Advances in AO Systems (fundamental limits)

Jared Males University of Arizona



JPL-MPIA Workshop, 2018-April-09



• Angel (1994)

Ground-based imaging of extrasolar planets using adaptive optics

J. R. P. Angel

Steward Observatory, University of Arizona, Tucson, Arizona 85721, USA

The detection of extrasolar planets by direct imaging presents an extraordinary technical challenge. They must be identified against background light scattered from a star close by and about a billion times brighter. It has been supposed that a near-perfect space telescope would be required to avoid atmospheric blurring. But by using adaptive optics operating at fundamental performance limits, the new generation of large ground-based telescopes has the potential to detect planets orbiting nearby stars.



- Angel (1994)
 - Predicted that 6.5 to 12 m ground-based telescopes would be characterizing planets orbiting nearby main-sequence stars in the coming decade.
 - Considered atmospheric speckles, which should average quickly
 - Did not yet know of the monsters called quasi-static speckles waiting just off the edge of the map...





- Guyon 2005
 - Analyzed the fundamental limits of AO
 - Primarily a spatial-PSD analysis, with simplified version of frozen-flow.



AO Limits



- Goal: develop a framework for analyzing the fundamental limit of ground-based contrast
 - Closed-loop control analysis
 - Consider full multi-layer turbulent atmosphere
 - Frozen flow, but not limited to this
 - Predict post-coronagraph contrast
 - Analyze the temporal behavior of speckles.

• With apologies: this requires math . . .



The limit of the planet:star flux ratio that we can detect & characterize is set by the variance of intensity in the focal plane:

$$\sigma_{tot}^2 = F_* \Delta t \left\{ I_c + I_{as} + I_{qs} + F_* \left[\tau_{as} \left(I_{as}^2 + 2[I_c I_{as} + I_{as} I_{qs}] \right) + \tau_{qs} \left(I_{qs}^2 + 2I_c I_{qs} \right) \right] \right\}$$

cf. Soummer et al., "Speckle Noise and Dynamic Range in Coronagraphic Images", ApJ 669:642 (2007)



$$\sigma_{tot}^2 = F_* \Delta t \left\{ I_c + I_{as} + I_{qs} + F_* \left[\tau_{as} \left(I_{as}^2 + 2[I_c I_{as} + I_{as} I_{qs}] \right) + \tau_{qs} \left(I_{qs}^2 + 2I_c I_{qs} \right) \right] \right\}$$

- F_* = photons/sec from the star
- $\Delta t = \text{total integration time}$
- I_c = intensity residual from the coronagraph & static aberrations
- I_{as} = intensity residual from atmospheric speckles
- τ_{as} = lifetime of atmospheric speckles
- I_{qs} = intensity residual from quasi-static speckles
- τ_{qs} = lifetime of quasi-static speckles

cf. Soummer et al., "Speckle Noise and Dynamic Range in Coronagraphic Images", ApJ 669:642 (2007)



Residual Variance - Photons

$$\sigma_{tot}^{2} = F_{*}\Delta t \left\{ I_{c} + I_{as} + I_{qs} + F_{*} \left[\tau_{as} \left(I_{as}^{2} + 2[I_{c}I_{as} + I_{as}I_{qs}] \right) + \tau_{qs} \left(I_{qs}^{2} + 2I_{c}I_{qs} \right) \right] \right\}$$

$$Photon Noise (Poisson statistics)$$





















AO Limits



$$\sigma_{tot}^{2} = F_{*}\Delta t \left\{ I_{c} + I_{as} + I_{qs} + F_{*} \left[\tau_{as} \left(I_{as}^{2} + 2[I_{c}I_{as} + I_{as}I_{qs}] \right) + \tau_{qs} \left(I_{qs}^{2} + 2I_{c}I_{qs} \right) \right] \right\}$$

- Coronagraph design is now quite advanced, (thanks WFIRST CGI!)
- It is not a wild assumption to ignore it, and that's what I'm going to do from now on.
- But: it is common to hear things like:

"well, we'll only ever achieve 1e-5 AO residual, so that's all we need from the coronagraph"

This is wrong. Because of pinning, coronagraph performance (I_c) must be much better than AO (I_{as}).

• Note also that I_c includes truly static aberrations, or (say) very long lived ones. So this also has implications for discussion of design and stability of ground-based instruments.



$$\sigma_{tot}^{2} = F_{*}\Delta t \left\{ I_{c} + I_{as} + I_{qs} + F_{*} \left[\tau_{as} \left(I_{as}^{2} + 2[I_{c}I_{as} + I_{as}I_{qs}] \right) + \tau_{qs} \left(I_{qs}^{2} + 2I_{c}I_{qs} \right) \right] \right\}$$





• Understanding the limits of ground-based HCI requires:

1) Predict residual atmospheric speckle contrast $\ I_{as}$

2) Predict residual atmospheric speckle lifetime au_{as}

3) Predict quasi-static speckle contrast

 I_{qs}

4) Predict quasi-static speckle lifetime





• AO control as a time-domain control problem is described by the temporal PSD



Fig 6 Calculated temporal PSDs in a 7-layer turbulence model at 4 different modes, corresponding to discrete spatial frequencies.

Males & Guyon, JATIS, 2018

These are the temporal PSDs, $\mathcal{T}_{mn}(f)$ of individual Fourier modes (or spatial frequencies) in the atmosphere.

Calculations assumed frozen-flow von Karman turbulence, with LCO median conditions.

 $\mathcal{T}_{mn}(f)$ describes the statistics of the modal amplitudes h_{mn}

The variance of the modal amplitudes $\langle |h_{mn}|^2 \rangle$ gives the contrast:

$$\left|I_{\Phi}(\vec{r})\right\rangle = \left(\frac{2\pi}{\lambda}\right)^{2} \left\langle |h_{mn}|^{2} \right\rangle \left[\text{PSF}(\vec{r} - \vec{k}_{mn}\lambda) + \text{PSF}(\vec{r} + \vec{k}_{mn}\lambda) \right]$$

AO Limits



Transfer Functions



- AO control acts on the input PSD.
- The output of the control system is described by its *transfer functions*

 $\mathcal{T}_{cl,mn}(f;g) = \mathcal{T}_{mn}(f) \left| \text{ETF}_{cl}(s;g) \right|^2 + \mathcal{T}_{ph,mn}(f) \left| \text{NTF}_{cl}(s;g) \right|^2$

ETF = *error transfer function*

NTF = *noise transfer function*

The residual variance, hence the contrast, is given by:

 $\left\langle h_{mn}^{2}\right\rangle (g)=\int_{0}^{f_{s}}\mathcal{T}_{cl,mn}(f;g)df$

 Goal: design system with variance minimizing ETF and NTF



Transfer Functions



- AO control acts on the input PSD.
- The output of the control system is described by its *transfer functions*

 $\mathcal{T}_{cl,mn}(f;g) = \mathcal{T}_{mn}(f) \left| \text{ETF}_{cl}(s;g) \right|^2 + \mathcal{T}_{ph,mn}(f) \left| \text{NTF}_{cl}(s;g) \right|^2$

ETF = *error transfer function*

NTF = *noise transfer function*

The residual variance, hence the contrast, is given by:

 $\left\langle h_{mn}^{2}\right\rangle (g)=\int_{0}^{f_{s}}\mathcal{T}_{cl,mn}(f;g)df$

 Goal: design system with variance minimizing ETF and NTF



AO Control System





AO Control System





- The control law calculates the command to apply to the DM based on the WFS measurements.
 - Simple Integrator: $\tilde{h}_i = \tilde{h}_{i-1} + g\Delta h_i$
 - General Integrator: $\tilde{h}(t_i) = \sum_{j=1}^J a_j \tilde{h}(t_{i-j}) + g \sum_{l=0}^L b_l \Delta h(t_{i-l})$
- Choosing an optimum set of coefficients is the subject of "predictive control", which many studies have considered.
 - E.g. Dessenne et al. (1998), Poyneer et al. (2007), Correia et al. (2017)
- We consider a method for determining the coefficients based on the Linear Prediction (LP) formalism using the input PSD.
 - See Males & Guyon (2018) for details . . .



Predictive Control In Action





MagA0-X





- NSF-MRI funded ExAO+Coronagraph
 - 2000 actuator BMC MEMS
 - 3.7 kHz Pyramid WFS
 - Suite of coronagraphs
 - vAPP (Leiden)
 - PIAACMC
 - Optimized for Vis-Near-IR
 - Young planets at $\text{H}\alpha$
- Integration in progress at UA
 - First-light spring 2019



MagA0-X





Residual Atmosphere: 6.5 m





G-MagAO-X





7x 3000 actuator MEMS

21,000 Actuators





Residual Atmosphere: 25.4 m





Contrast due to residual atmospheric speckles on a 25.4 m telescope. see Males & Guyon, JATIS, 2018



- Make use of all available information Sensor Fusion
 - Not just WFS
 - Accelerometers, FPWFS, etc.
- Empirical Orthogonal Functions (EOF)
 - A time-and-space PCA of all available information
 - Going back in time
 - See Guyon & Males 2017.



```
On sky proof of concept:
Guyon et al., in prep
On-sky results
```

 Loop running at 2kHz, filter computed every 50sec
 54 consecutives 0.5s images (26 sec exposure), 3 mn apart Same star, same exposure time, same intensity scale
 ← Std dev
 OFF = integrator, gain 20%

conditions: 1.5" seeing, ~35mph wind

AO Limits



Step 2: "Predict residual atmospheric speckle lifetime"



- Now we're on step #2
- We'll define the statistical speckle lifetime by
 - Rate of increase of variance with time: $\sigma^2(t) = \sigma_o^2 \frac{t}{t}$
 - Or, equivalently, improvement in variance of the mean: $<|a_0|^2>=rac{\sigma_o^2}{t/ au}$

• Historical note: this development started with ACESat (PIs Belikov and Bendek), trying to understand the power of post-processing over very long observations.



Variance of the Mean

- Noll (1976) calc. of statistics of a process given PSD
- Used Zernikes for 2D Kolmogorov PSD
 - Variance & co-var of amplitudes

Zernike polynomials and atmospheric turbulence*

Robert J. Noll The Perkin-Elmer Corporation, Norwalk, Connecticut 06856 (Received 3 October 1975)

This paper discusses some general properties of Zernike polynomials, such as their Fourier transforms, integral representations, and derivatives. A Zernike representation of the Kolmogoroff spectrum of turbulence is given that provides a complete analytical description of the number of independent corrections required in a wave-front compensation system.

J. Opt. Soc. Am., Vol. 66, No. 3, March 1976

Variance of process after correction of N Zernike modes.

TABLE IV. Zernike-Kolmogoroff residual errors (Δ_{J}) . (D is the aperture diameter.)	
$\Delta_1 = 1.0299 \ (D/r_0)^{5/3}$	$\Delta_{12} = 0.0352 (D/r_0)^{5/3}$
$\Delta_2 = 0.582 \ (D/r_0)^{5/3}$	$\Delta_{13} = 0.0328 \ (D/r_0)^{5/3}$
$\Delta_3 = 0.134 \ (D/r_0)^{5/3}$	$\Delta_{14} = 0.0304 \ (D/r_0)^{5/3}$
$\Delta_4 = 0.111 (D/r_0)^{5/3}$	$\Delta_{15} = 0.0279 \ (D/r_0)^{5/3}$
$\Delta_5 = 0.0880 \ (D/r_0)^{5/3}$	$\Delta_{16} = 0.0267 \ (D/r_0)^{5/3}$
$\Delta_6 = 0.0648 \ (D/r_0)^{5/3}$	$\Delta_{17} = 0.0255 \ (D/r_0)^{5/3}$
$\Delta_7 = 0.0587 (D/r_0)^{5/3}$	$\Delta_{18} = 0.0243 \ (D/r_0)^{5/3}$
$\Delta_8 = 0.0525 (D/r_0)^{5/3}$	$\Delta_{19} = 0.0232 \ (D/r_0)^{5/3}$
$\Delta_9 = 0.0463 (D/r_0)^{5/3}$	$\Delta_{20} = 0.0220 \ (D/r_0)^{5/3}$
$\Delta_{10} = 0.0401 \ (D/r_0)^{5/3}$	$\Delta_{21} = 0.0208 \ (D/r_0)^{5/3}$
$\Delta_{11} = 0.0377 (D/r_0)^{5/3}$	
$\Delta_J \sim 0.2944 J^{-\sqrt{3}/2} (D/r_0)^{5/3}$	(For large J)

AO Limits



- We can repeat Noll's analysis in 1-D with Legendre polynomials.
 - Express time-series as expansion in Legendre polynomials

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} I(t) P_n\left(\frac{2}{T}t\right) dt$$

- Apply temporal PSD of process governing the time-series.
 ... bunch of math ...
- Derive covariance of coefficients:

$$< a_n^* a_{n'} >= -i^n i^{n'} \frac{\sqrt{2n+1}\sqrt{2n'+1}}{2T} \int_0^\infty \frac{J_{n+\frac{1}{2}}(2\pi k)J_{n'+\frac{1}{2}}(2\pi k)}{k} \mathcal{T}\left(\frac{2}{T}k\right) dk$$

- Which gives variance of coefficient **n**:

$$<|a_n|^2>=rac{2n+1}{2T}\int_0^\infty rac{J_{n+\frac{1}{2}}^2(2\pi k)}{k}\mathcal{T}\left(rac{2}{T}k
ight)dk$$



• We can derive an expression for the correlation length of any process given its temporal PSD:

$$\tau = \lim_{T \to \infty} \frac{\int_0^\infty \frac{J_{\frac{1}{2}}^2(2\pi k)}{k} \mathcal{T}\left(\frac{2}{T}k\right) dk}{2\int_0^{f_{max}} \mathcal{T}(f) df} \quad \text{Males et al, in prep.}$$

• Sanity check: if you plug in a white noise PSD, T(f) = constant, you get:

$$<|a_0|^2>=\frac{\sigma_o^2\Delta t}{T}=\frac{\sigma_o^2}{N}$$

Note: an alternative way to derive tau is to use the Wiener-Khinchin theorem to get the autocorrelation, and find tau as the integral of the A.C. (see Fitzgerald & Graham 2006). In practice, I've found this to be quite hard to implement with numerical PSDs.



• Getting to speckle intensity from Fourier amplitude

$$I_s \approx 2\left(\frac{\pi}{\lambda}\right)^2 \left(h_{mn}^{+2} + h_{mn}^{-2}\right) \left[\text{PSF}(\vec{r} - \vec{k}_{mn}\lambda) + \text{PSF}(\vec{r} + \vec{k}_{mn}\lambda)\right]$$

• From the closed-loop control analysis, we have the PSD of h, not h^2

- Brute force it:
 - 1) generate 2x random correlated time-series of h w/ closed-loop residual PSD of the Fourier mode amplitude
 - 2) calculate the periodogram: $\mathcal{T}_{I_s}(f) \propto \left| \mathcal{F} \left\{ h_{mn}^{+\,2} + h_{mn}^{-\,2} \right\} \right|^2$
 - 3) repeat N times and average



Speckle Lifetimes



Comparison to Macintosh et al (2005) "Crossing-Time Speckles"



Speckle Lifetimes



- Macintosh et al. (2005) ignored control dynamics (they acknowledged this)
- Concept of "crossing time speckle" is less useful with action of control law on PSDs
- Predictive Control significantly shortens speckle lifetime → faster averaging of noise (but still well above white noise)



Step 3: "Predict quasi-static speckle contrast" Step 4: "Predict quasi-static speckle lifetime"



Quasi-Static Speckles



• Surface map based on MagAO-X design specs

~1e-4 raw contrast





Quasi-Static Speckles





Quasi-static speckles with lifetimes of order 1 to 10 minutes commonly reported; e.g. Hinkley et al (2007), Martinez et al (2012), Milli et al (2016)





- Aperture Photometry
 - Measurement of background requires referencing a different spatial location → subject to speckle noise.

- High Dispersion Coronagraphy Snellen+ (2013), Mawet+ (2017), Wang+ (2017)
 - Measurement of continuum happens in-situ (across very small $\Delta\lambda$) \rightarrow not subject to speckle noise





- Aperture Photometry
 - Measurement of background requires referencing a different spatial location → subject to speckle noise.
- High Dispersion Coronagraphy Snellen+ (2013), Mawet+ (2017), Wang+ (2017)
 - Measurement of continuum happens in-situ (across very small $\Delta\lambda$) \rightarrow not subject to speckle noise





$$\frac{S/N_{HDC}}{S/N_{AP}} = \frac{\sqrt{\eta_{sp}}}{\left[1 - \frac{\Delta\lambda}{W_{\lambda}} \sum^{N_l} q_l\right]} \times \sqrt{\frac{\Delta\lambda N_{l,eff}}{W_{\lambda}}} \times \sqrt{1 + C_H F_* W_{\lambda} \tau_{sl}}$$

Comparison of S/N for the HDC vs AP techniques.

If this ratio is > 1 then we should do HDC.

Males et al., in prep





Males et al., in prep



- Let's assume nothing works all that well:
 - Achieve only 10x the LP contrast
 - Residual is long lived speckles (~10 mins)
 - 3 λ /D IWA coronagraph
- And assume GMT and TMT
 - Observe known-from-RV planet hosts
 - In 25%-ile conditions for LCO and MKO
 - Science and WFS both @ 800 nm
 - 10% throughput, with noiseless detectors



HDC w/ Reflected Stellar Lines



AO Limits



GSMT HDC Targets (reflected light albedo)



AO Limits



Control of Quasi-Static Speckles



- What if we implement a 1 Hz speckle control loop?
 - Analyze it with same tools we used for atmosphere



- On-line control of quasi-static speckles is not a new idea
- How fast can we run?
- Example: speckle nulling at SCExAO
 - Martinache et al (2014)
 - 30 Hz, using 30 probe frames
 - So getting to 1 Hz



0.03 0.06 0.09 0.12 0.15 0.18 0.21 0.24

06 0.012 0.018 0.024 0.030 0.036 0.042 0.048 0.054 0.060

On-sky demo of speckle nulling at ~1 Hz by Martinache et al (2014)



Control of Quasi-Static Speckles



- What if we implement a 1 Hz speckle control loop?
 - Analyze it with same tools we used for atmosphere
 - I_{qs} 1e-4 \rightarrow 1e-9
 - τ_{qs} 10 min \rightarrow 1 sec

• Now AP will be more efficient than HDC



- Let's assume it all works really well:
 - Achieve the LP contrast prediction
 - Residual is short lived speckles (10 ms)
 - 1λ /D IWA coronagraph
- And assume GMT and TMT (same as before)
 - Observe known-from-RV planet hosts
 - In 25%-ile conditions for LCO and MKO
 - Science and WFS both @ 800 nm
 - 10% throughput, with noiseless detectors



GSMT AP Targets (reflected light albedo)



AO Limits



GSMT AP Targets (reflected light albedo)



AO Limits



What Is Our Limit On The Ground?



speckle limited \rightarrow Using HDC

speckle limited \rightarrow Using AP

• I have not considered Post-Processing in "fundamental" limits: $\sigma_{tot}^2 = F_* \Delta t \left\{ I_c + I_{as} + I_{qs} + F_* \left[\tau_{as} \left(I_{as}^2 + 2[I_c I_{as} + I_{as}] \right) + \frac{\tau_{qs} \left(I_{qs}^2 + 2I_c I_{qs} \right)}{I_{qs}} \right] \right\}$

> P.P. will only remove the long-lived terms — much better to remove Iqs before taking our images

- But, for long-lived speckles that we haven't controlled, post-processing is the key to reducing their spatial variance.
- We should be able to cast the post-processing step into the temporal frequency domain as a filter.
 - See a first attempt at this in ODI (Males, Belikov, Bendek 2015).
- Any ideas on how to do this generally?

- Predictive Control offers great promise
 - Much better contrast (>10x on 8th mag star, > 1000x on bright stars)
 - See Correia 2017 for a different approach which reaches same conclusion
 - Significantly shortens speckle lifetime
 - But it's hard...
 - Requires excellent calibration of system
 - WFS gain (which we know is variable)
 - Subtle details of system transfer functions really matter

- We should be able get the atmosphere out of the way
 - GMT raw contrast (@800nm and 25% conditions):
 - 1e-7 on 5th mag star
 - 1e-6 on an 8th mag star
 - 5-10 ms speckle lifetimes, will average!

- Comes down to in-instrument quasi-static speckles
- Should we be talking about moving HCIT to LCO & MKO?
 - If we can achieve optimum control of the atmosphere we will only be limited by instrumental aberrations...
 - Is that level of raw contrast motivating enough?