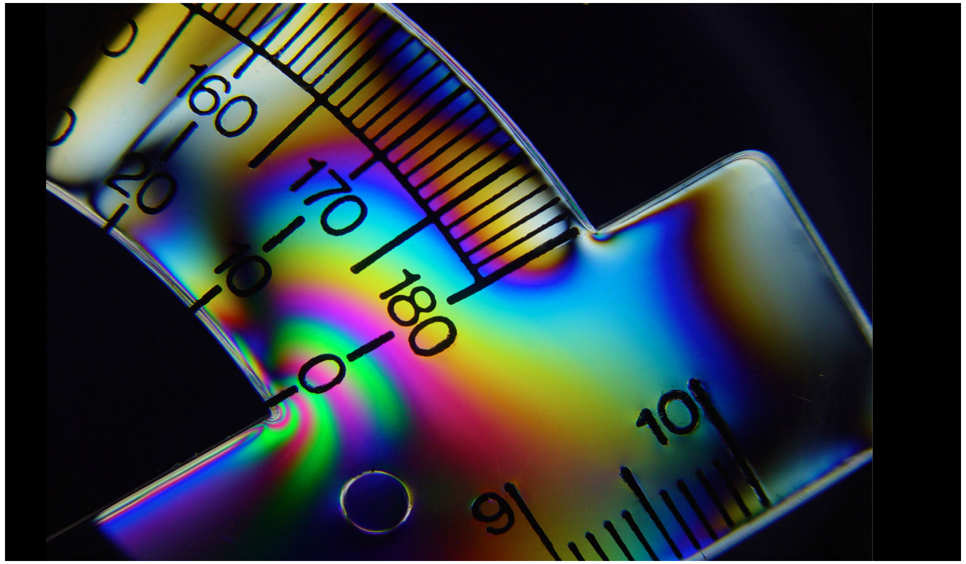
# The role of polarization in Coronagraphy

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**Protractor is transparent to the eye but has polarization aberrations**

Breckinridge, Lam & Chipman (2015) *Polarization Aberrations in Astronomical Telescopes: The Point Spread Function*, PASP **127**:445-468

# Polarization aspects of exoplanet science

- Polarimetry map the exoplanet neighborhood: Gas & Dust
- Polarimetry of unresolved planets
- Internal polarization modifies shape of the PSF

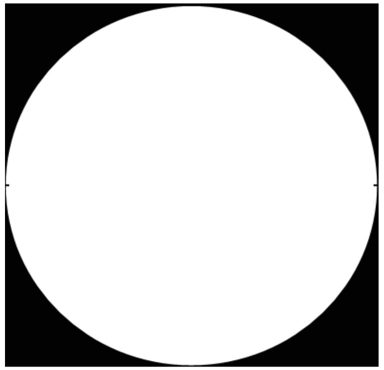
# Physical optics says:

- Unpolarized white light can be represented by 2 orthogonal polarizations
- For convenience we select 2 linear orthogonal polarizations
  - Perpendicular Y and
  - Parallel X

# Role of vector waves in image formation

For zero OPD error $W(x,y)$=0.0

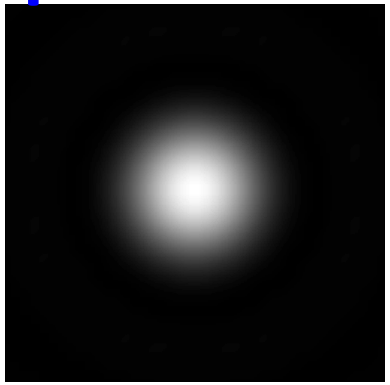
**Exit pupil**



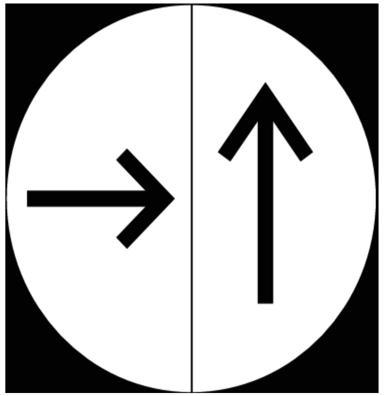
***No Polarizer***

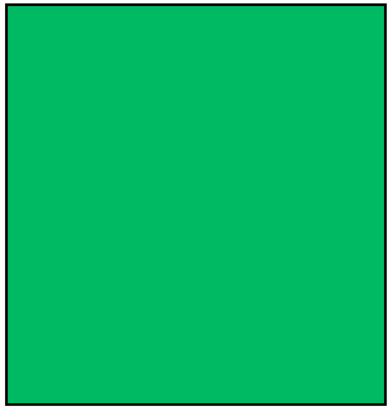
Resolution is position angle **<u>in</u>**dependent

**Image plane PSF**



To represent internal polarization in the extreme we add two perpendicular linear polarizers





# Role of vector waves in image formation

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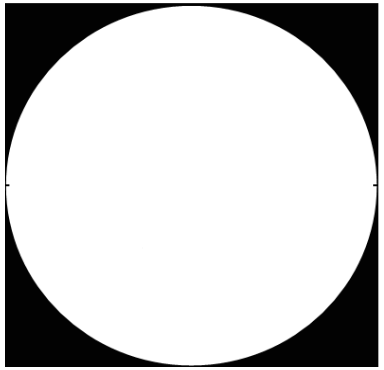
**Exit pupil**

**Image plane PSF**



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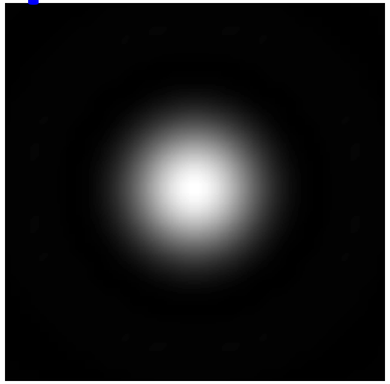

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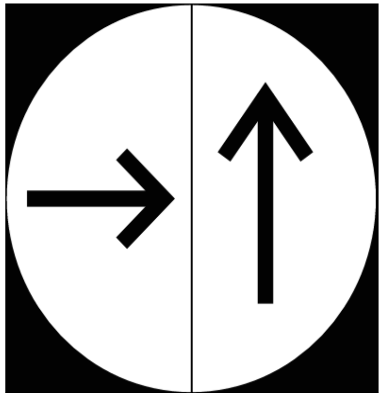
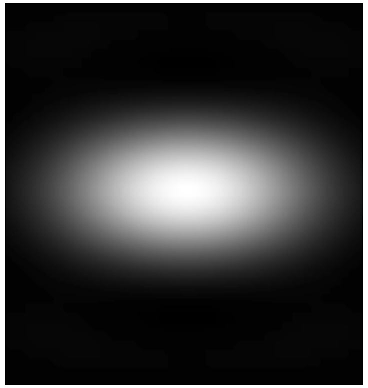


Resolution is position angle **<u>de</u>**pendent

The PSF is the incoherent sum of two "D" apertures



# Fresnel (1823) equations & definitions

$$N_0 = 1.$$

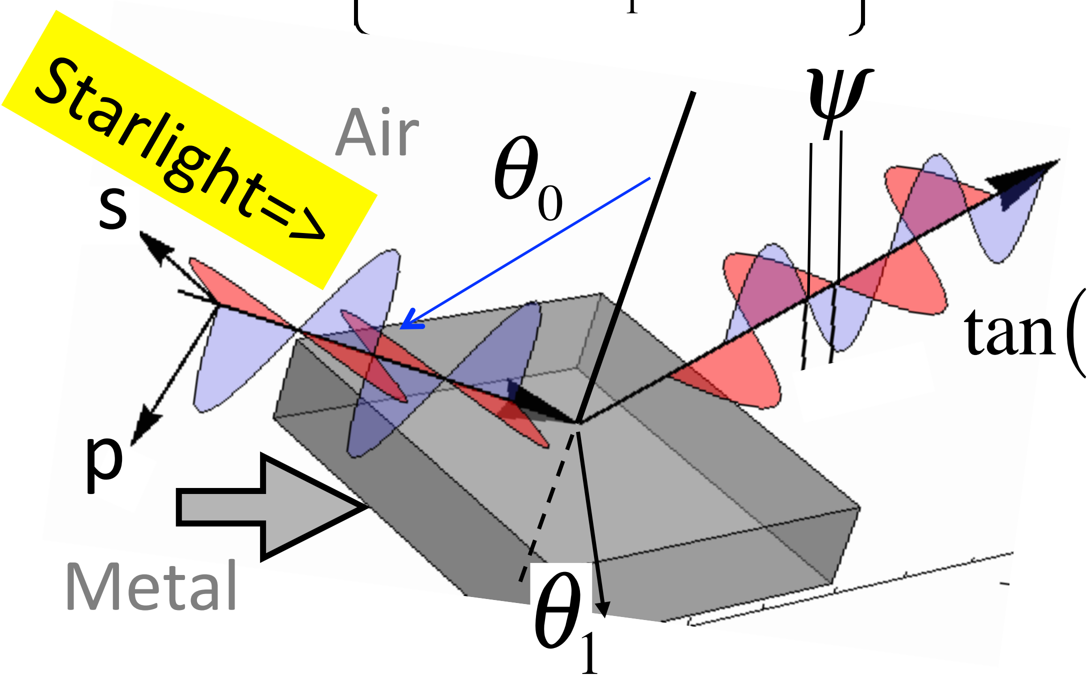
For metals, index is complex: $N_1 = n_1 - ik_1$

$$\theta_1 = \arccos\left\{ \frac{\sqrt{N_1^2 - N_0^2 \sin^2\theta_0}}{N_1} \right\}$$

$$r_p = \frac{\tan(\theta_0 - \theta_1)}{\tan(\theta_0 + \theta_1)}$$

$$r_s = \frac{-\sin(\theta_0 - \theta_1)}{\sin(\theta_0 + \theta_1)}$$

$$\left.\begin{matrix} \theta_1 \\ r_p \\ r_S \end{matrix}\right\} Complex$$

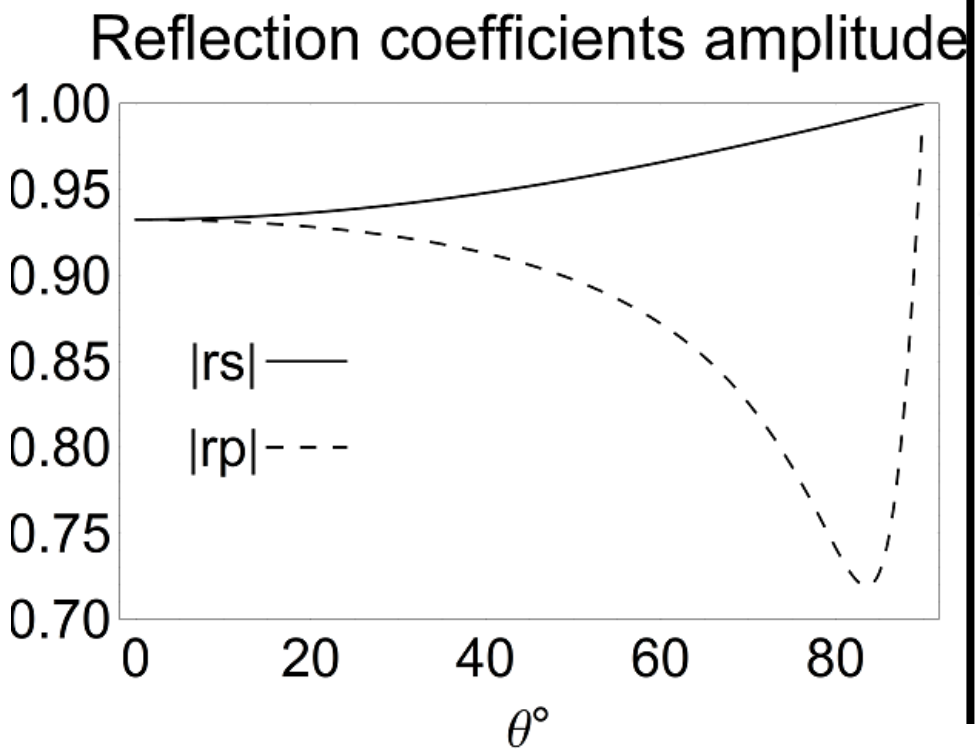


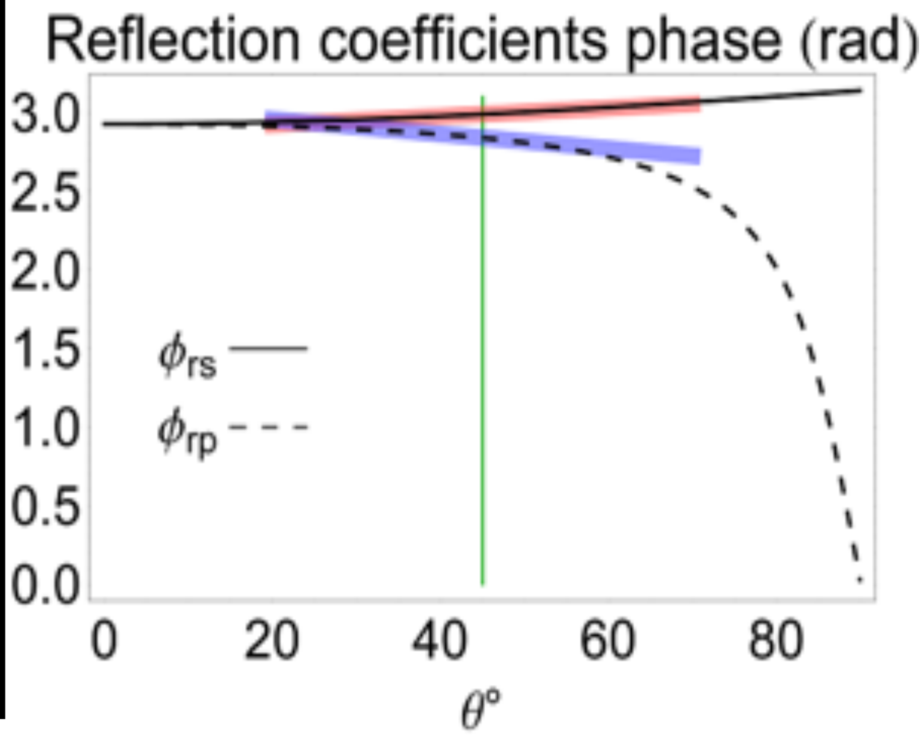

$$\boxed{\frac{r_s - r_p}{r_s + r_p} = \text{Diattenuation}}$$

$$\tan(\psi) = \tan(\phi_S - \phi_P) = |r_p| / |r_s|$$

$$\boxed{\psi \text{ is called Retardance}}$$

# Reflection coefficients (A & $\phi$) for Al @ 800 nm; $N_1 = 2.80 + 8.45i$




The two polarization aberrations are

$$\frac{r_s - r_p}{r_s + r_p} = \text{Diattenuation \& Retardance} \left(\tan\psi = |r_p| / |r_s|\right)$$

## For a white-light, incoherent point in object space a ray is mapped through the optical system.

White-light source

$$\left.\begin{aligned} U_X &= A_X e^{i\phi_X} \\ U_Y &= A_Y e^{i\phi_Y} \end{aligned}\right\}$$

Telescope & Instrument

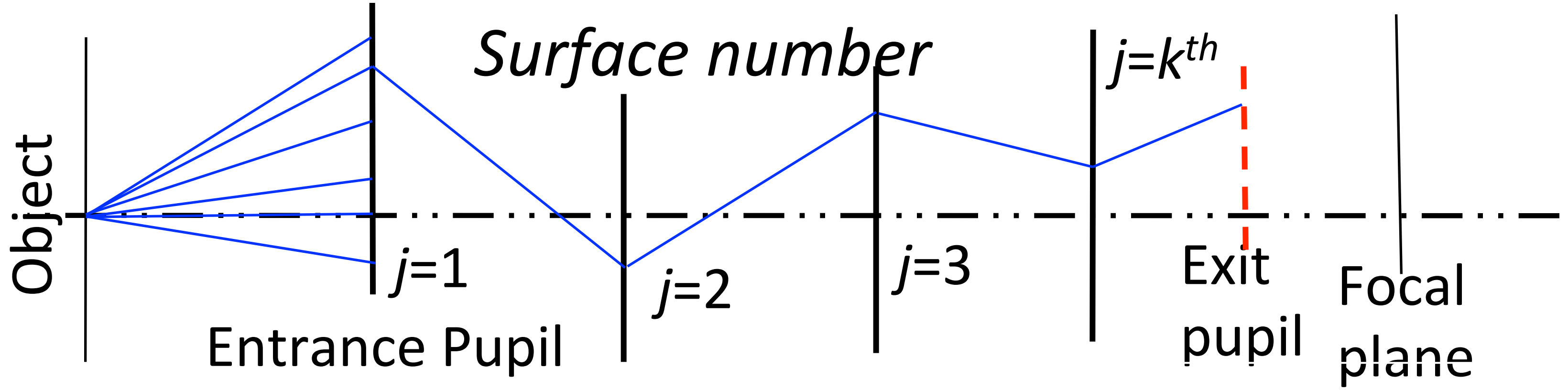
Focal plane

$$\left\{\begin{aligned} &A_{XX} e^{i\phi_{XX}} \\ &A_{XY} e^{i\phi_{XY}} \\ &A_{YY} e^{i\phi_{YY}} \\ &A_{YX} e^{i\phi_{YX}} \end{aligned}\right.$$

**Rearrange these to give:**

$$\begin{vmatrix} A_{XX}\, e^{i\phi_{XX}} & A_{XY}\, e^{i\phi_{XY}} \\ A_{YX}\, e^{i\phi_{YX}} & A_{YY}\, e^{i\phi_{YY}} \end{vmatrix} \equiv \begin{vmatrix} J_{XX} & J_{XY} \\ J_{YX} & J_{YY} \end{vmatrix} \equiv \mathbf{J}_{ExitPupil}$$

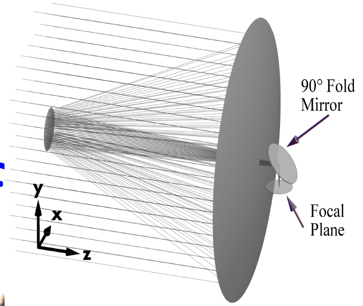
# How to calculate the PSF for each polarization




Based on the direction cosine at each surface and the physical properties of each surface $(n-ik)$ we use the Fresnel equations to calculate the **amplitude change** and the **phase change** for each ray at each surface

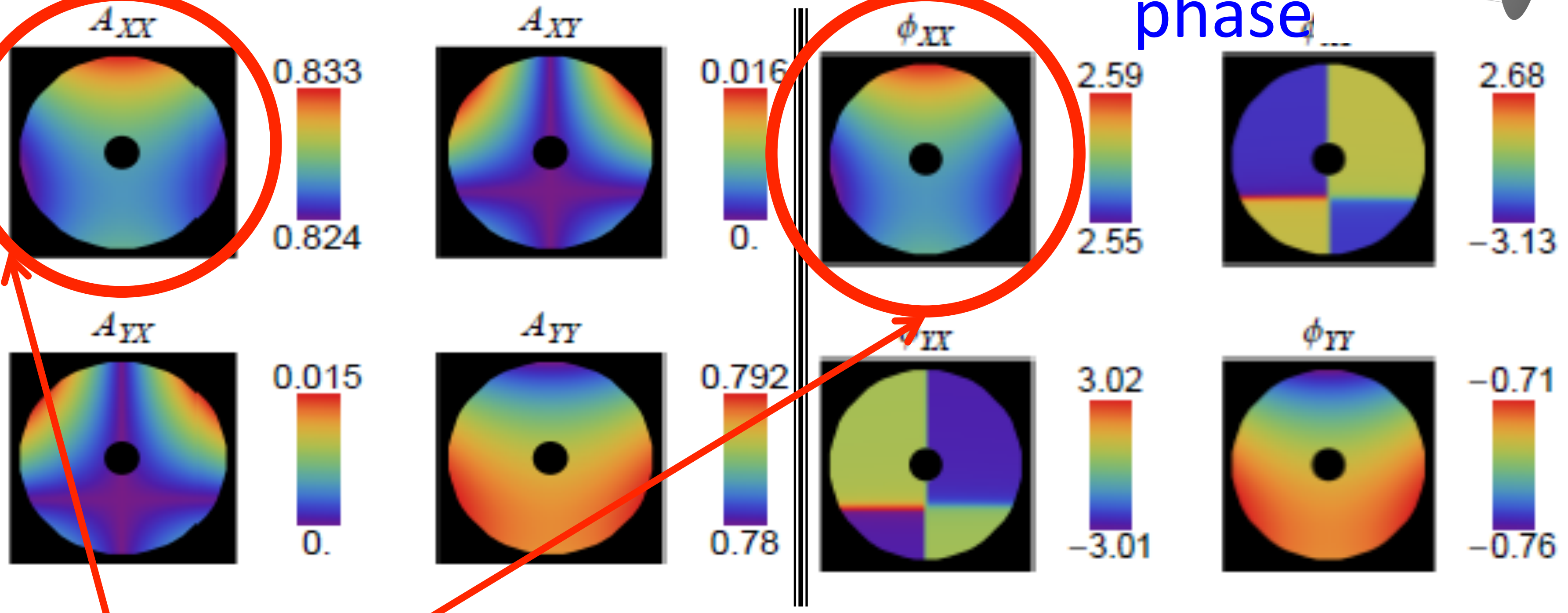
**Compute the multiplicative amplitude and cumulative phase for both the $\|$ and the $\perp$ light for each ray traced across the entrance pupil & map these 4 complex arrays onto the exit pupil.**

# Jones pupil for a 3-mirror bent Cass




Amplitude normalized

Radians of phase




A/O cannot Correct phase

$$\begin{vmatrix} A_{XX}\,e^{i\phi_{XX}} & A_{XY}\,e^{i\phi_{XY}} \\ A_{YX}\,e^{i\phi_{YX}} & A_{YY}\,e^{i\phi_{YY}} \end{vmatrix} \equiv \begin{vmatrix} J_{XX} & J_{XY} \\ J_{YX} & J_{YY} \end{vmatrix}$$

**Jones vector**

$\equiv \mathbf{J}_{ExitPupil}$

# Habex retardance & diattenuation maps

## Primary mirror Al + 25nm $MgF_2$

Retardance $(\psi)$

Max 30 milli-radians

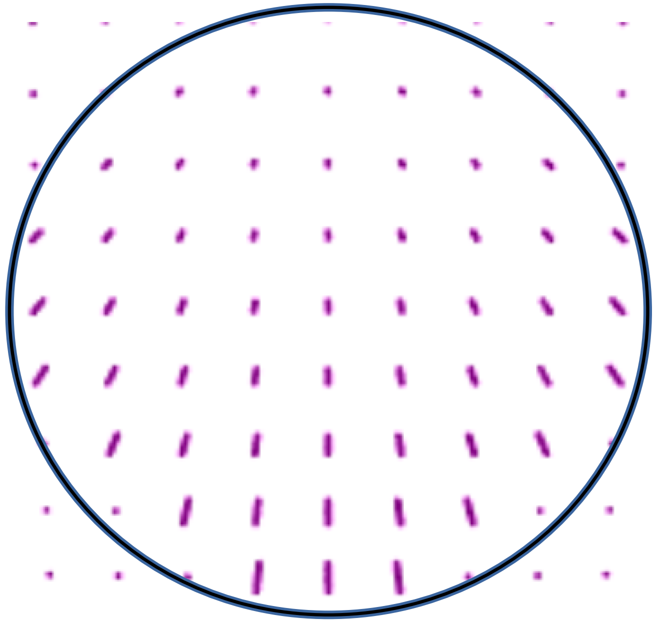
Min 0 radians

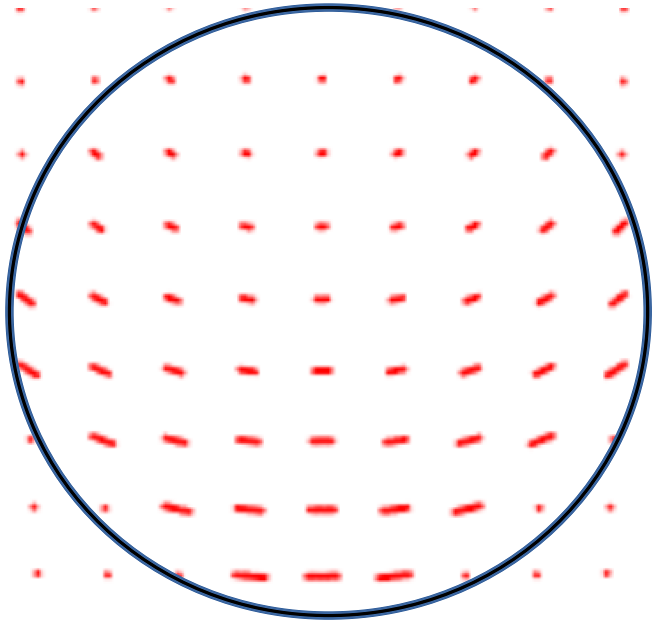
Diattenuation

Max 0.002

Min 0

$$D = \frac{|r_s|^2 - |r_p|^2}{|r_s|^2 + |r_p|^2}$$

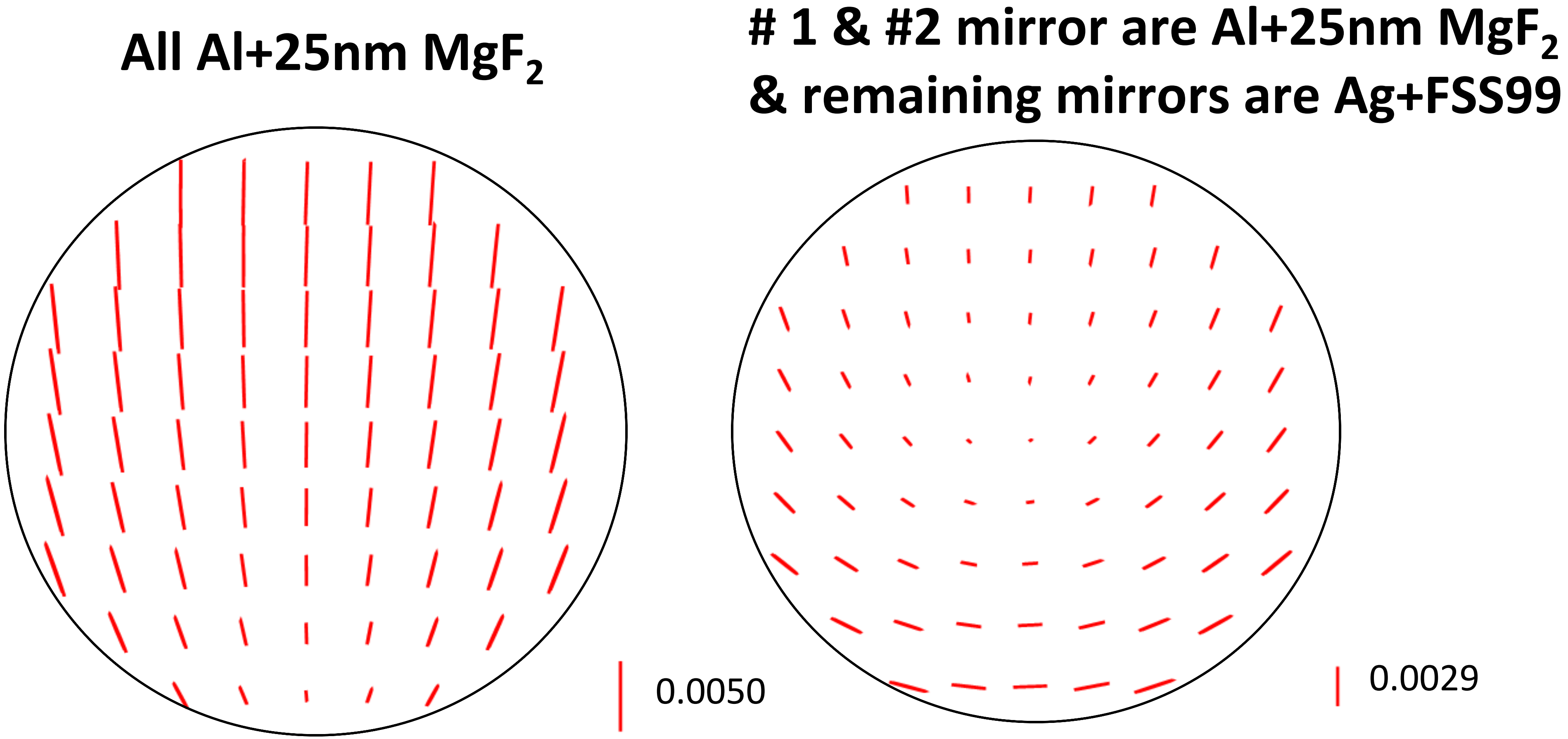




M1

30 milli-radians of retardance @ 450 nm is $\psi = 2.15\text{-nm or } 2150\text{-pm}$

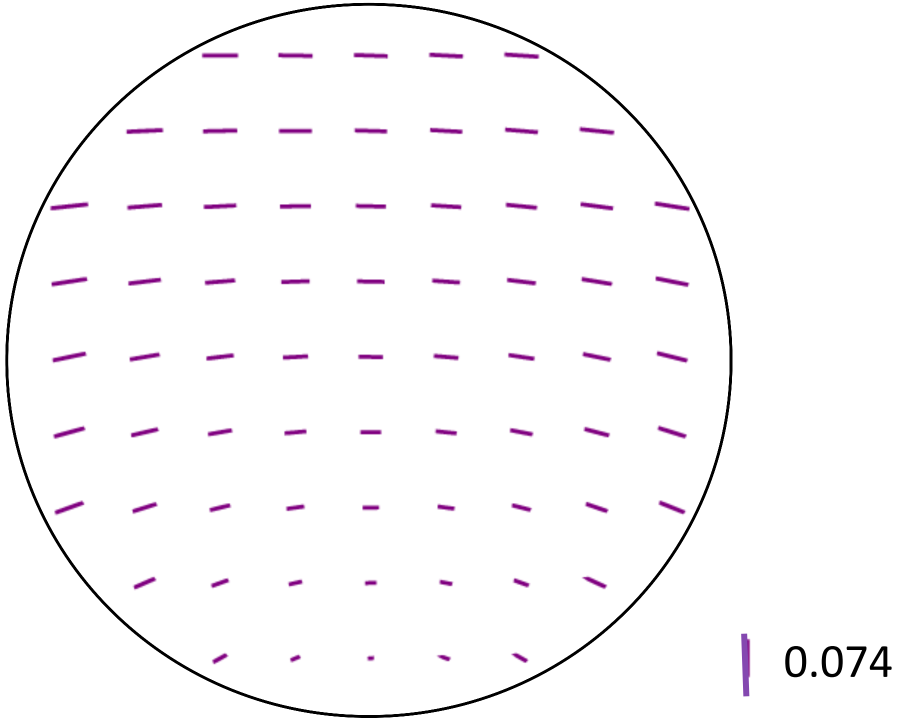
# Cumulative Diattenuation (11 mirrors) across the last mirror before the focal plane mask




@ 500nm

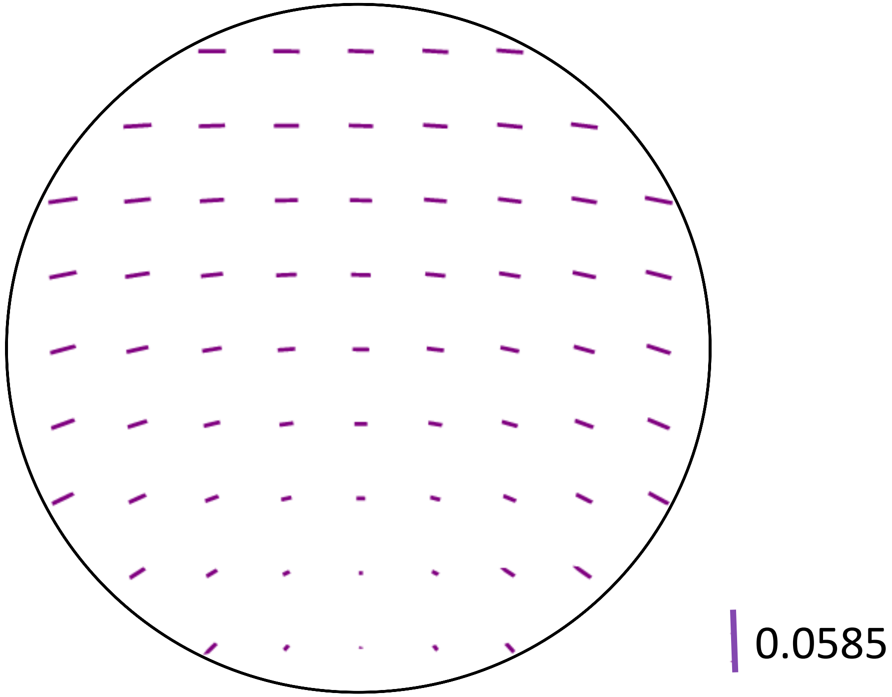
# Cumulative retardance (11 mirrors) across the exit pupil to the focal plane mask

**All Al+25nm $MgF_2$**




0.074 radians =
0.0118 waves

**# 1 & #2 mirror are Al+25nm $MgF_2$ & remaining mirrors are Ag+FSS99**

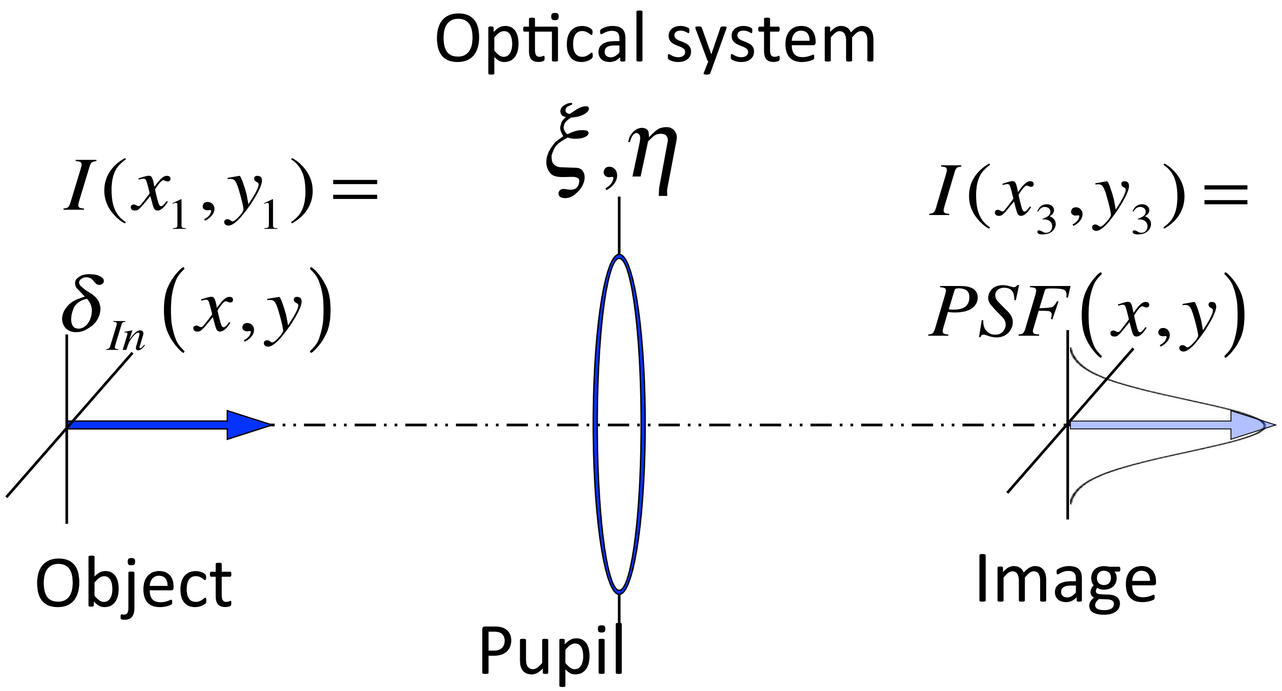



0.0585 radians =
0.0093 waves

@ 500nm

# Back up

# Definition of Point Spread Function




$$I_{image}(x,y) = I_{Object}(x,y) \otimes PSF$$

$$\text{where } PSF = \mathcal{F}\left[P(\xi,\eta)\right]$$

The PSF is the spatial frequency intensity

Impulse response of the optical system

# The vector transmittance of the telescope is

$$\vec{\tau}_2(\xi_2,\eta_2)=\left|\begin{matrix} J_{XX} & J_{YX} \\ J_{XY} & J_{YY} \end{matrix}\right|_{\xi,\eta}$$

The telescope/coronagraph system complex transmittance across the exit pupil depends on the vector of the electromagnetic field at point $(\xi,\eta)$ within the exit pupil. The complex electric field at the image plane, for an on-axis unpolarized star of unit brightness follows from the Fresnel Kirchoff diffraction integral and is written:

$$u_3(x_3,y_3)=$$

$$K\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|\begin{matrix} J_{XX} & J_{YX} \\ J_{XY} & J_{YY} \end{matrix}\right|\exp\left(-\frac{2\pi}{\lambda f}[x_3\xi_2+y_3\eta_2]\right)\right\}d\xi d\eta=$$

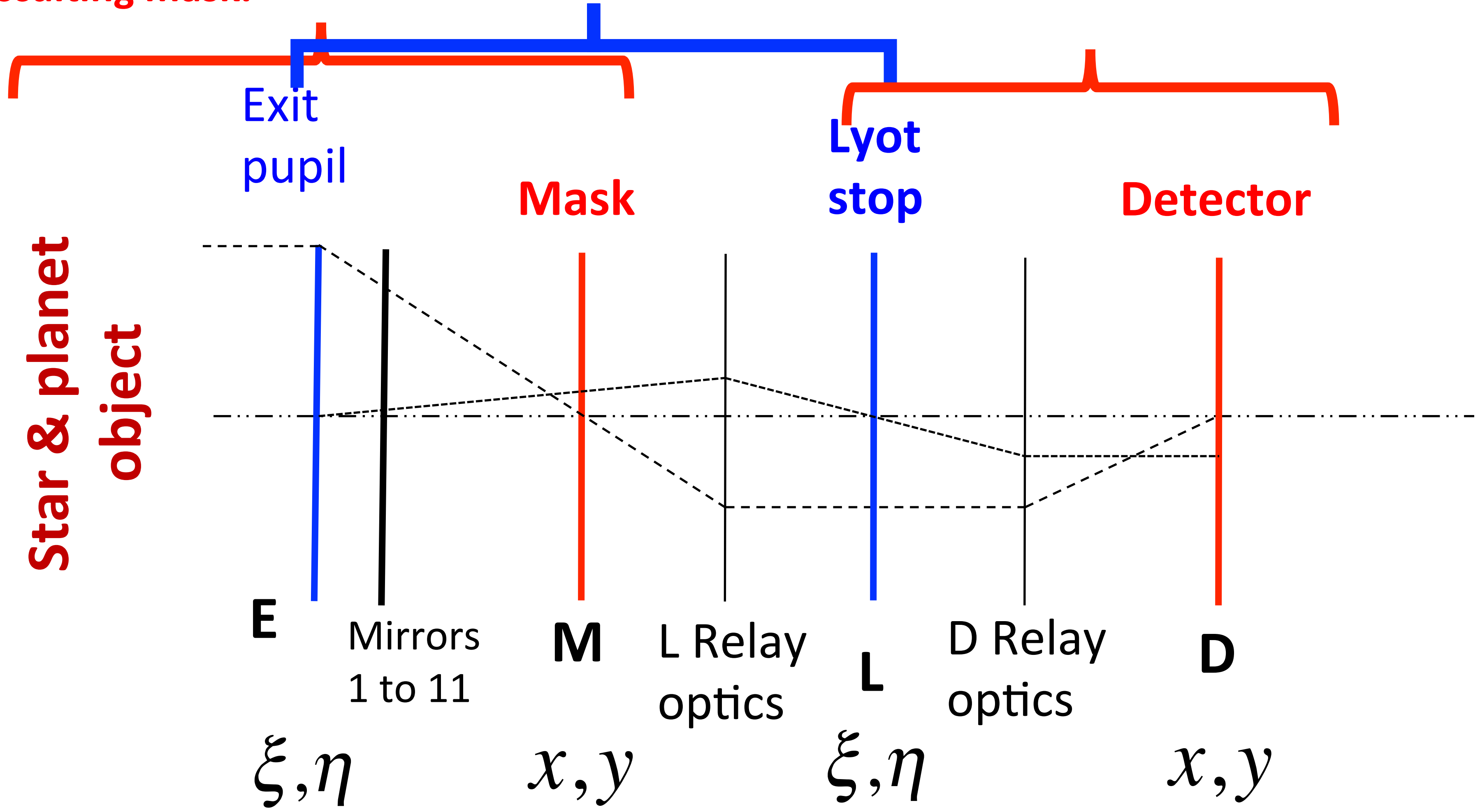
$$K\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[J_{XX}+J_{YY}+J_{YX}+J_{XY}]\exp\left(-\frac{2\pi}{\lambda f}[x_3\xi_2+y_3\eta_2]\right)d\xi d\eta\right\}$$

# The HabEx Lyot coronagraph divides into three optical subsystems

**#1 delivers an image (complex field) to the occulting mask.**

#2 the **L relay optic** maps the exit pupil field E to the **Lyot stop field**

#3 the **D relay optic** maps the field at the Lyot stop L into an image on the detector




**Polarization ray trace physical surfaces**

**&**

**FFT in free space to propagate between exit pupils and image planes**

# Vector wave image formation

$$\vec{U}_3(x_3,y_3)=$$

$$K\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[\vec{U}_2^{-}(\xi_2,\eta_2)]\bullet\underline{\vec{\tau}_2(\xi_2,\eta_2)}\exp\left\{-j\frac{2\pi}{\lambda f}(x_3\xi_2+y_3\eta_2)\right\}d\xi_2\,d\eta_2\right]$$

In astronomical telescopes and instruments the term $\vec{\tau}_2(\xi_2,\eta_2)$ is a **vector** and $\vec{U}_3(x_3,y_3)$ depends on **BOTH** the polarization properties of the **source** & **the telescope/instrument end to end optical system**.

**Pupil transmittance complex Jones matrix**

$$\vec{\tau}_2(\xi_2,\eta_2)=\begin{vmatrix} J_{XX} & J_{XY} \\ J_{YX} & J_{YY} \end{vmatrix}$$

Where $J_{XX}$ is $\vec{X}$ light *in* $\vec{X}$ light *out* and $J_{XY}$ is the $\vec{X}$ light *in* that has been projected into $\vec{Y}$

# How do we calculate the vector PSF?

The complex field at the focal plane is given by

$$U_3(x_3,y_3)=\mathcal{F}\{J_{XX}+J_{YY}+J_{XY}+J_{YX}\}$$

The intensity is then

$$I_3(x_3,y_3)=|U_3(x_3,y_3)|^2=|\mathcal{F}\{J_{XX}+J_{YY}+J_{XY}+J_{YX}\}|^2$$

E. Wolf [Theory Of Coherence & Polarization Of Light (Cambridge2007)] shows that these four fields are not correlated and are therefore incoherent.

The intensity at the focal plane is then the incoherent sum:

$$I_3(x_3,y_3)=|\mathcal{F}(J_{XX})|^2+|\mathcal{F}(J_{YY})|^2+|\mathcal{F}(J_{YX})|^2+|\mathcal{F}(J_{XY})|^2$$

The telescope PSF is the linear (uncorrelated) superposition of these 4 separate PSF’s