"New Trends in Astrodynamics and Applications - V"

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The Sun as a Gravitational Lens : A Target for Space Missions Reaching 550 AU to 1000 AU

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Gravitational Lens of the Sun

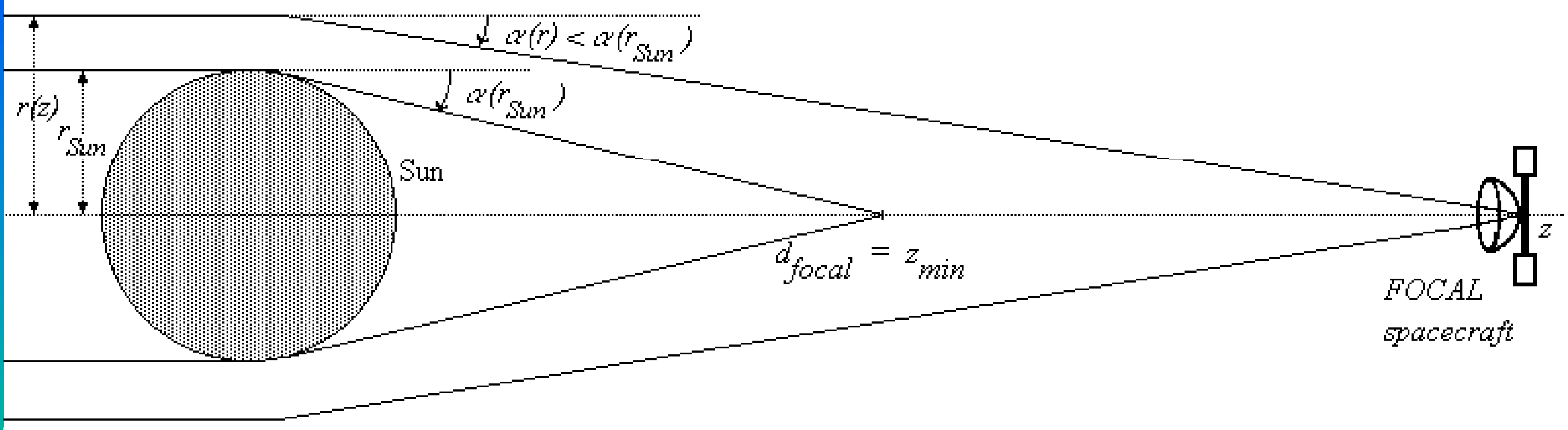


Figure 1:

Basic geometry of the gravitational lens of the Sun:

the minimal focal length at 550 AU

and the FOCAL spacecraft position.

Gravitational Lens of the Sun

- The geometry of the Sun gravitational lens is easily described: incoming electromagnetic waves (arriving, for instance, from the center of the Galaxy) pass *outside* the Sun and pass within a certain distance *r* of its center.
- Then a basic result following from General Relativity shows that the corresponding *deflection angle (r)* at the distance *r* from the Sun center is given by (Einstein, 1907):

$$\alpha(r) = \frac{4GM_{Sun}}{c^2 r}.$$

Gravitational Lens of the Sun

- Let's set the following parameters for the Sun:
  1. Assumed Mass of the Sun: $1.9889164628 \cdot 10^{30}$ kg, that is μSun = 132712439900 kg^3s^{-2}
  2. Assumed Radius of the Sun: 696000 km
  3. Sun Mean Density: 1408.316 kgm^{-3}
  4. Schwarzschild radius of the Sun: 2.953 km

  One then finds the BASIC RESULT:

MINIMAL FOCAL DISTANCE OF THE SUN:

548.230 AU ~ 3.17 light days ~

13.86 times the Sun-to-Pluto distance.

Larger and larger Telescopes...

- Wherever in space there are intelligent creatures like us, they will be driven to explore and understand our universe, just as we do. We and they wish to see to the farthest depths of space with the greatest clarity allowed by the laws of nature. To this end, we build, at great expense, ever more powerful telescopes of all kinds on Earth, and now in space.
- As each civilization becomes more knowledgeable, they will recognize, as we now have recognized, that each civilization has been given a single great gift: a lens of such power that no reasonable technology could ever duplicate or surpass its power. This lens is the civilization's star. In our case, our Sun.
- The gravity of each such star acts to bend space, and thus the paths of any wave or particle, in the end creating an image just as familiar lenses do.

Every Star IS a Telescope !!!

- This lens can produce images which would take perhaps thousands of conventional telescopes to produce. It can produce images of the finest detail of distant stars and galaxies.
- Every civilization will discover this eventually, and surely will make the exploitation of such a lens a very high priority enterprise.
- One wonders how many such lenses are being used at this moment in time to scan the universe, capturing a flood of information about both the physical and biological realities of our time.
- ***Frank Drake, 1999, from his Foreward to Claudio's book.***

Infinity of Focuses > 550 AU

- There is an infinity of focuses from 550 AU outward in any direction. Thus, 550 AU is actually the minimal focal sphere.
- In the practice, we won't have to stop a spacecraft just at 550 AU, but we just let it go.
- The further the spacecraft goes beyond 550 AU, the better it is. In fact, radio waves impinging on the spacecraft at distances higher than 550 AU will have to cross less and less dense layers of the Solar Corona.
- The Solar Corona is difficult to model. Essentially, because of the radially decreasing electron density, the Solar Corona acts as a divergent lens opposing the convergent lens of gravity.
- We shall study the Corona later. Just the Naked Sun for now.

Gain of Any Star as a Lens

- The "gravity lens" concept means that the Sun (and any other massive celestial body) is an antenna since it can increase the intensity of the signal, by virtue of its deflection.
- We define the Gain associated to any star, G_{star}, as the ratio between the intensity of the signal in presence of the star compared to the intensity of the signal without the star. It can be proven that, along the focal axis, one has

$$G_{star}(\lambda) = 4\pi^2 \frac{r_g}{\lambda} = \frac{8\pi^2 G M_{star}}{c^2} \cdot \frac{1}{\lambda}$$

The gain is constant along the focal axis but is wavelength-dependent. There, r_g is the Schwarzschild radius of the star.

Sun Gain at Some Frequencies

- In the following table we remind on-axis GAIN of the gravitational lens of the naked Sun lens for seven important frequencies:

Line	Neutral Hydrogen	OH maser	Water maser	Ka Band	CMB peak	**Visible red**	**Visible Violet**
Frequency **ν (GHz)**	1.420	1.6	22	32	160	**4.3 10^5**	**5.5 10^5**
Wavelength **λ (cm)**	21	18	1.35	0.937	0.106	**700 nano m**	**400 nano m**
Naked Sun Gain (dB)	**57.4**	**57.9**	**69.3**	**71.46**	**80.40**	**112.22**	**114.65**

Off Axis Gain for any Star

- Off axis, when the spacecraft is at a distance ρ from the focal axis, and when it is at a distance z from the star, *the gain* can be proved to be:

$$G_{star}(\lambda, \rho, z) = 4\,\pi^2\,\frac{r_g}{\lambda} \cdot J_0^2\left(\frac{2\,\pi\,\rho}{\lambda}\sqrt{\frac{2\,r_g}{z}}\right)$$

where $J_0(x)$ is the Bessel function of order zero and argument x

- The *total gain* for the combined (Star + receiving antenna) system is:

$$G_{Total}(\lambda) = G_{star}(\lambda) \cdot G_{antenna}(\lambda)$$

and so it increases with the CUBE of the frequency (next slide).

Total Gain of Star+Spacecraft

- If a spacecraft (S/C) has an antenna with radius $r_{antenna}$ and efficiency $k_{antenna}$, the spacecraft antenna gain is given by

$$G_{antenna}(\lambda) = 4\pi \frac{A_{physical} \cdot k_{antenna}}{\lambda^2} = \frac{4\pi^2 r_{antenna}^2 \cdot k_{antenna}}{\lambda^2}$$

and is proportional to the inverse of λ^2.

- The *total gain* for the combined (Star+S/C receiving antenna) system is proportional to the inverse of λ^3 and is:

$$G_{Total}(\lambda) = G_{star}(\lambda) \cdot G_{antenna}(\lambda) = \frac{32\pi^4 G M_{star} r_{antenna}^2 \cdot k_{antenna}}{\lambda^3}$$

Sun comes BEFORE interstellar!

- ***Sun's Focus Comes FIRST.***
- ***Interstellar Target Comes SECOND.***

1) The Sun's gravity focus is ***MUCH CLOSER*** than the target star, actually hundreds or thousand of times closer according to the target star (for α Cen it is 253 times closer than 1000 AU, where the "true" focus is found by taking the CORONA into account.
2) ***BEFORE*** any interstellar probe is launched towards a nearby star, we need a highly magnified radio-map of whatever lies around that star. And this can be achieved only by sending a probe to the opposite direction to let the Sun magnify!
3) It is much ***CHEAPER*** to reach 550 AU or 1000 AU than hundreds of AU, and it takes so much time less!

The Solar Corona

- So far, we have been concerned with the Naked Star Gravitational Lens, due to a spherical distribution of Mass.

- However, above the surface of the Sun, the Corona extends into space across distances that are comparable to the Sun radius, and the coronal effects may only complicate the physical picture of the Sun as a gravitational lens.

- How does the Sun Corona affect the electromagnetic waves convergence? How does the Sun Corona affect the electromagnetic waves gain?

The Baumbach-Allen Model

- The story of the Baumbach-Allen model of the Corona began in the 1930's, when the German astronomer S. Baumbach worked out and published a formula yielding the density of electrons in the Coronal Plasma. Baumach's formula reads:

$$N_{electrons}(b) = const \cdot \left[2.952 \cdot 10^3 \left(\frac{r_{Sun}}{b} \right)^{16} + 2.28 \cdot 10^2 \left(\frac{r_{Sun}}{b} \right)^{6} + 1.1 \cdot \left(\frac{r_{Sun}}{b} \right)^{2} \right]$$

- *b* is of course the "impact parameter" (the radial distance from the Sun's center) and is measured in units of the Sun radius.
- In 1947, the Australian astronomer C. W. Allen confirmed the validity of the above equation but re-interpreted one of the terms in Baumbach's formula proving what is now called the Baumbach-Allen model of the Corona, where the plasma density around the Sun varies as a combination of and power laws, from the photosphere to about 10 solar radii.

Corona Deflection Angle

- In 1998, starting from the Baumbach-Allen model, Andersson and Turyshev of JPL gave in the following *empirical* formula for the *deflection angle* $\theta_{plasma}(b,\nu)$ caused by the coronal plasma effects and *opposed to the action of gravity:*

$$\vartheta_{plasma}(b,\nu) = \left(\frac{\nu_0}{\nu}\right)^2 \left[2.952 \cdot 10^3 \left(\frac{r_s}{b}\right)^{16} + 2.28 \cdot 10^2 \left(\frac{r_s}{b}\right)^6 + 1.1 \cdot \left(\frac{r_s}{b}\right)^2 \right]$$

- being $\nu_0 = 6.36$ MHz

Focusing Requirements

- Solving the two equation system,

$$\vartheta_{gravity+plasma}(b,\nu) \approx \frac{b}{D_{gravity+plasma}}$$

$$\vartheta_{gravity+plasma}(b,\nu) = \vartheta_{gravity}(b) - \vartheta_{plasma}(b,\nu)$$

a critical frequency $\nu_{critical}$ can be found, such that there is convergence if and only if

$$\nu(b) > \nu_{critical}$$

Critical Frequency & More…

- The critical frequency is

$$\nu_{critical}(b) = \sqrt{\frac{\nu_0 \cdot r_s}{2r_{sc}}\left[2952 \cdot \left(\frac{r_s}{b}\right)^{15} + 228 \cdot \left(\frac{r_s}{b}\right)^{5} + 1.1 \cdot \frac{r_s}{b}\right]}$$

- Therefore, the ***EFFECTIVE MINIMAL FOCAL DISTANCE*** turns out to be

$$D_{gravity+plasma}(b,\nu) \approx \frac{\dfrac{b^2}{2r_{sc}}}{1 - \dfrac{\nu^2_{critical}(b)}{\nu^2}}$$

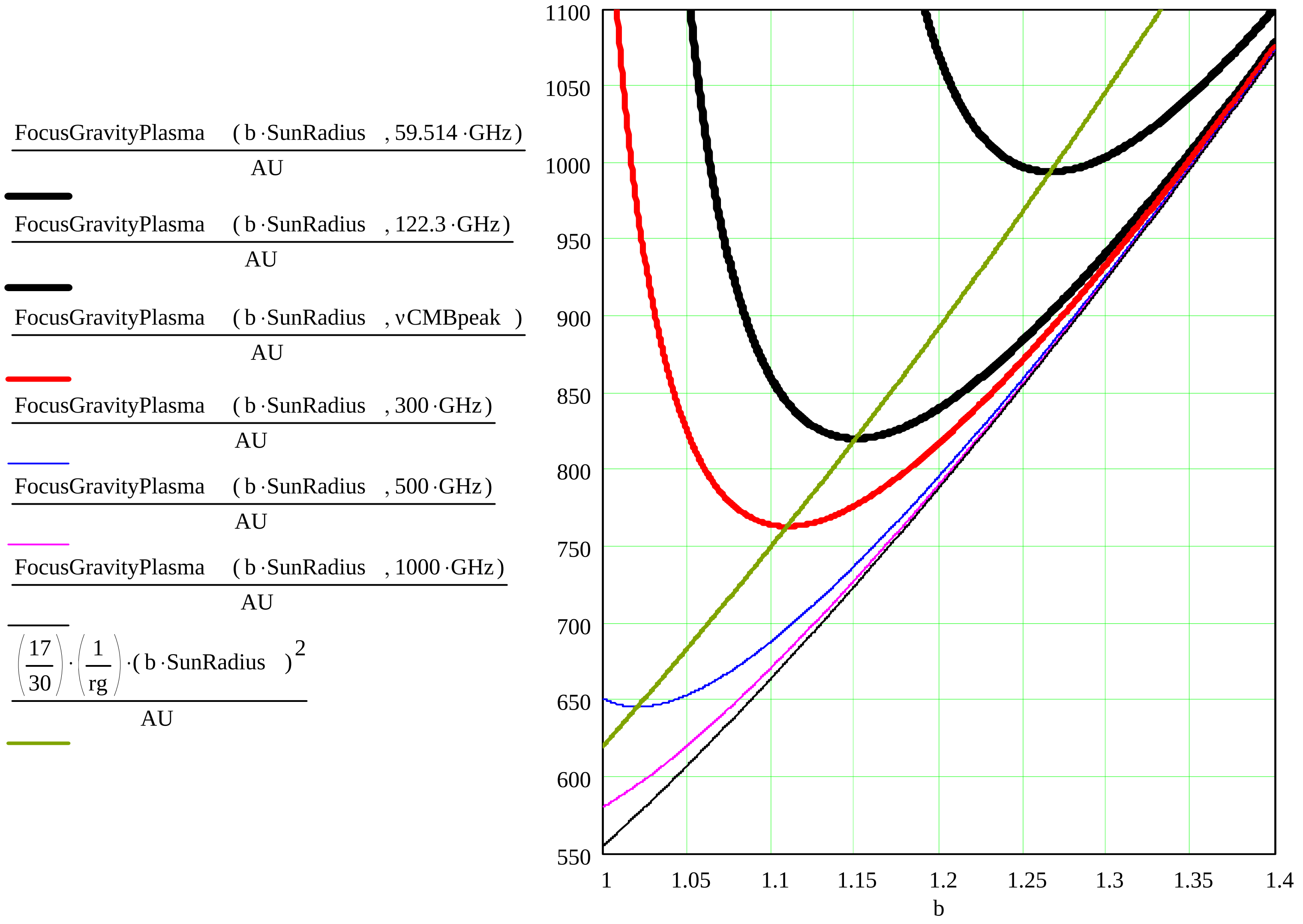
- Where $\nu_{critical}(r_s)$ = 122 GHz is the critical frequency of the Schwarzschild radius of the Sun.

Focal Dist. with the CORONA

Assume that ν > 122.361 GHz, and consider a diagram as follows:

- On the horizontal axis we have the “impact parameter” *b* i.e. the distance at which the bypassing waves graze the Sun’s surface.
- On the vertical axis we have the focal distance (in AU) at which are focused just those radio waves that graze the Sun with impact parameter *b*, i.e. the ***EFFECTIVE MINIMAL FOCAL DIST.***
- The second thick, solid curve from the top of the following figure corresponds to radio waves having the “critical” frequency of 122.361 GHz, at which no more focusing occurs because the outward deflection due to the coronal plasma counterbalances the inner deflection due to gravity. *We are only interested to radio waves having frequency higher than* of 122.361 GHz.

Effective Minimal Focal Dist.



Geometric Locus of Minima

- We see that *the effective minimal focal distance decreases for increasing frequencies*, and nearly reaches the “gravity only” value of 550 AU for 1000 GHz ore more.
- The equation of the geometric locus of all the minima can be proven to be the following parabola:

$$z_{\min}(b_{\min}) = \frac{17}{30} \times \frac{b_{\min}^2}{r_g}$$

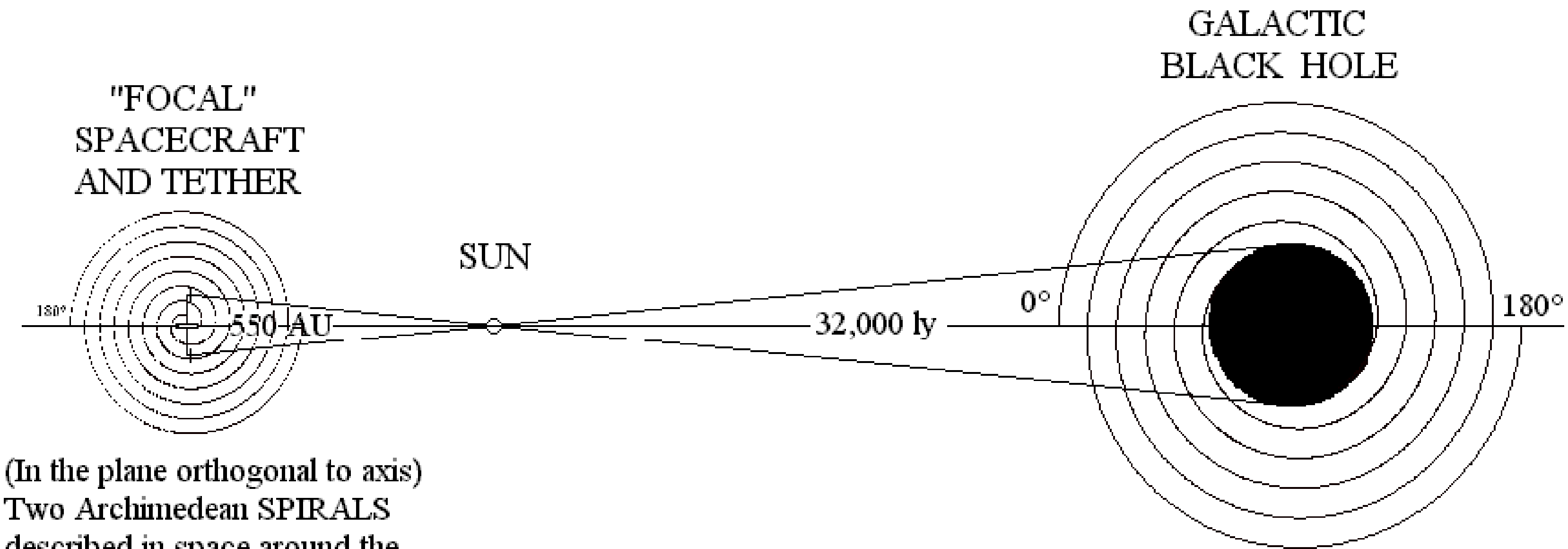
ASTRODYNAMICS: Max Speed

SOLAR SYSTEM EJECTION PROBLEM :

- ***Find the sequence of optimal FLYBYS within the solar system such that***
- ***THE SUN FLYBY IS THE LAST ONE and***
- ***the Focal probe LEAVES the solar system at the MAXIMUM POSSIBLE SPEED…***
- ***and in the direction OPPOSITE to the target star.***
- This problem was first considered by K. Ehricke in 1972.
- Today's Astrodynamics would find better solutions!

INTERFEROMETRY in space

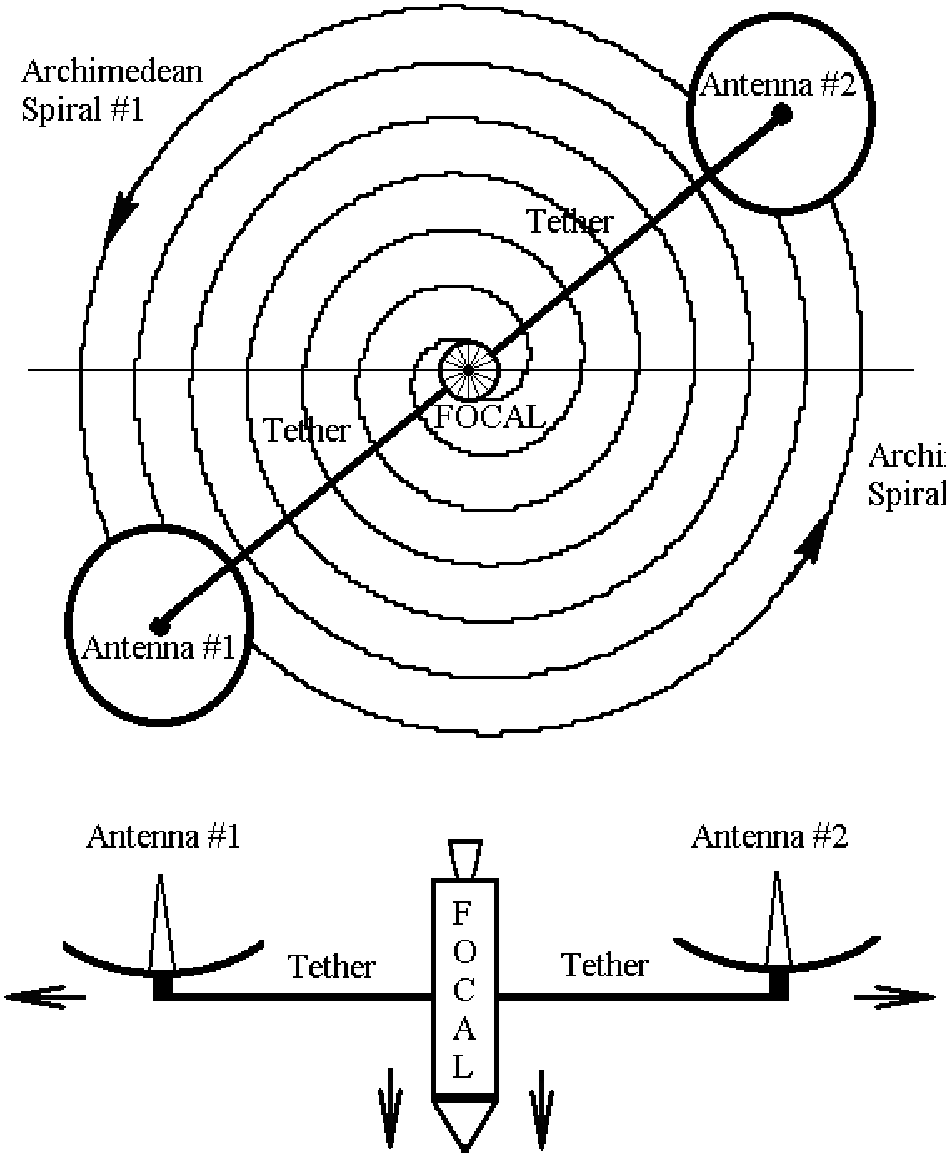
"FOCAL" Spacecraft made up by TWO ANTENNAE TIED BY A TETHER ~ 2 km Long



(In the plane orthogonal to axis)
Two Archimedean SPIRALS described in space around the "FOCAL" spacecraft by each of two antennae tied to each other by a TETHER longer than 1.6 km.

(In the plane orthogonal to axis)
The corresponding two Archimedean SPIRALS described 32,000 light years away from the Sun, at the Galactic Center, around the gigantic Galactic Black Hole. Assuming its mass to be a million times the mass of the Sun, then its Schwarzschild radius equals 0,02 AU.

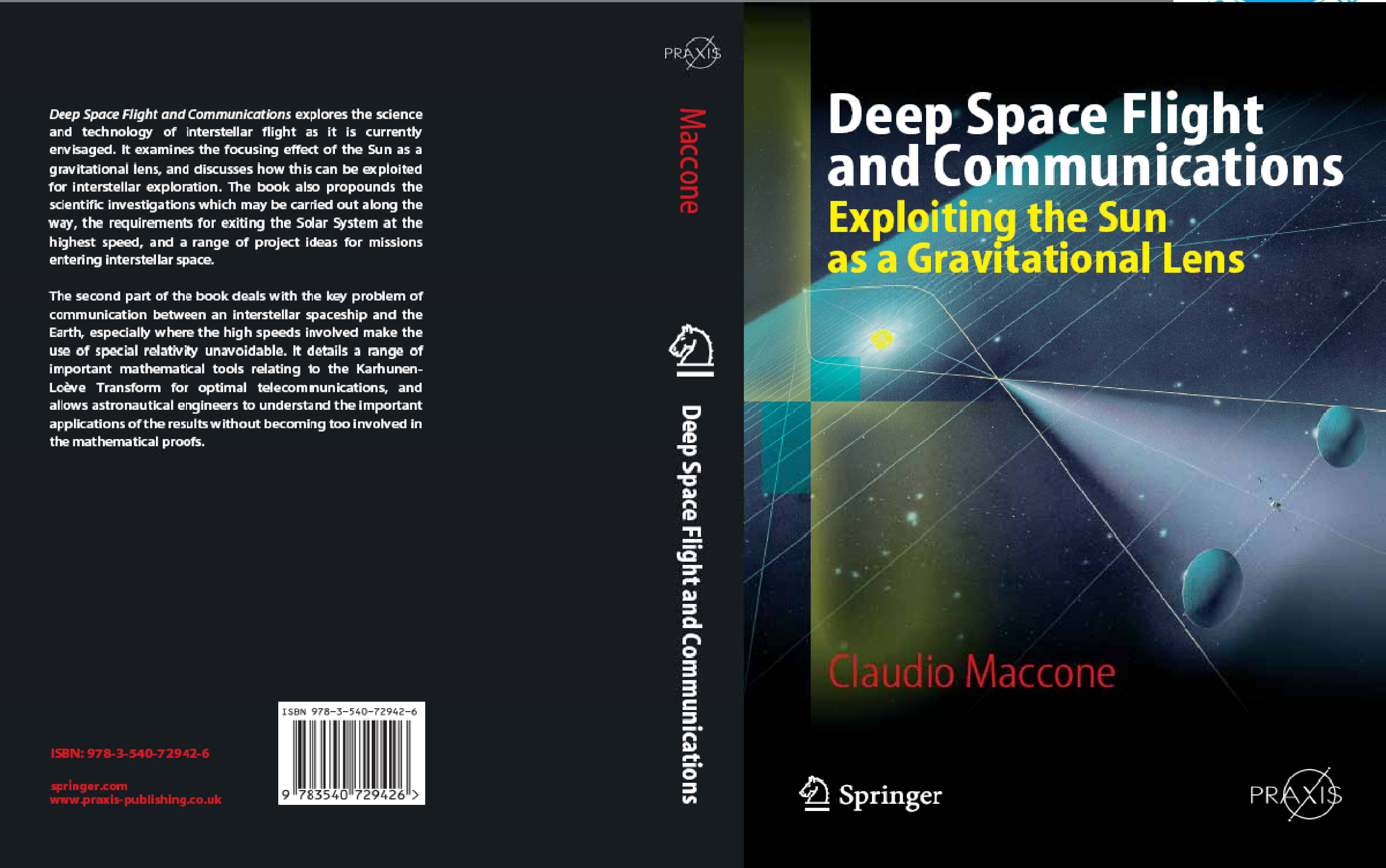
INTERFEROMETRY in space



Deep Space LINK: KLT vs. FFT

	KLT	FFT	
↑	Works well for both wide and narrow band signals	Rigorously true for narrow band signals only	↓
↑	Works for both stationary and non-stationary input stochastic processes	Works OK for stationary input stochastic processes only	↓
↑	Is defined for any finite time interval	Is plagued by the “windowing” problems	↓
↓	Needs high computational burden: no “fast” KLT	Fast algorithm FFT	↑

BOOK by Claudio Maccone



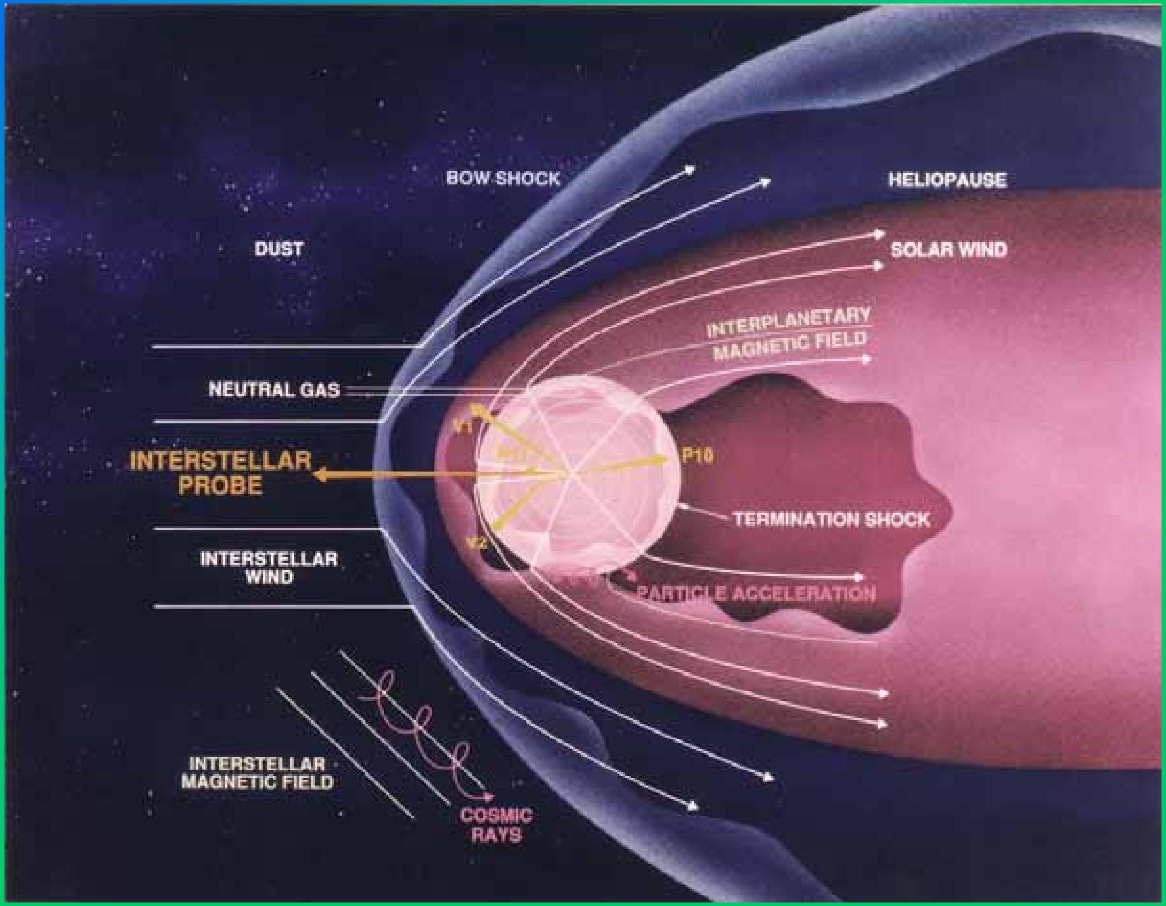
NASA Interstellar Probe (ISP)

Jet Propulsion Laboratory
California Institute of Technology

Interstellar Probe

Exploring the Interstellar Medium and the Boundaries of the Heliosphere

- Interstellar Probe
- Exploring the Interstellar Medium and the Boundaries of the Heliosphere



ESA Interstellar Probe

Proposal for an Interstellar Heliospheric Probe /
Heliospheric Boundary Explorer Mission

in response to ESA's Call for Mission Proposals within the
Cosmic Vision 2015-2015 Programme

Submitted by:

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on behalf of the IHP/HEX consortium:

Janhunen's Electric Sail (2006)

INTERNATIONAL ACADEMY OF ASTRONAUTICS

Missions to the outer solar system and beyond

FIFTH IAA SYMPOSIUM ON REALISTIC NEAR-TERM ADVANCED SCIENTIFIC SPACE MISSIONS

Aosta, Italy, July 2-4, 2007

THE ELECTRIC SAIL - A NEW PROPULSION METHOD WHICH MAY ENABLE FAST MISSIONS TO THE OUTER SOLAR SYSTEM

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1. PHYSICAL PRINCIPLES OF ELECTRIC SAIL

The electric sail (E-sail) is a new propulsion invention [1] which is similar to the solar sail, but it uses the solar wind dynamic pressure instead of the radiation pressure. The electric sail is conceptually similar to Zubrin's magnetic sail [2] except that it uses an electrostatic field instead of a magnetostatic field for deflecting the solar wind proton flow and extracting momentum from it.

Alpha Centauri A as a Gravitational Lens

- The following data apply to Alpha Centauri A:
  1. Mass of Alpha Centauri A: 2.2351443209 · 10^{30} kg, that is μ = 1.1238 μSun.
  2. Radius of Alpha Centauri A: 821280 km, that is 1.18 times the Sun radius.
  3. Alpha Centauri A Mean Density: 963.259 kgm^{-3}
  4. Schwarzschild radius of Alpha Centauri A: 3.318 km
  5. Minimal Focal Distance of naked Alpha Centauri A: 679.262 AU ~ 3.92 light days ~ 17.17 times the Sun-to-Pluto distance.
  6. What is the Effective Minimal Focal Distance ?

Thanks !