetary evolution timescale

\[
t_{\text{tide}} \approx \frac{3}{2} \frac{\Omega_m}{\Omega_p - \Omega_m} t_{\alpha}
\]

• Effective tidal $Q$ is usually much smaller than frequency-averaged $Q$

\[
Q_{\text{ResLock}} = \frac{9 k_2}{2} \frac{M_m}{M_p} \left( \frac{R}{a} \right)^5 \left[ \frac{\omega_{\alpha}}{m \Omega_m^2 t_{\alpha}} - \frac{\Omega_p}{\Omega_m^2 t_p} \right]^{-1}
\]
$t_\alpha = 50$ Gyr
$t_\alpha = 50 \text{ Gyr}$
Nakajima et al. 2018
Evolutionary History

• In equilibrium tidal theory, migration rate strongly dependent on semi-major axis

\[ Q \equiv 3k_2 \frac{M_m}{M_p} \left( \frac{R_p}{a_m} \right)^5 \Omega_m t_{\text{tide}} \]

• According to resonance locking, tidal migration rate is only weakly dependent on semi-major axis

\[ t_{\text{tide}} \approx \frac{3}{2} \frac{\Omega_m}{\Omega_p - \Omega_m} t_{\alpha} \]
Mean motion resonances occur when:

\[(j + 1)\Omega_2 - j\Omega_1 \approx 0\]

For pair of moons tidally driven outward by inner moon and locked in MMR, angular momentum deficit develops, and eccentricity/inclination must increase.

Due to non-spherical gravitational potential, resonances are split into multiple components:

- Eccentricity type resonances excite eccentricity:
  \[2\lambda' - \lambda - \omega\]
  \[2\lambda' - \lambda - \omega'\]

- Inclination type resonances excite inclination:
  \[4\lambda' - 2\lambda - \Omega - \Omega'\]
Zhang & Nimmo 2009
Tidal Heating
Tidal Heating

• Tidal heating rate via eccentricity tides is

\[ \dot{E}_{\text{heat}} = \frac{21}{2} k_1 \frac{G M_p^2 R_1^5}{Q_1 a_1^6} \Omega_1 e_1^2 \]

• Eccentricity is boosted by mean motion resonance but damped by tidal heating. At equilibrium eccentricity, these effects balance

\[ e_{eq}^2 = \frac{1}{7(j - 1)} \frac{M_1 M_2}{M^2} \left( \frac{R_p}{R_1} \right)^5 \frac{Q_1}{Q_{p,1}} \frac{k_p}{k_1} \]

• If inner moon pushes outer moon outward via mean motion resonance, tidal heating rate of inner moon is

\[ \dot{E}_{\text{heat},1} \approx \frac{1}{j - 1} \frac{|E_{2,\text{orb}}|}{t_{\text{tide}}} \]

For Enceladus: \[ \dot{E}_{\text{heat}} \approx 50 \text{ GW} \]
Spawning Moons from Rings

Charnoz et al. 2011
Standard Lore

• Tides drive moons outward, MMRs encountered, tidal heating ensues

New Paths Forward

• Effective tidal Q is different for each moon, and varies with time (dynamical tides)

• Tidal evolution occurs on planetary evolution timescale (resonance locking)

• Substantial migration of outer moons (e.g., Titan & Callisto)

• Rapid migration, ring-driven migration, late formation of inner moons
Thanks!