Physical optics of image formation coronagraphs:

The Point Spread Function

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Presented also at the ExoPAG (2014), GSFC, STSci, & U of R Institute of Optics

Keck Institute for Space Studies

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Papers on Wiki

J. B. Breckinridge and B. Oppenheimer, Polarization Effects in Reflecting Coronagraphs for White Light Applications in Astronomy, ApJ, 600, pp 1091, 2004

J. B. Breckinridge, Wai Sze T. Lam and R. A. Chipman, Polarization Aberrations in Astronomical Telescopes: The Point Spread Function, PASP, 127**:445–468** May 2015

Russell A. Chipman, Wai Sze T. Lam and James B. Breckinridge Polarization Aberration in Astronomical Telescopes, Proc. SPIE 9613-16, Polarization Science and Remote Sensing VII , San Diego, CA 2015

Wavefront errors

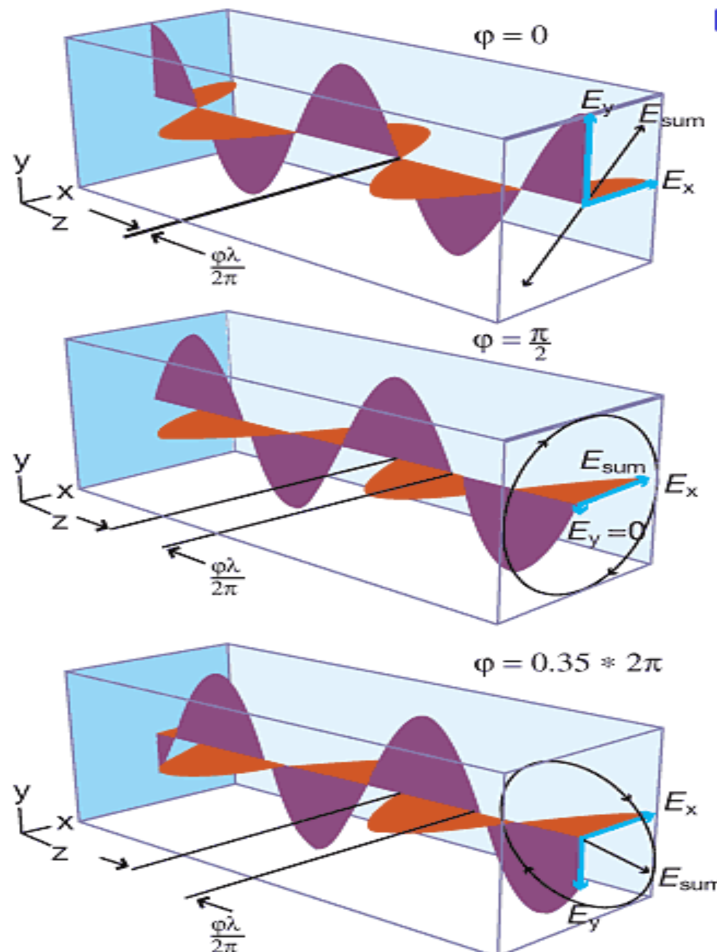
- Correcting for geometric wavefront errors is a necessary but not sufficient to guarantee a "perfect" PSF.
- Polarization wavefront errors [polarization aberrations {10^{14} not 10^{3} Hz}]
- Diffraction from secondary support structures & segment edges
- As built opto-mechanical 3-D layout

Aberrations

It is easy to visualize surface OPD geometric wavefront

HST primary 1982
8 years before launch

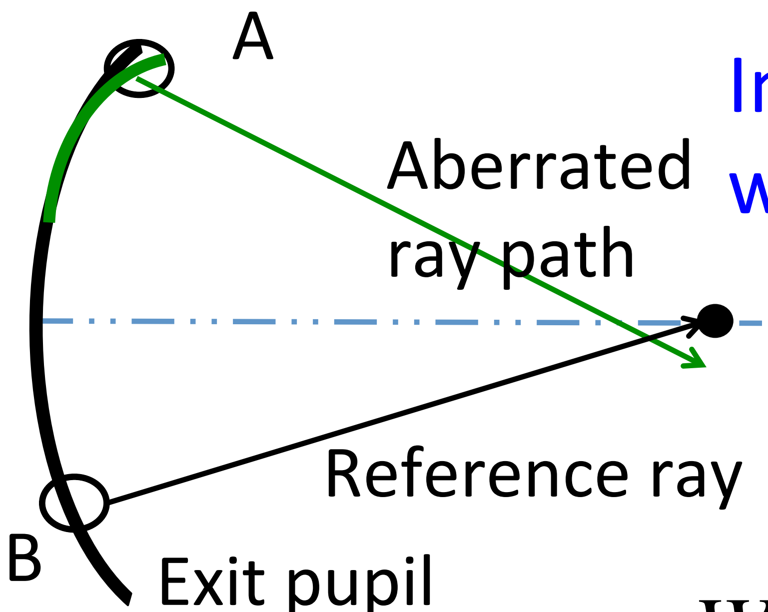
Challenge to visualize polarization aberrations



Geometric aberrations

$$\text{Wavefront error}\ (W) = \frac{\text{reference ray path} - \text{ray path}}{\lambda} = \frac{\text{OPD}}{\lambda}$$

For all points *x,y* across the exit pupil



In Space, with no atmosphere, we can come close to

$$W(x,y) = 0$$

But . . .

$$W(x,y) = 0 \not=> \text{perfect image}$$

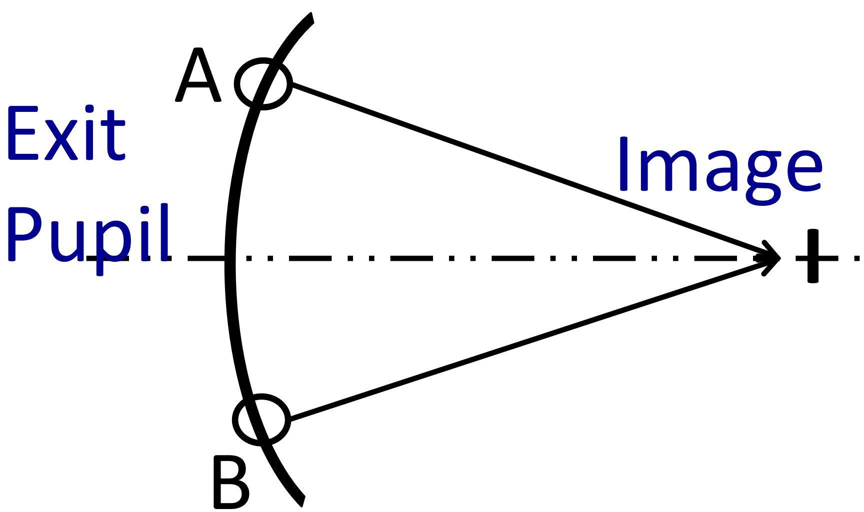
Need to examine polarization aberrations

Polarization aberrations

Polarization determines image quality.

E & M fields from regions A and B need to be correlated (the SAME polarization state) to form the pixels in an image

Geometric wavefront error
W=0.0



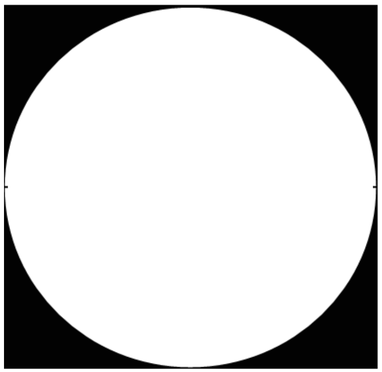
Models that use vector representation of fields are necessary

More later=>

Why vector waves are needed: polarization role in image formation

For zero OPD error $W(x,y)=0.0$

Exit pupil

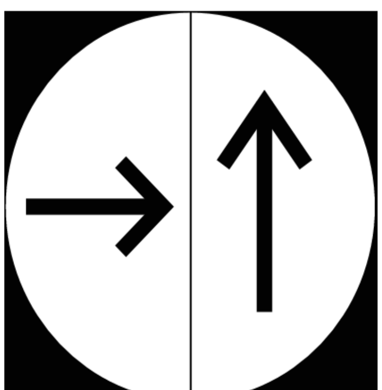


No Polarizer

Resolution is position angle **<u>in</u>**dependent

Image plane PSF

To represent internal polarization in the extreme we add two perpendicular linear polarizers



Resolution is position angle **<u>de</u>**pendent

The PSF is the incoherent sum of two "D" apertures

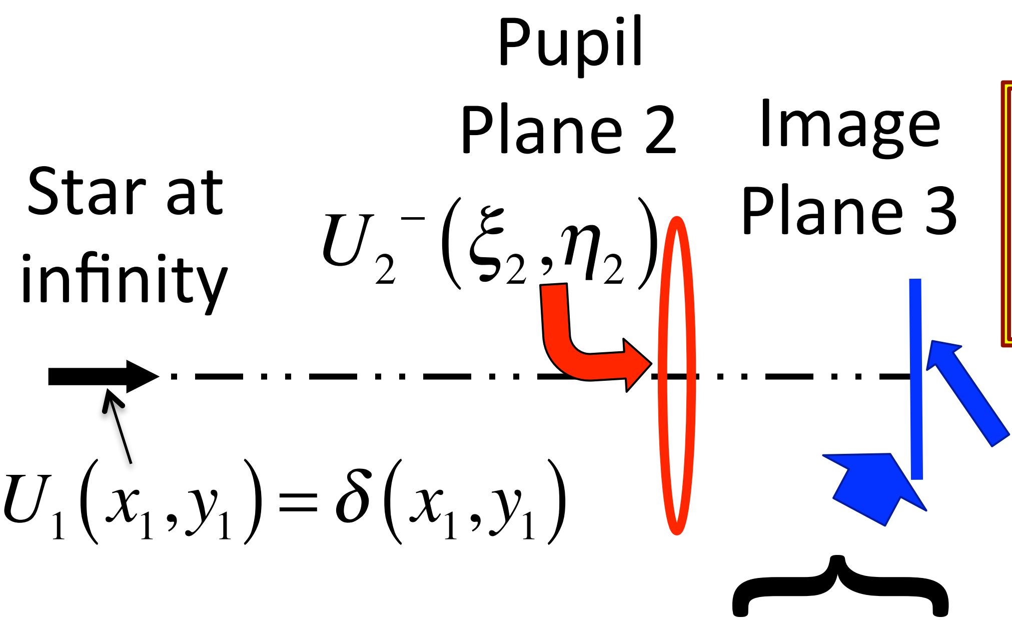
Observations

- Orthogonally polarized light does not interfere to to contribute to an image.
- The shape of the point spread function depends on how polarization changes across the exit pupil.

Questions?

- What are the sources of instrument polarization in astronomical telescopes?
- What is the magnitude of the effect?
- What is the impact?

Propagate the **field** through the system to find the **complex scalar field** at the focal plane $U_3(x_3,y_3)$



x,y : image plane

ξ,η : pupil plane

$U_1(x_1,y_1)=\delta(x_1,y_1)$

$I_3(x_3,y_3)=\left|U_3(x_3,y_3)\right|^2$

$$U_3(x_3,y_3)=$$
$$\mathrm{K}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[U_2^-(\xi_2,\eta_2)]\cdot\tau_2(\xi_2,\eta_2)\cdot\exp\left\{-j\frac{2\pi}{\lambda f}(x_3,\xi_2+y_3,\eta_2)\right\}d\xi\, d\eta$$

Where $\tau_2(\xi_2,\eta_2)=A_2(\xi_2,\eta_2)+i\phi_2(\xi_2,\eta_2)$

Vector wave image formation

$$\vec{U}_3(x_3,y_3) =$$

$$\mathrm{K}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[\vec{U}_2^{\,-}(\xi_2,\eta_2)]\bullet\underline{\vec{\tau}_2(\xi_2,\eta_2)}\exp\left\{-j\frac{2\pi}{\lambda f}(x_3,\xi_2+y_3,\eta_2)\right\}d\xi_2\,d\eta_2\right]$$

In astronomical telescopes and instruments the term $\vec{\tau}_2(\xi_2,\eta_2)$ is a **vector** and $\vec{U}_3(x_3,y_3)$ depends on **BOTH** the polarization properties of the **source** & **the telescope/instrument**.

Pupil transmittance complex Jones vector

$$\vec{\tau}_2(\xi_2,\eta_2) = \begin{vmatrix} J_{XX} & J_{XY} \\ J_{YX} & J_{YY} \end{vmatrix}$$

Where J_{XX} is $\vec{\mathrm{X}}$ light *in* $\vec{\mathrm{X}}$ light *out* and J_{XY} is the $\vec{\mathrm{X}}$ light *in* that has been projected into $\vec{\mathrm{Y}}$

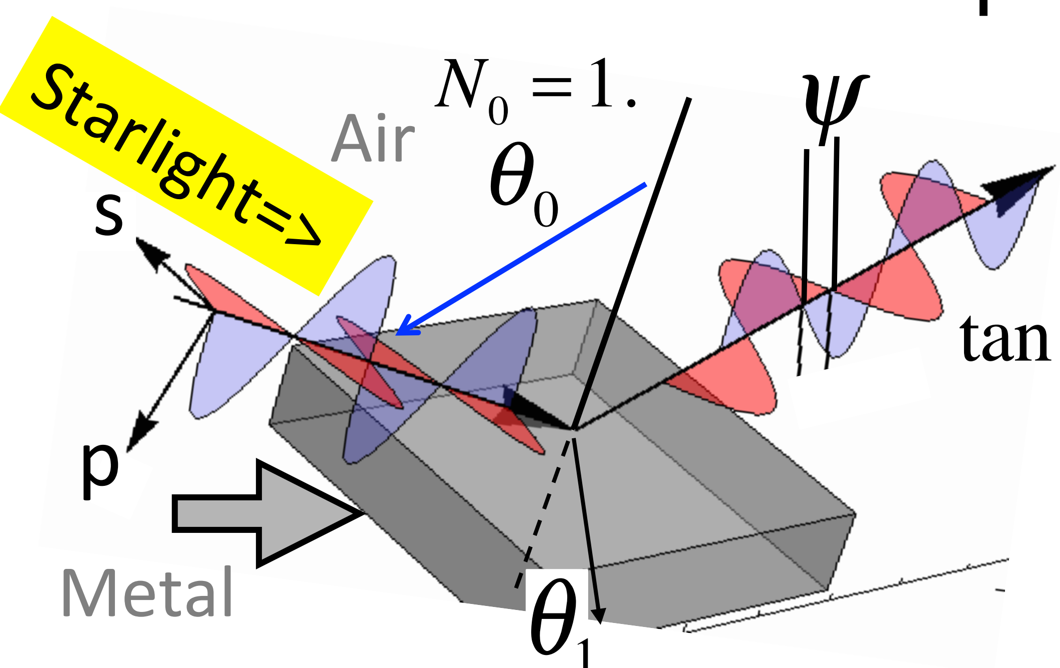
"Fresnel (1823) equations" derived from Maxwell's equations

For metals: $N_1 = n_1 - ik_1$

$$\theta_1 = \arccos\left\{\frac{\sqrt{N_1^2 - N_0^2 \sin^2\theta_0}}{N_1}\right\}$$

$$r_p = \frac{\tan(\theta_0 - \theta_1)}{\tan(\theta_0 + \theta_1)}$$

$$r_s = \frac{-\sin(\theta_0 - \theta_1)}{\sin(\theta_0 + \theta_1)}$$

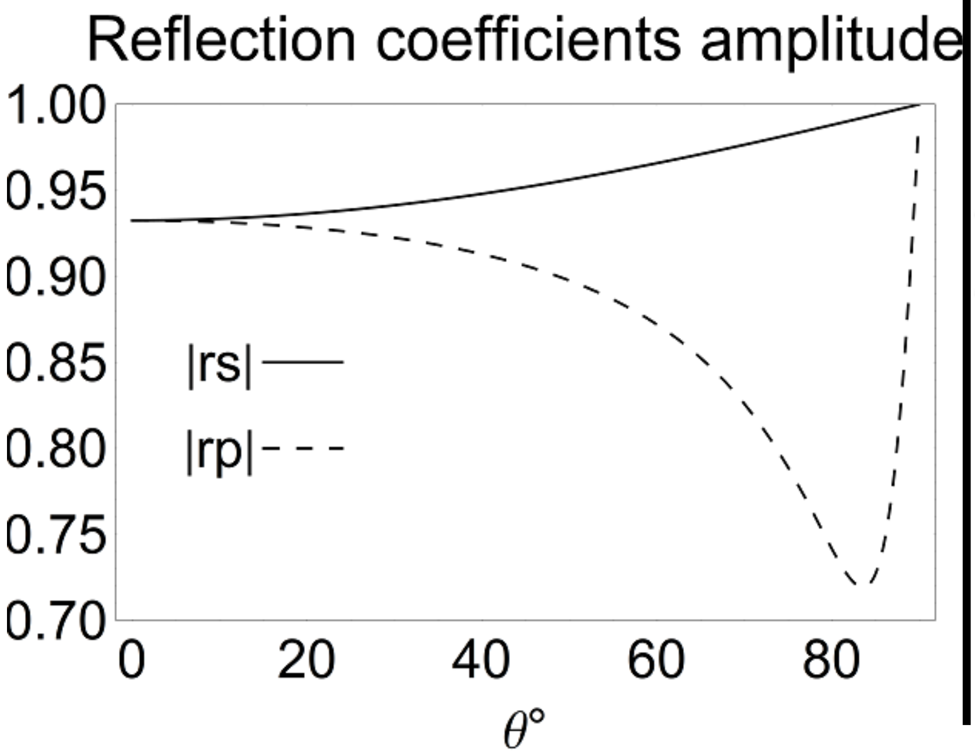


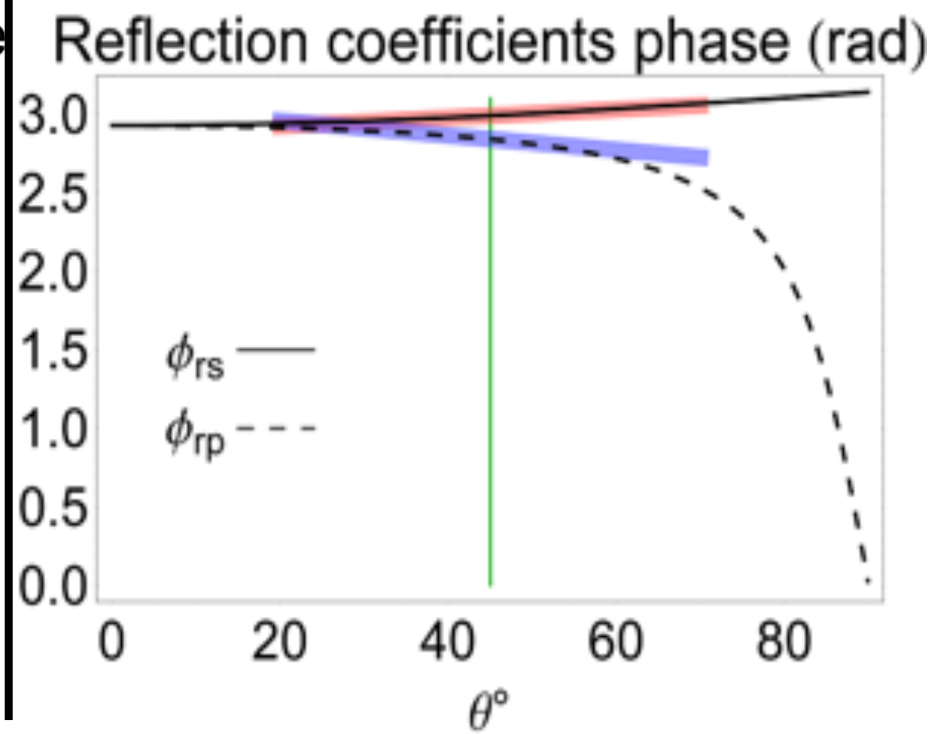
$$\frac{r_s - r_p}{r_s + r_p} = \text{diattenuation}$$

$$\tan(\psi) = \tan(\phi_S - \phi_P) = |r_p| / |r_s|$$

ψ is called retardance

Reflection coefficients (A & ϕ) for Al @ 800 nm; $N_1 = 2.80 + 8.45i$





The two polarization aberrations are

$$\frac{r_s - r_p}{r_s + r_p} = \text{diattenuation and retardance} \left(\tan\psi = \left|r_p\right| / \left|r_s\right|\right)$$

Decompose white-light (star) into polarization components

- **We select any orthonormal basis set for ray trace**
- Select the easiest for for intuition
- Component perpendicular ($\perp$, or Y or p) **&**
- Component parallel ($\parallel$, or X, or s)

White-light source

$$s_{ray} = A_X e^{-i\phi_X}$$

$$p_{ray} = A_Y e^{-i\phi_Y}$$

→ Telescope & Instrument →

Focal plane

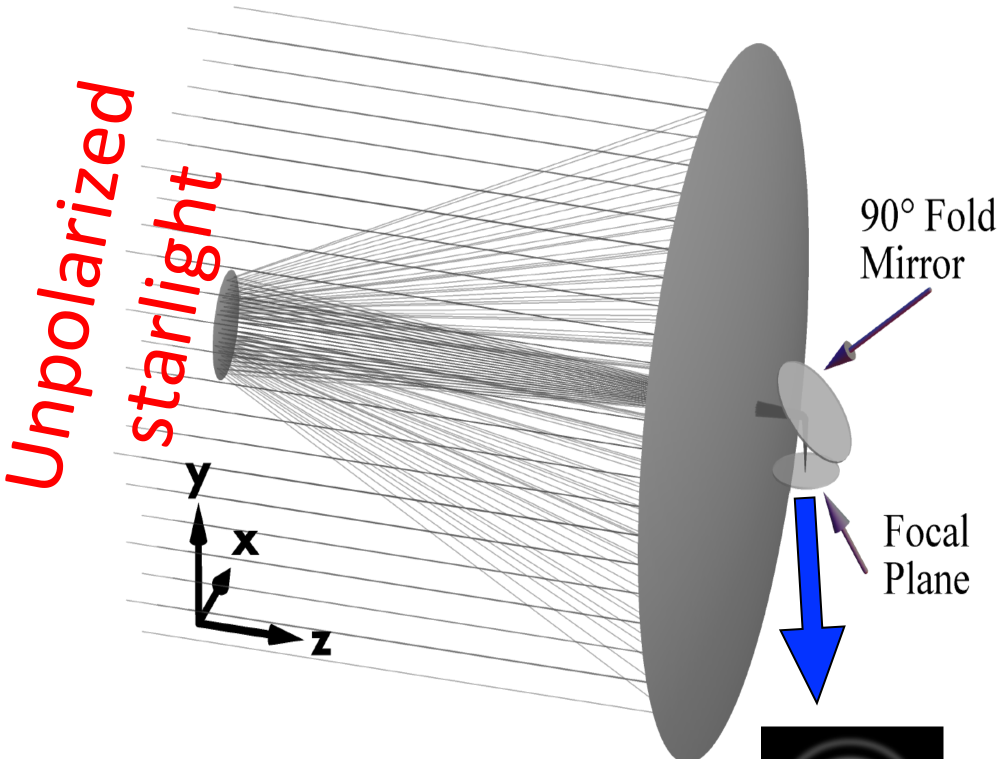
$$A_{XX} e^{-i\phi_{XX}}$$

$$A_{XY} e^{-i\phi_{XY}}$$

$$A_{YY} e^{-i\phi_{YY}}$$

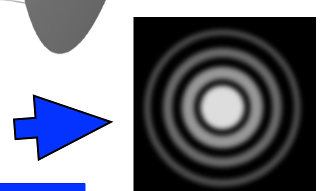
$$A_{YX} e^{-i\phi_{YX}}$$

Polarization ray trace a 3-element minimally complicated (no A/R coat, one fold) layout



2.4 meter F#=1.2
aluminum coated
mirrors & F#=8 focus

Curvatures on the primary and secondary optimized for *W(x,y)* = 0.



To design an optimum mask for exoplanets => model the focal plane electric field accurately.

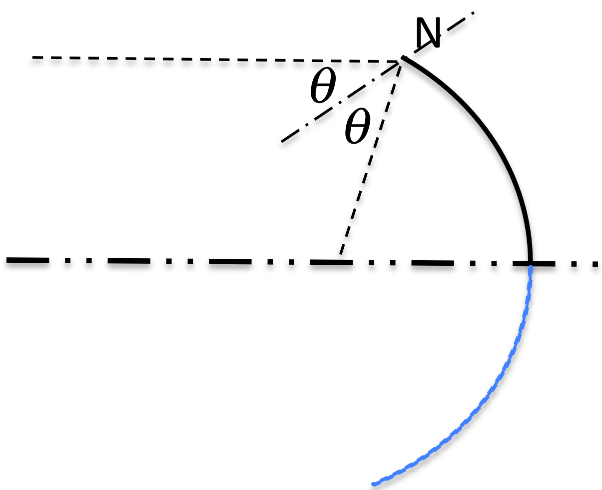
We will find that $I(x,y)$ is the sum of 4 complex PSF's

Fresnel effects

For an unobscured coronagraph the angles are steeper because the primary is off axis. The $f\#$ is calculated using the "parent" primary mirror.

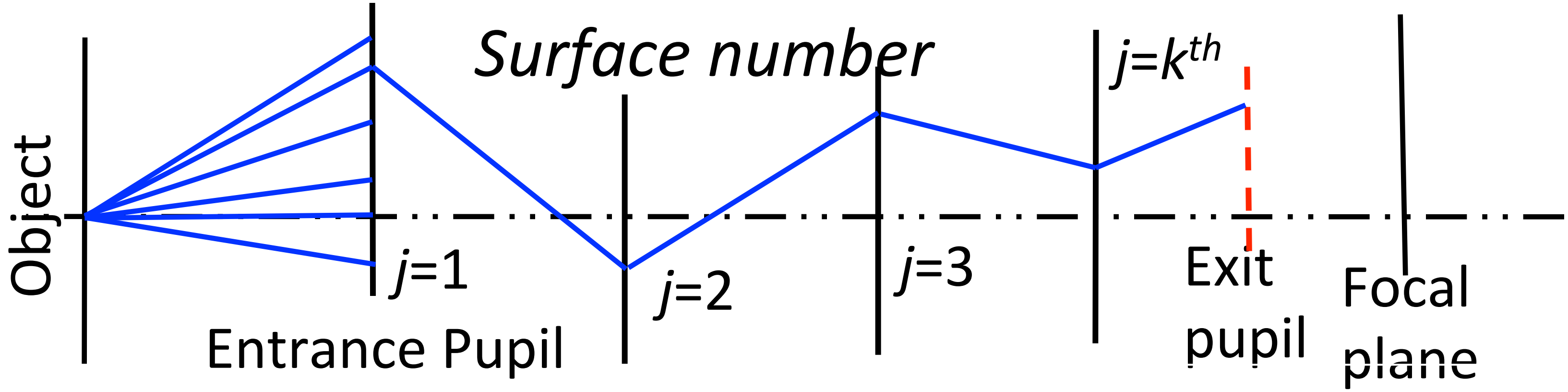
$$\alpha = \arctan\left[\frac{1}{2(f\#)}\right]$$

$$2\theta = \alpha$$



F#	Radians alpha	Degrees alpha	Degrees from normal theta
0.8	0.56	32.01	16.00
0.9	0.51	29.05	14.53
1	0.46	26.57	13.28
1.2	0.39	22.62	11.31
1.4	0.34	19.65	9.83
1.6	0.30	17.35	8.68
1.8	0.27	15.52	7.76
2	0.24	14.04	7.02
2.2	0.22	12.80	6.40
2.4	0.21	11.77	5.88

How to calculate the PSF for each polarization

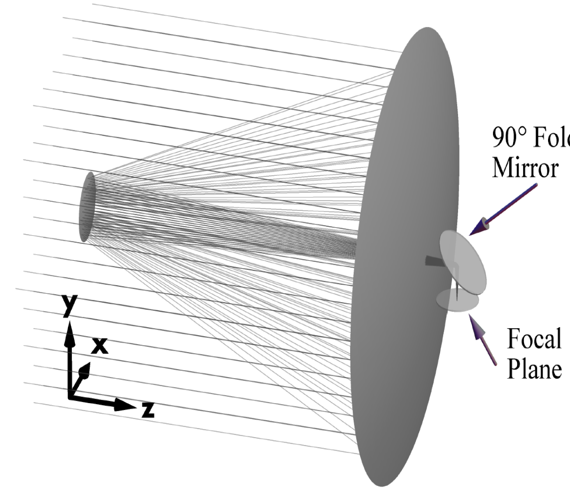


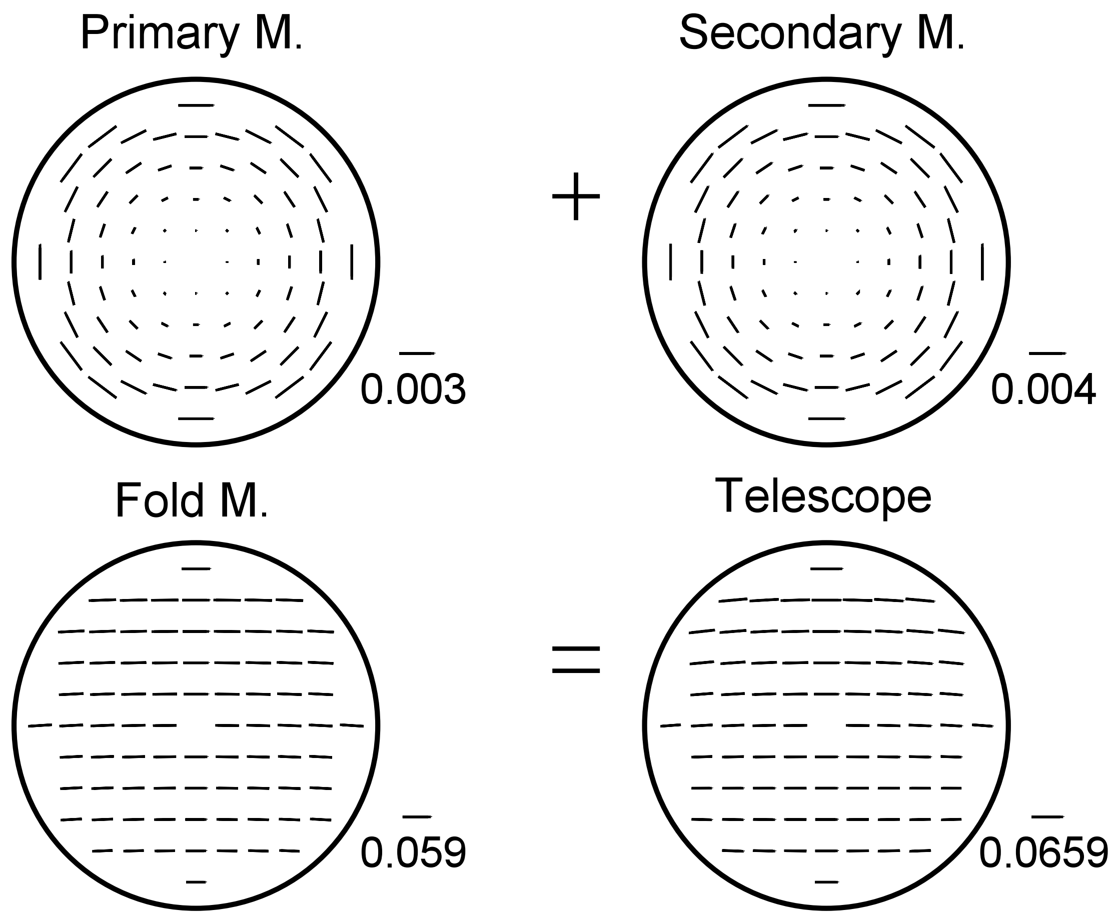
Based on the direction cosine at each surface and the physical properties of each surface $(n - ik)$ we use the Fresnel equations to calculate the **amplitude change** and the **phase change** for each ray at each surface

Compute the multiplicative amplitude and cumulative phase for both the $\|$ and the $\perp$ light for each ray traced across the entrance pupil & map these 2 complex arrays onto the exit pupil.

$$\frac{r_s(\xi,\eta)-r_p(\xi,\eta)}{r_s(\xi,\eta)+r_p(\xi,\eta)}$$

diattenuation face-on surface maps





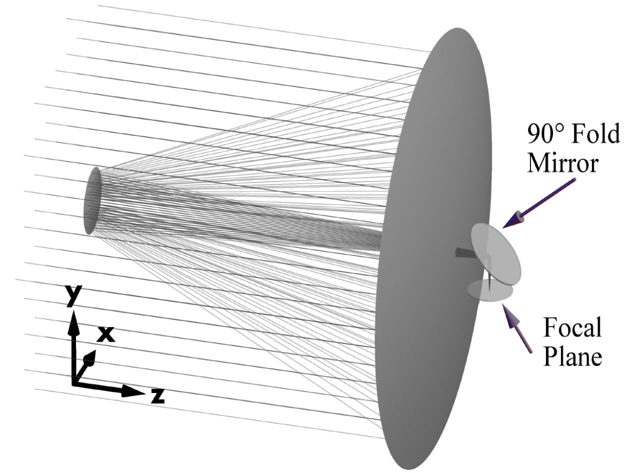
Primary is F/# =1.2 and the F/# at the focal plane is 8.

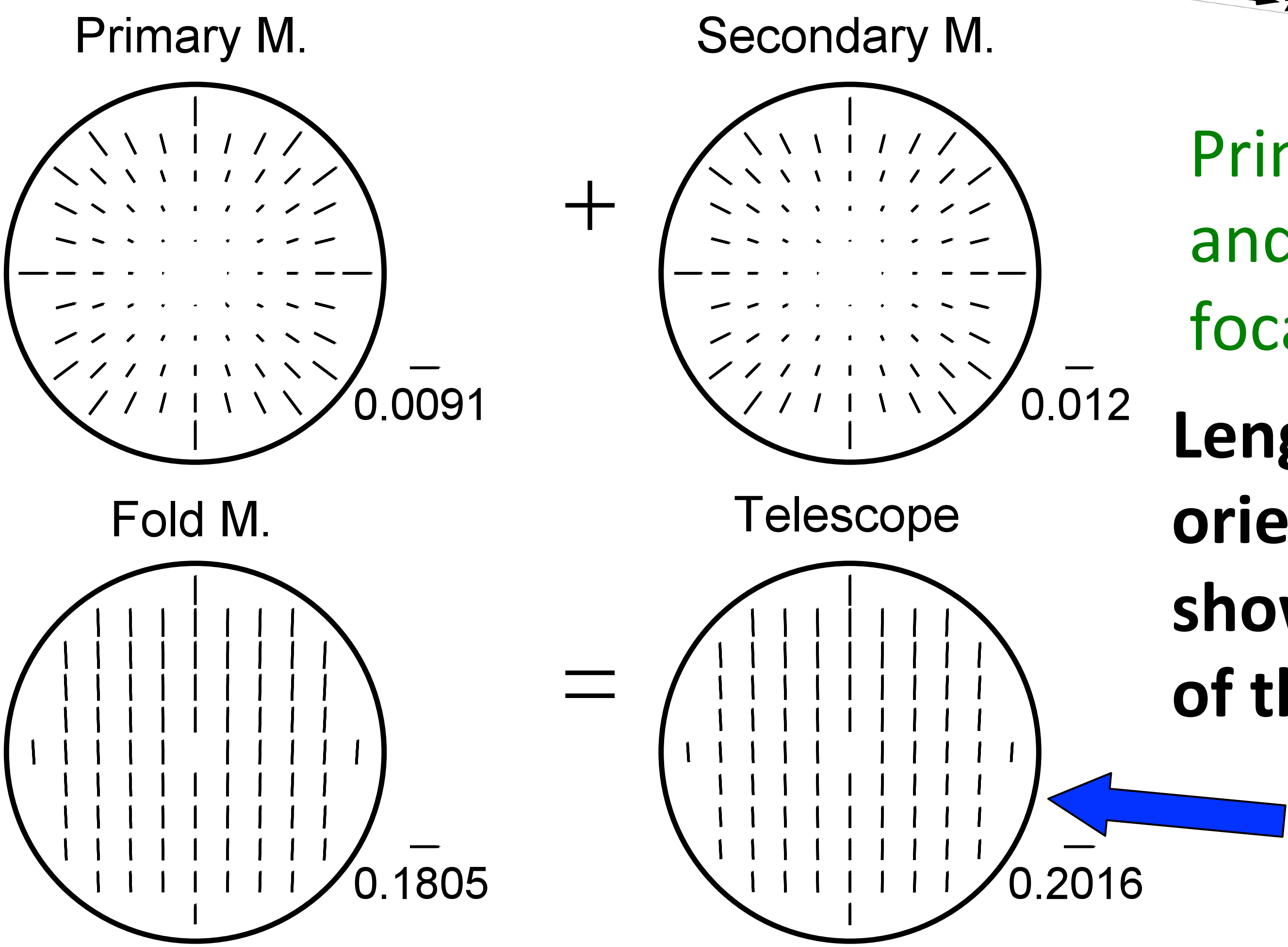
Length of the line & orientation shows the vector of the diattenuation

Exit pupil

$$\tan\left(\psi(\xi,\eta)\right)=\tan\left(\phi_S(\xi,\eta)-\phi_P(\xi,\eta)\right)$$

retardance face-on surface maps





Primary is F/# =1.2 and the F/# at the focal plane is 8.

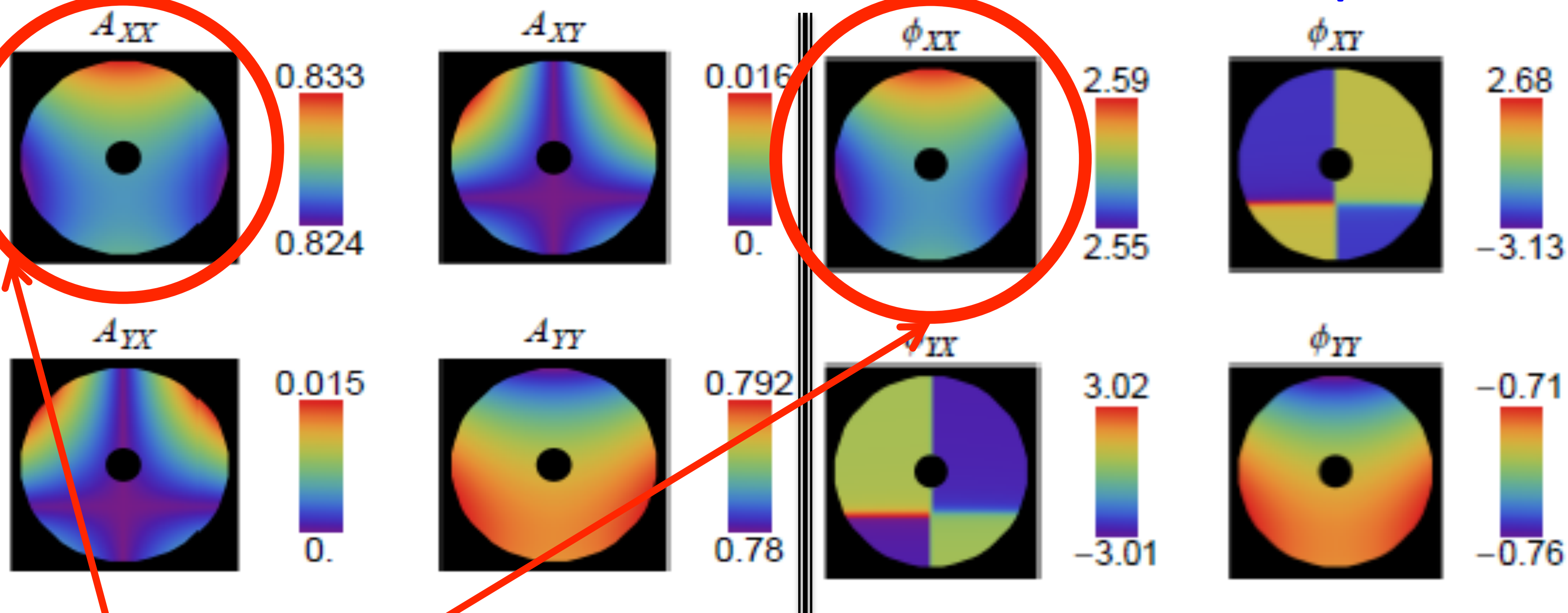
Length of the line & orientation shows the vector of the retardance

Exit pupil

Map & group the functions

Amplitude normalized

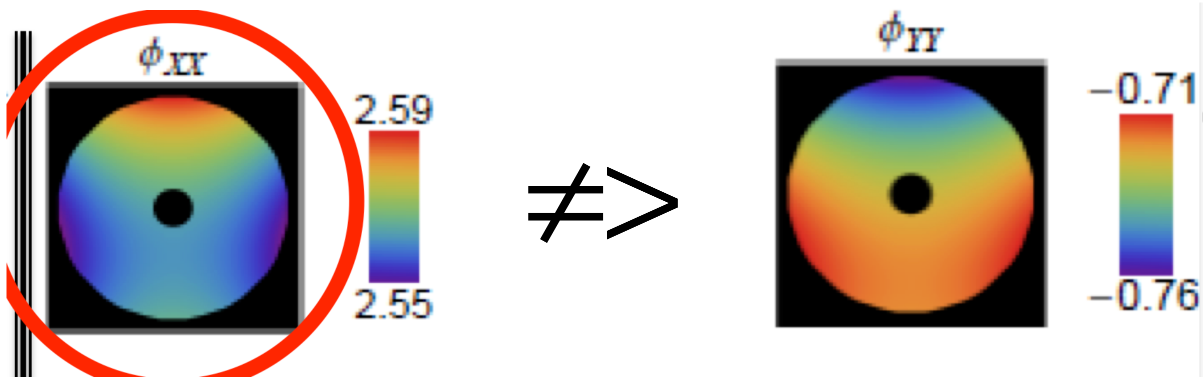
Radians of phase



$$\begin{vmatrix} A_{XX}\, e^{i\phi_{XX}} & A_{XY}\, e^{i\phi_{XY}} \\ A_{YX}\, e^{i\phi_{YX}} & A_{YY}\, e^{i\phi_{YY}} \end{vmatrix} \equiv \begin{vmatrix} J_{XX} & J_{XY} \\ J_{YX} & J_{YY} \end{vmatrix} \equiv \mathbf{J}_{ExitPupil}$$

Jones vector

Polarization dependent wedge



- The orthogonally polarized components contain different wavefront aberrations, which differ by approximately 32 milliwaves.
- A single A/O system cannot correct for both polarizations simultaneously
- Wedge between the two gives .6 milli arc seconds shear

s ray is 9% brighter than the p ray

How do we calculate the PSF?

- The **electric field** at the focal plane is given by

$$U_3(x_3, y_3) =$$

$$K\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\begin{vmatrix} J_{XX} & J_{XY} \\ J_{YX} & J_{YY} \end{vmatrix}\exp\left\{-j\frac{2\pi}{\lambda f}(x_3,\xi_2 + y_3,\eta_2)\right\}d\xi_2\, d\eta_2\right]$$

And the focal plane intensities are given by

$$I_3(x_3, y_3) = \left|U_3(x_3, y_3)\right|^2 =$$

$$\left|\mathcal{F}(J_{XX})\right|^2 + \left|\mathcal{F}(J_{YY})\right|^2 + \left|\mathcal{F}(J_{YX})\right|^2 + \left|\mathcal{F}(J_{XY})\right|^2$$

The telescope PSF is the linear (uncorrelated) superposition of these 4 PSF’s

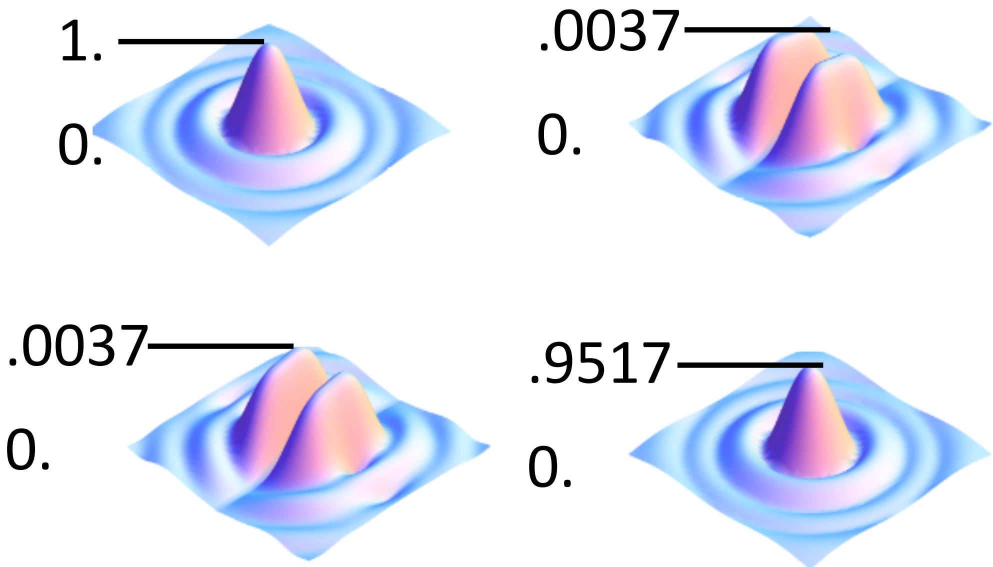
Propagate the exit pupil field to map the 4 independent co-propagating PSF's to the image plane

Amplitude Response Matrix

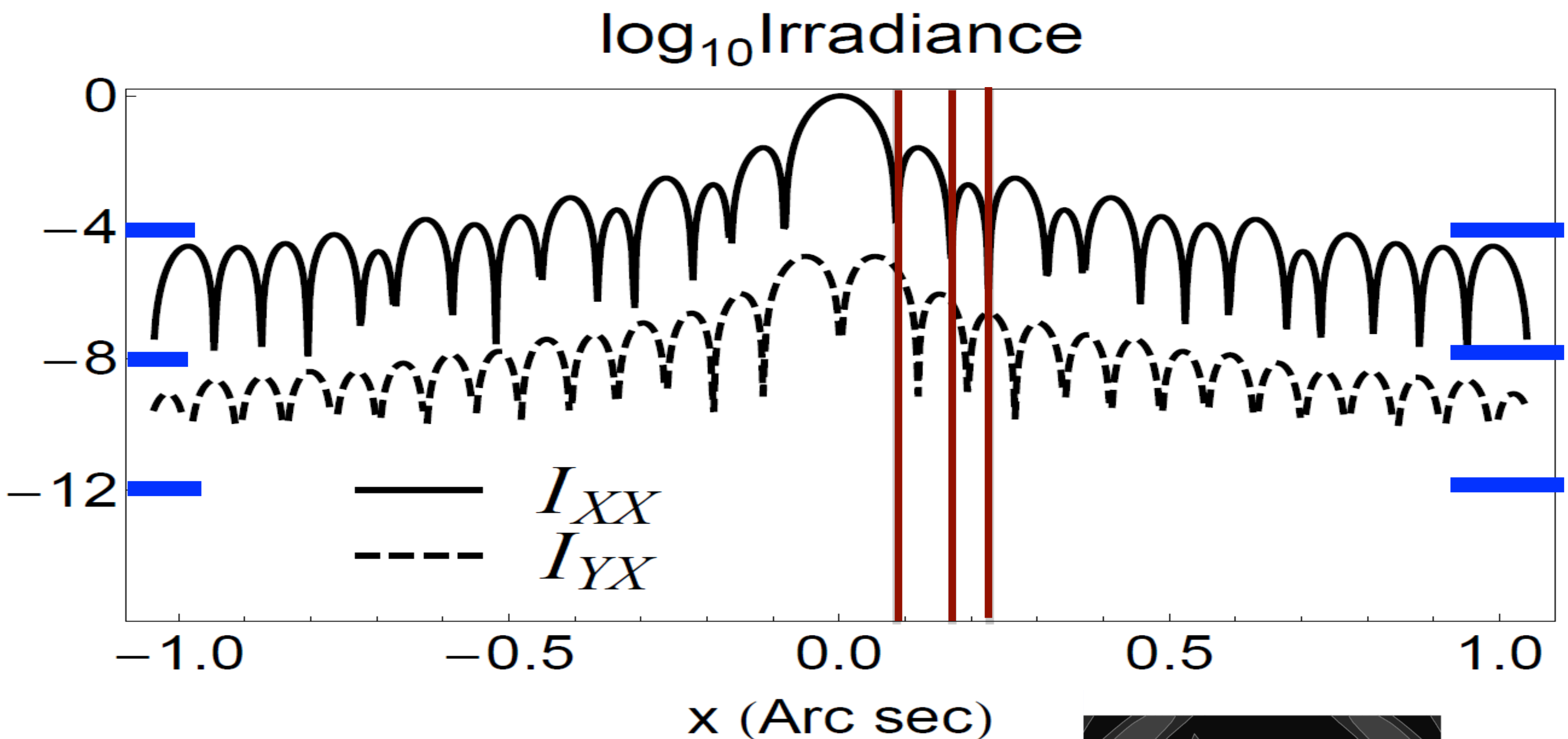
$$= \mathbf{ARM} = \begin{vmatrix} \Im\left[J_{XX}(x,y)\right] & \Im\left[J_{XY}(x,y)\right] \\ \Im\left[J_{YX}(x,y)\right] & \Im\left[J_{YY}(x,y)\right] \end{vmatrix}$$

What does the focal plane look like?

3.7E^{-03} in amplitude is 1.4E^{-05} in intensity



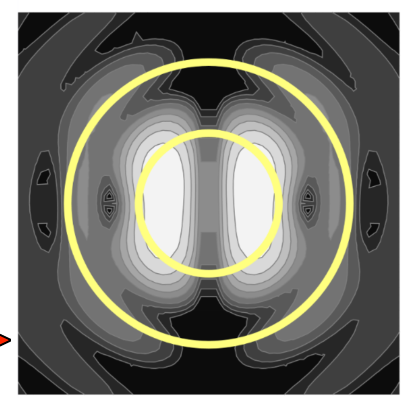
Polarization PSF (I_{XX}) & the "ghost" PSF (I_{YX}) for the 2.4 meter telescope - note the **"zeros" do not line up**



I_{XX} I_{XY}

I_{YX} I_{YY}

Face-on ghost PSF

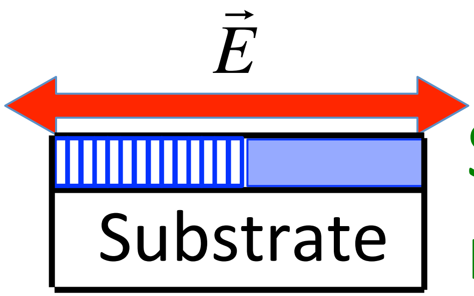


Polarization reflectivity anisotropy => changes polarization across wavefront surface

Flavio Horowitz, 1983 & Smith/Purcell 1953

- Anisotropy is produced by the coating processes used for large telescope mirrors

$\vec{E}$ incident sees a different conductivity in the substrate depending on whether the wave is reflecting from an amporphous or the columnar structure



Side view of reflecting film

Left columnar (crystal)
Right amorphous micro-structure

$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Summary for this telescope

- 32 milli-waves difference in the wavefront aberrations (tilt, coma, astigmatism, spherical, etc.) between $\|$ and $\perp$
- Shift between the PSF's for *X* and *Y* is 0.625 masec
- *X* and *Y* show a 9% difference in intensity reflectance

Summary for this telescope (cont'd)

- Light coupled from one polarization forms a separate faint and much larger PSF not superposed on J_{XX} and the J_{YY}
- => complex field may spill over the edges of a mask that is designed assuming scalar diffraction.
  - Radius of 90% encircled energy:

$$r_{XX} = r_{YY} = 0.15 \text{ arcsec and}$$

$$r_{XY} = r_{YX} = 0.36 \text{ arcsec}$$

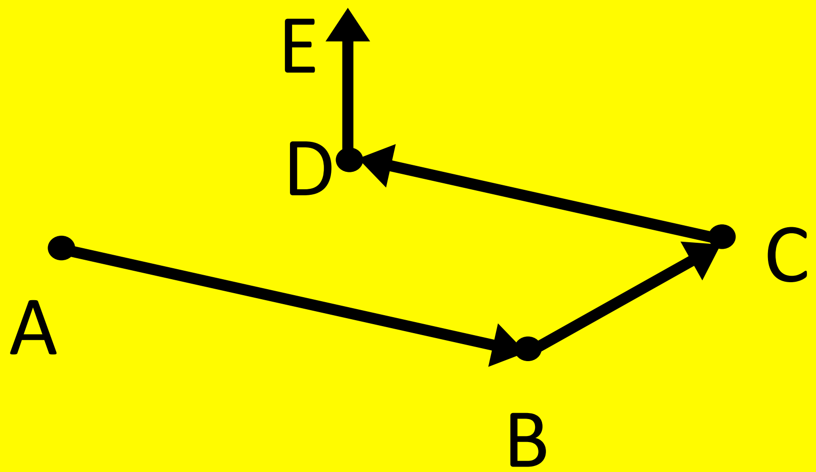
Summary for this telescope

- **Intensity of the ghost PSF increases as the square of the number of reflections {Breckinridge et. al 2015}**
- Unpolarized sources exit partially polarized into an instrument.
- The telescope coatings cause polarization variations throughout the PSF, particularly into the diffraction rings to complicate polarization measurements of exoplanets and debris rings in coronagraphs.

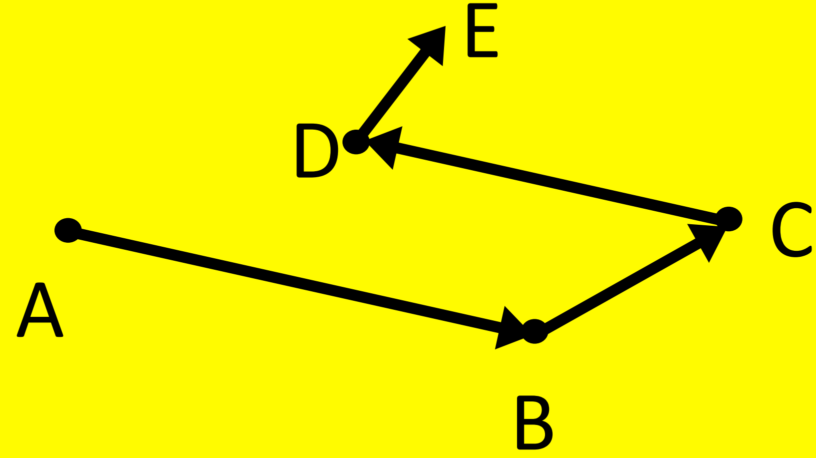
Why current test beds see very high contrast?

- Ground test beds have few to no fold mirrors & high F#'s, unlike many space and ground-based telescope systems
- Our work was at 800nm. N+iK depends on wavelength
- Test beds function at narrow bandwidths
- Industry uses proprietary dielectric overcoated metal mirrors – we used no overcoat

Polarization cross talk is increased by a change of the Eigenstate of the of the propagating wavefront; tilted mirrors



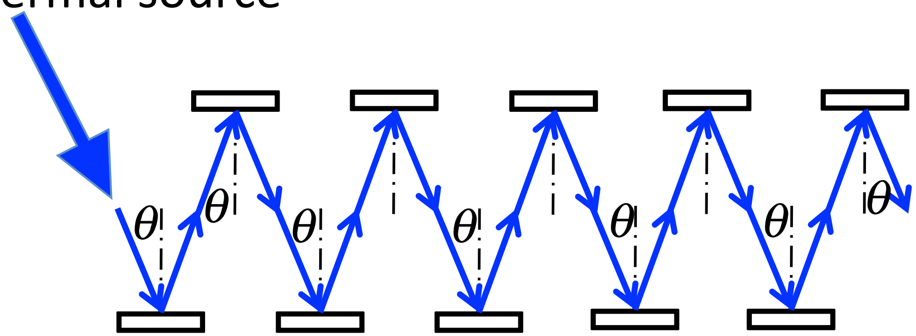
All angles 90-degrees => then the Eigenstate of the final wavefront are mixed $\perp$ and $\|$



Mirror D now sends beam into a compound angle and the cross product terms increase

Case one (1)

White-light, thermal source



$A(\lambda)_{s,\,out}\exp(i\phi(\lambda)_{s,\,out})$ and
$A(\lambda)_{p,\,out}\exp(i\phi(\lambda)_{p,\,out})$

$A(\lambda)_{incident}\exp\left[i\phi(\lambda)_{Incident}\right]$

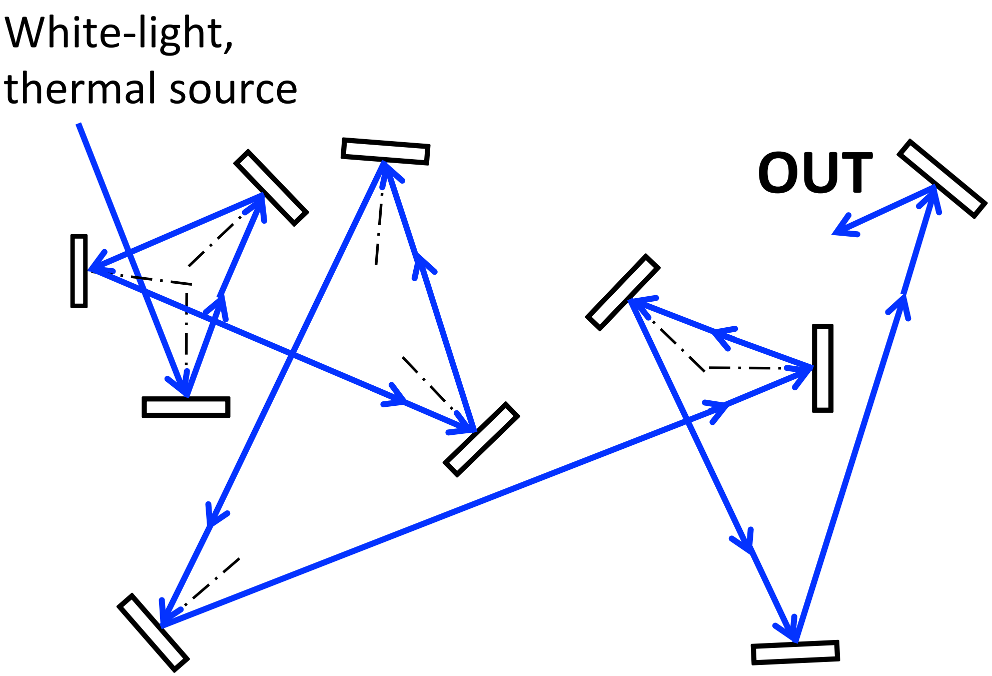
For $\theta = 15$ degrees; 10 reflections as shown; $600 \le \lambda \le 720$;
$A(\lambda)_{incident} = 1.00$; $\phi(\lambda)_{Incident} = 0.0$
please calculate
$A(\lambda)_{s,\,out}$ & $\phi(\lambda)_{s,\,out}$ and
$A(\lambda)_{p,\,out}$ & $\phi(\lambda)_{p,\,out}$

LRLRLRLR

Assume all co-planar
and bare aluminum

Case (2)

Riding along on the Pointing vector,
The reflected beam always exits the
surface to the observers left



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Assume all co-planar
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Several ways to mitigate these effects One is to build a phase plate

$$\mathbf{J}_{\mathrm{T+Cgph}} = \begin{pmatrix} J_{XX} & J_{YX} \\ J_{XY} & J_{YY} \end{pmatrix} \equiv \begin{pmatrix} A_{XX}\, e^{i\phi_{XX}} & A_{YX}\, e^{i\phi_{YX}} \\ A_{XY}\, e^{i\phi_{XY}} & A_{YY}\, e^{i\phi_{YY}} \end{pmatrix}$$

To minimize the polarization effects, we need to develop a corrective optical element whose Jones pupil, $\mathbf{J}_{\mathrm{corrector}}$ has the property:

$$\mathbf{J}_{\mathrm{System}} = \left(\mathbf{J}_{\mathrm{T+Cgph}}\right) \cdot \left(\mathbf{J}_{\mathrm{Corrector}}\right) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mitigation: spatially variable retarder plate

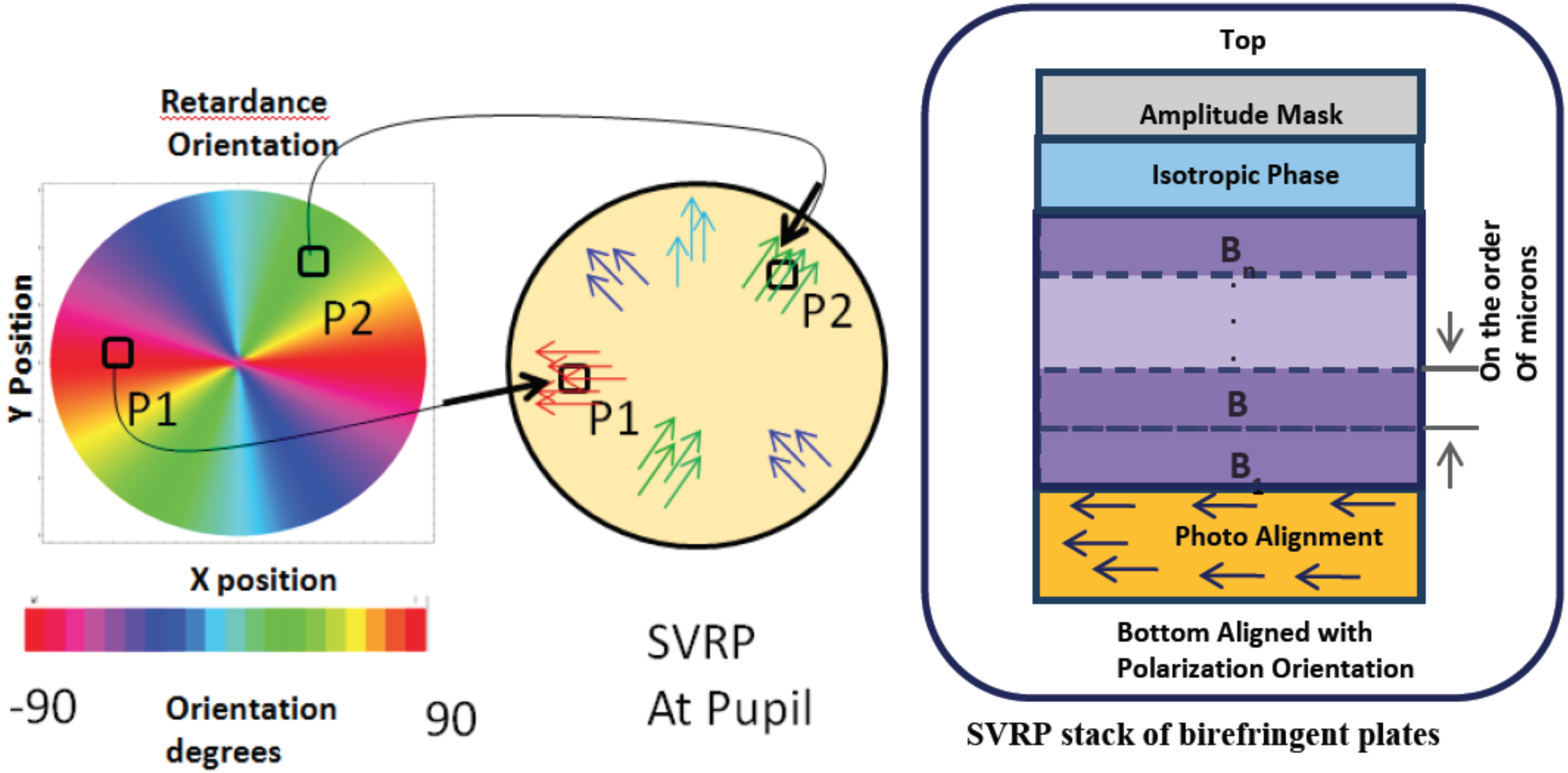

Figure 4. Photo alignment layer of a SVRP plate. (a) shows a spatially variable retarder plate (SVRP) face on (x,y) with retardance direction indicated by the colors shown in the stripe below which maps color into orientation in degrees as shown. (b) shows two particular regions, P1 and P2 which have two different polarization states and orientation are shown. (c) shows a diagram of the typical stack or sandwich. The bottom layer is a layer of homogeneous dielectric oriented to the polarization direction. Birefringent layers of B_1, B_2...B_n will be deposited with thickness layers and specific process recipe calculated and optimized to compensate for the Fresnel polarization of light reflected form the telescope.

Thank you

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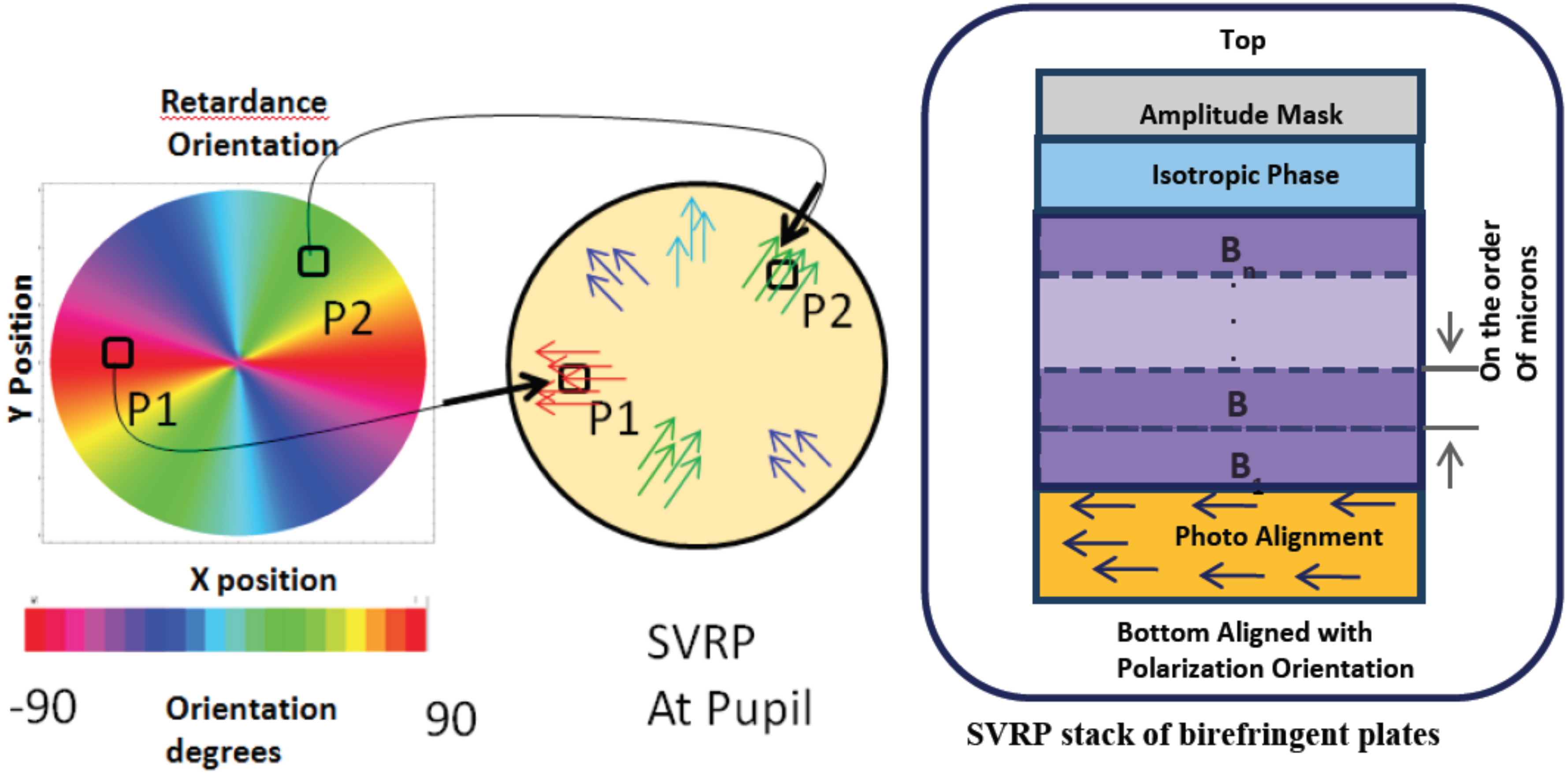

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Linear polarizers do not mitigate these effects!

- A Wollaston beam splitter (WBS) prism placed over the focal plane does not unscramble the co-propagating mixed polarized signals.
  - They were mixed up-stream in the optical path
- Since the beams are deviated in a Wollaston, the Eigenstates are projected onto a rotated coordinate system & the power in the off-diagonal elements is increased.

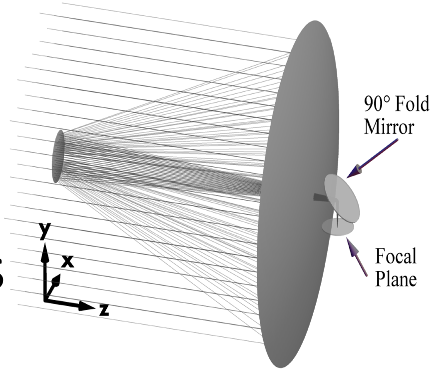
System polarization operator

- The 4x4 Mueller matrix is often used as the linear operator to describe how an optical system operates on an incoming beam of light

$$\begin{vmatrix} I \\ Q \\ U \\ V \end{vmatrix}_{IMAGE} = \begin{vmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{vmatrix}_{SYSTEM} \times \begin{vmatrix} I \\ Q \\ U \\ V \end{vmatrix}_{OBJECT}$$

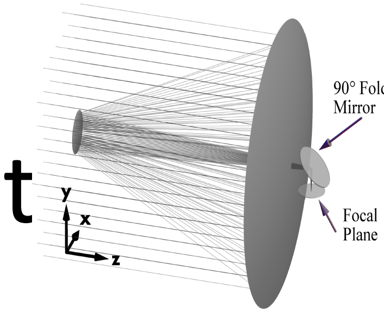
- The next charts show the 4x4 Mueller matrix for the Cassegrain telescope with fold mirror shown on page 24.

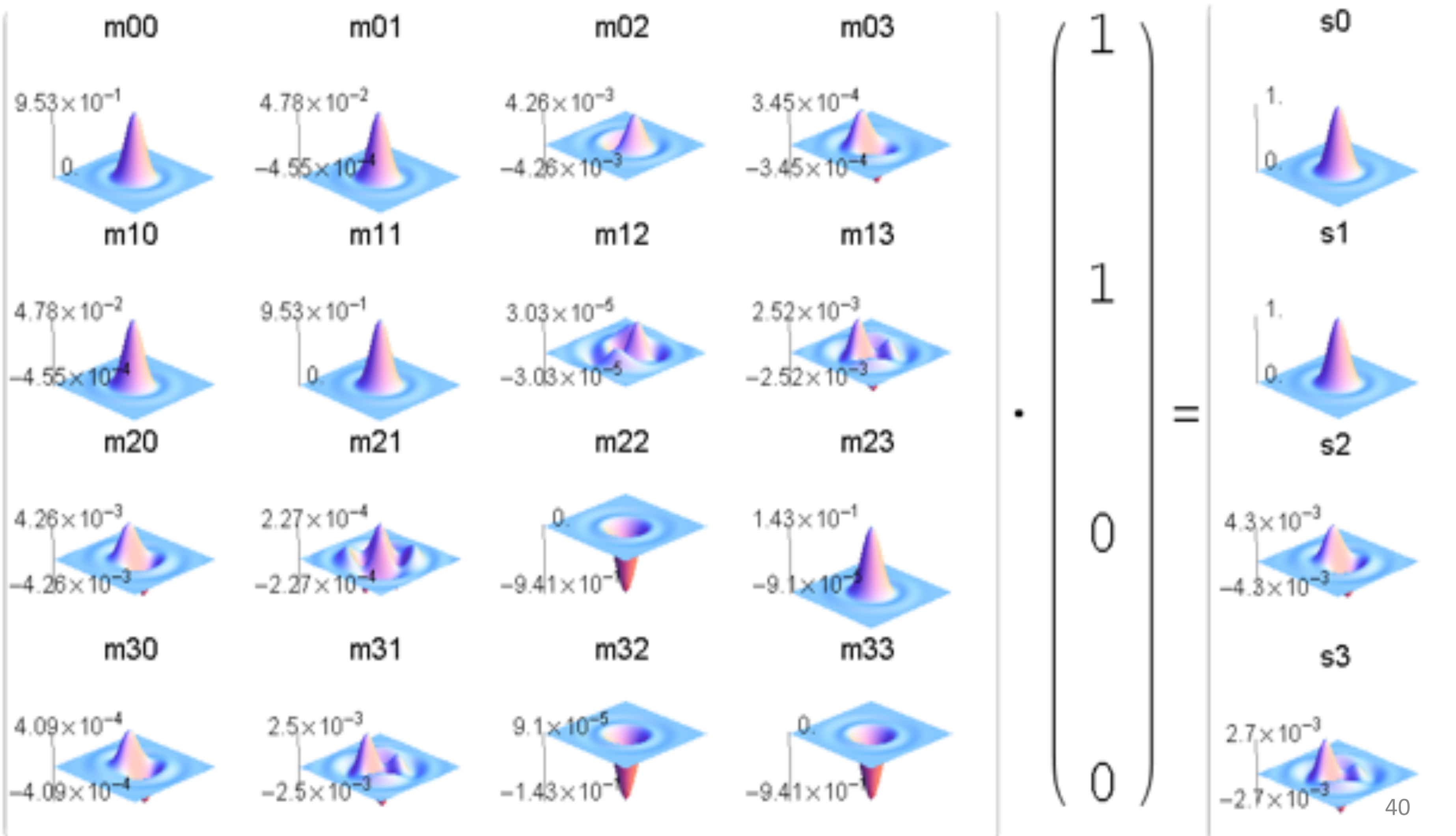
Comment



- The Mueller matrix is a 4x4 matrix of real numbers represent the properties of a telescope, device or instrument.
- A 1x4 column matrix represents the Stokes vector which describes the polarization content of a beam of light.
- When this beam of light strikes a surface or passes through a polarization filter the polarization state of the beam is changed.
- On the page after next we see the 4 x 4 Mueller matrix operator that represents the Cassegrain & fold mirror telescope we see in Chart 24.
- Each of the elements in the 4 x 4 matrix makes some contribution to the field. Some are insignificant.
- These contributions are plotted within each element to provide the reader with an intuitive understanding of how the aberrations evolve.

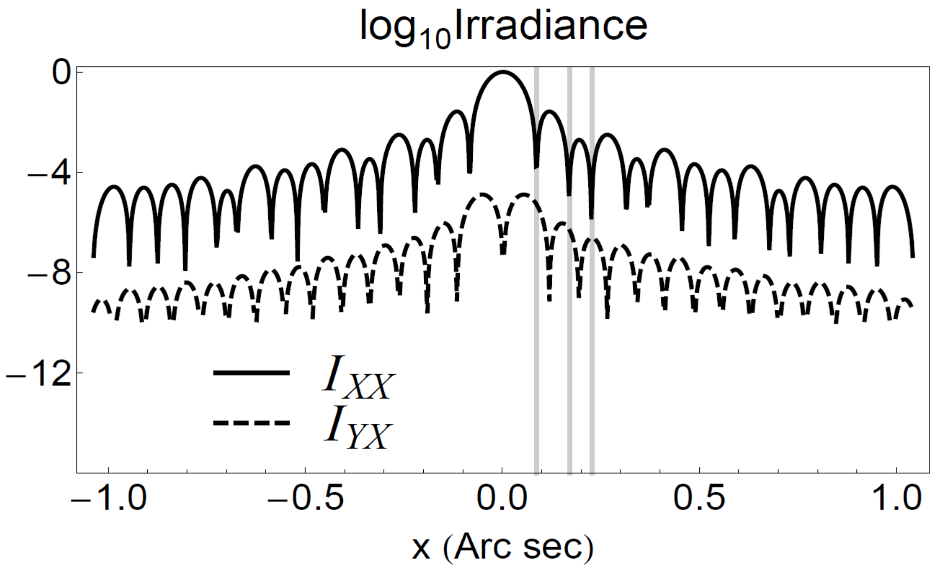
Convert the ARM to a Mueller Matrix and look at x-polarized light



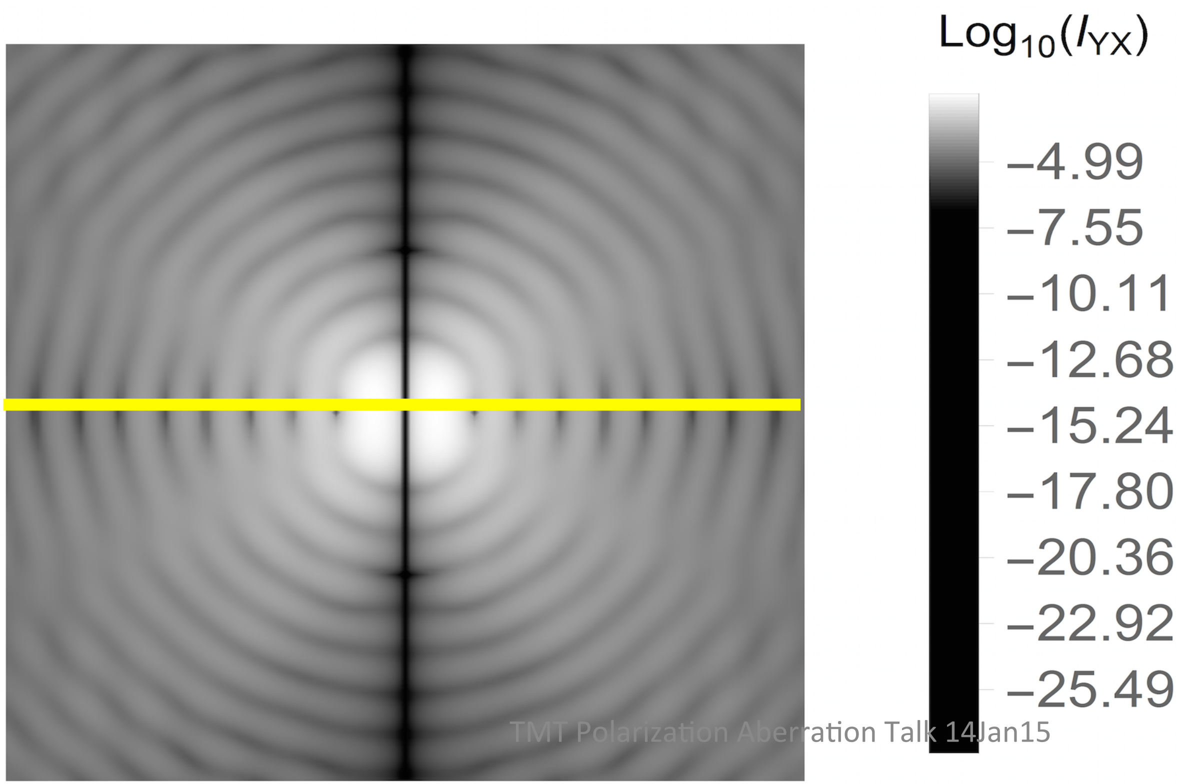


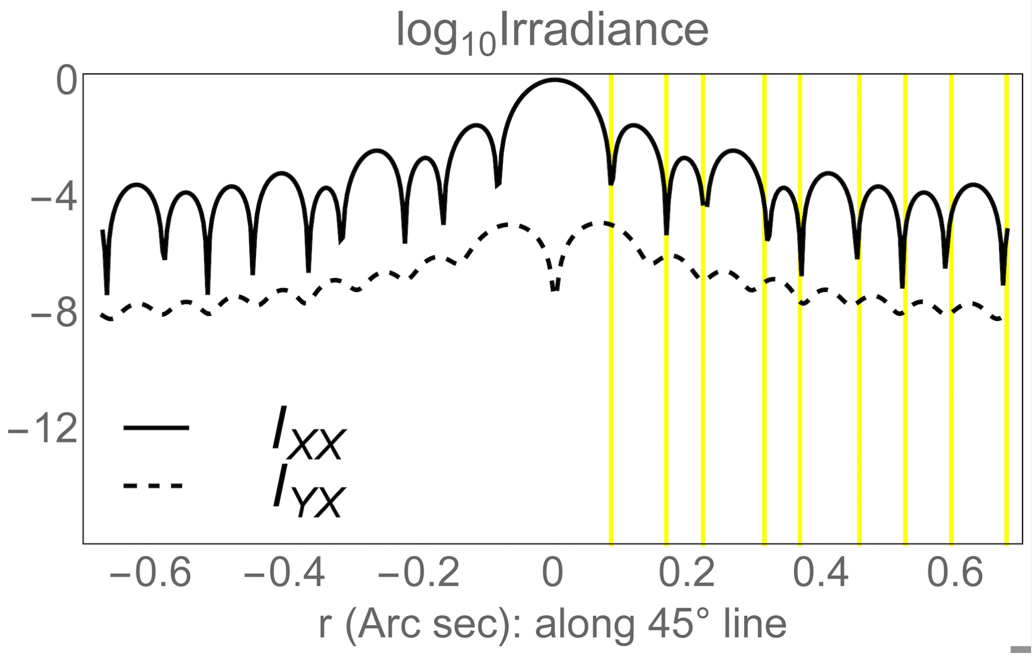
Linear polarizers do not mitigate these effects!

- A Wollaston beam splitter (WBS) prism placed over the focal plane does not unscramble the co-propagating mixed polarized signals.
  - They were mixed up stream in the optical path
- Since the beams are deviated in a Wollaston, the Eigenstates are projected onto a rotated coordinate system & the power in the off-diagonal elements is increased.

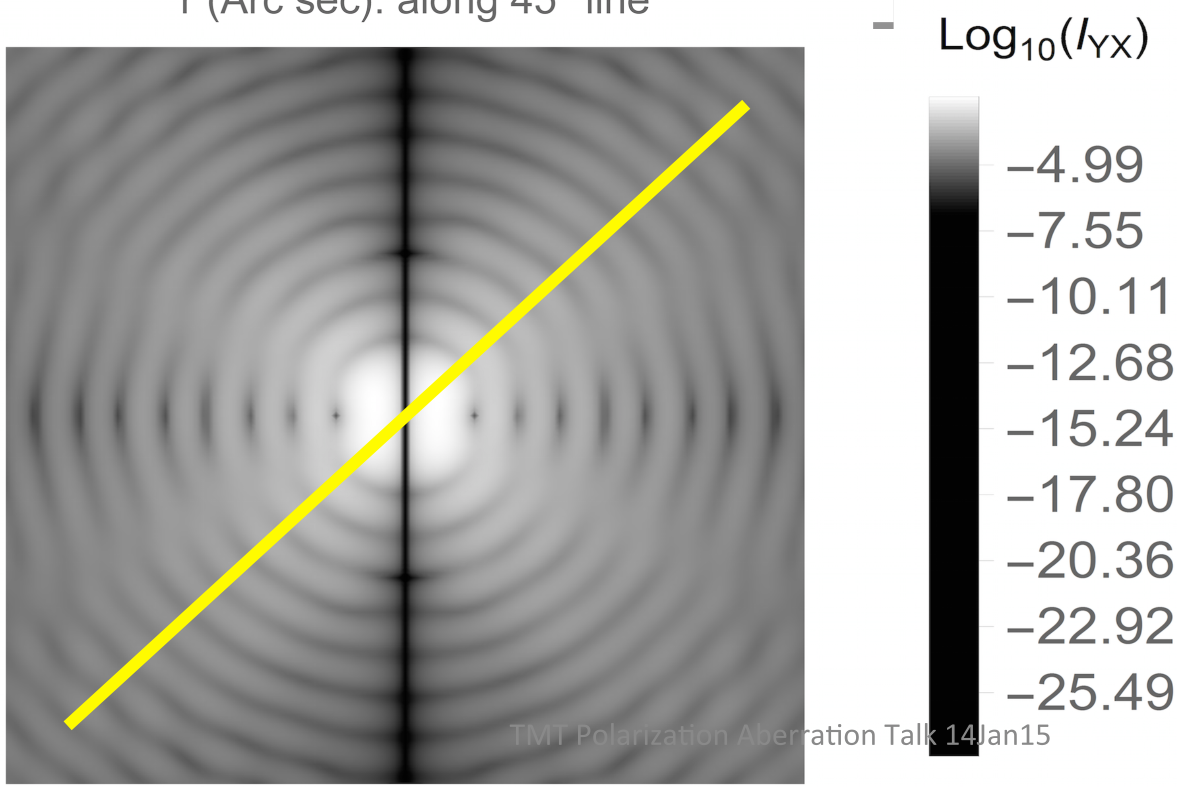


$\text{Log}_{10}\ I_{YX}$ at a 0 degree slice

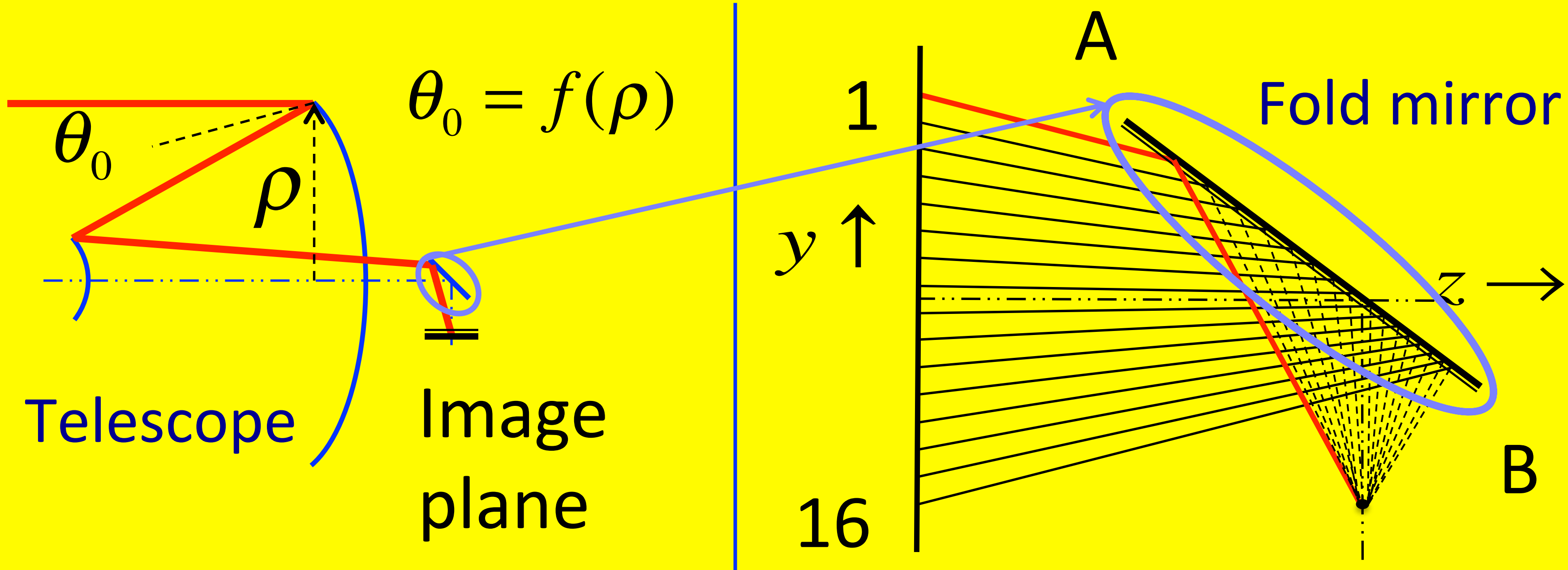




Log_{10} I_{YX} at a 45 degree slice



Polarization depends on incidence angle



The incident rays march across the pupil strike the mirror at different angles, depending on radius

Incident rays march across the fold mirror striking at decreasing angles from the top down

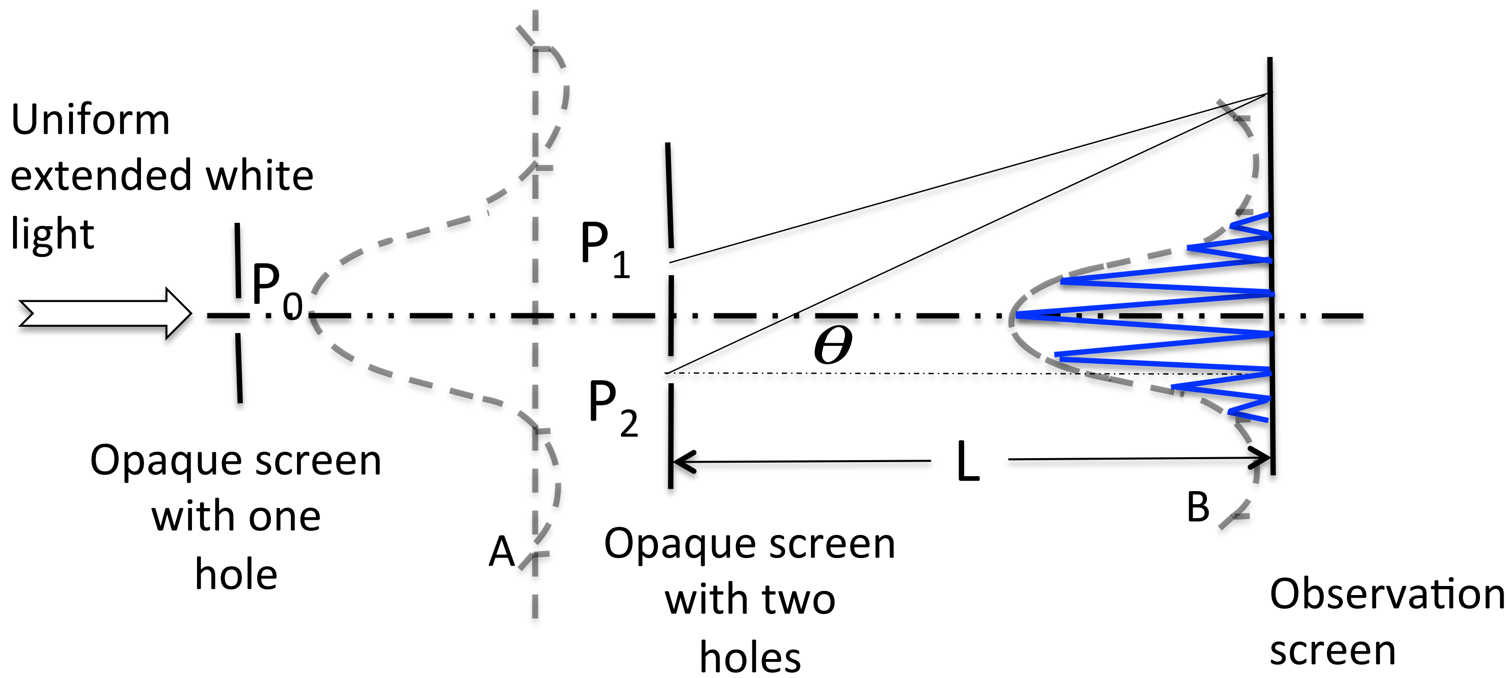
New work

- Measure the polarization reflectivity anisotropy and its spatial scale on a large astronomical telescope mirror
- Select a practical coronagraph design and calculate contrast using vector wavefronts
- Refine models to calculate vector diffraction around masks and stops
- Once we have contrast = *f*(polarization), then search for practical mitigation approaches

New work

- Develop a coronagraph test bed that emulates a practical system, measure the polarization aberrations and validate the models
- Explore a spatially variable wave plate which will correct "as-built" telescope systems.
- How much internal polarization can we have and still achieve the 10^{-11} extinction needed for terrestrial exoplanets?
- Determine the requirements on the physical properties of the surfaces, # of mirrors, angles, masks, transmittance, etc.
- Design and develop masks and stops to optimize terrestrial exoplanet characterization in the presence of polarization aberrations

Double slit experiment



- Curve B is the diffraction pattern from holes P1 and P2
  - Spacing of the fringes underneath curve B is related to the separation of the holes P_1 and P_2
  - Visibility (contrast) of these fringes underneath curve B is given by the degree of correlation (coherence) of the fluctuating electromagnetic fields between P_1 and P_2.