A talk at the KISS Workshop
"Science and Enabling Technologies to Explore the Interstellar Medium (ISM)"
September 8, 2014, Caltech

The Solar Gravitational Lens:

It is out there… can we use it?

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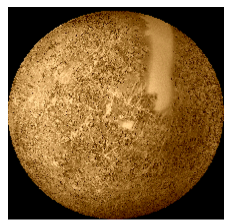
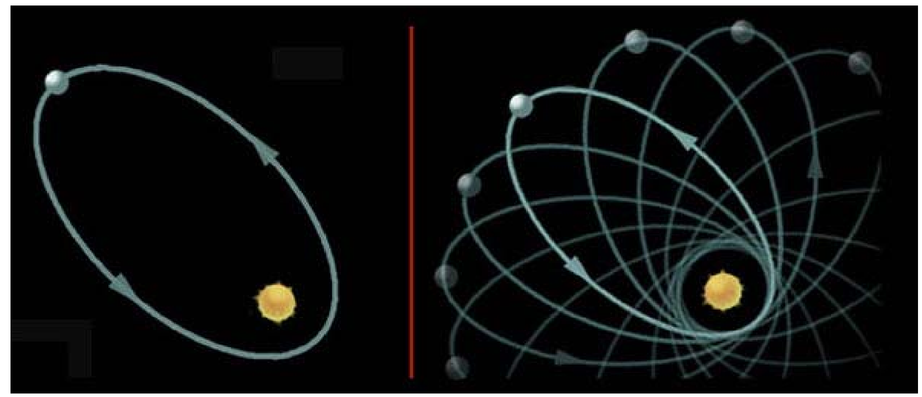
Triumph of Mathematical Astronomy in 19th Century

Discovery of Neptune: 1845

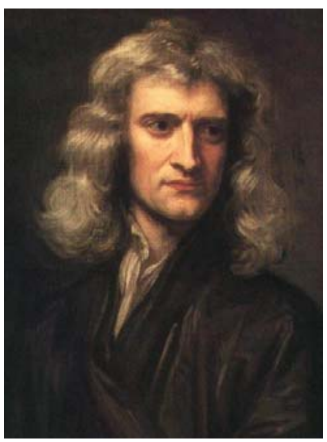
Urbain LeVerrier
(1811-1877)



- 1845: the search for Planet-X:
  - Anomaly in the Uranus' orbit → Neptune
  - Anomalous motion of Mercury → Vulcan



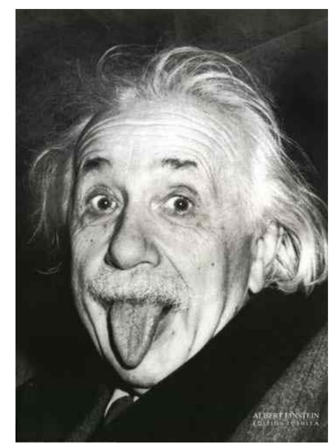
Newtonian Gravity General Relativity



Sir Isaac Newton
(1643-1727)

- Anomalous precession of Mercury's perihelion :
  - 43 arcsec/cy can not be explained by Newton's gravity
- Before publishing GR, in 1915, Einstein computed the expected perihelion precession of Mercury
  - When he got out 43 arcsec/cy – a new era just began!!

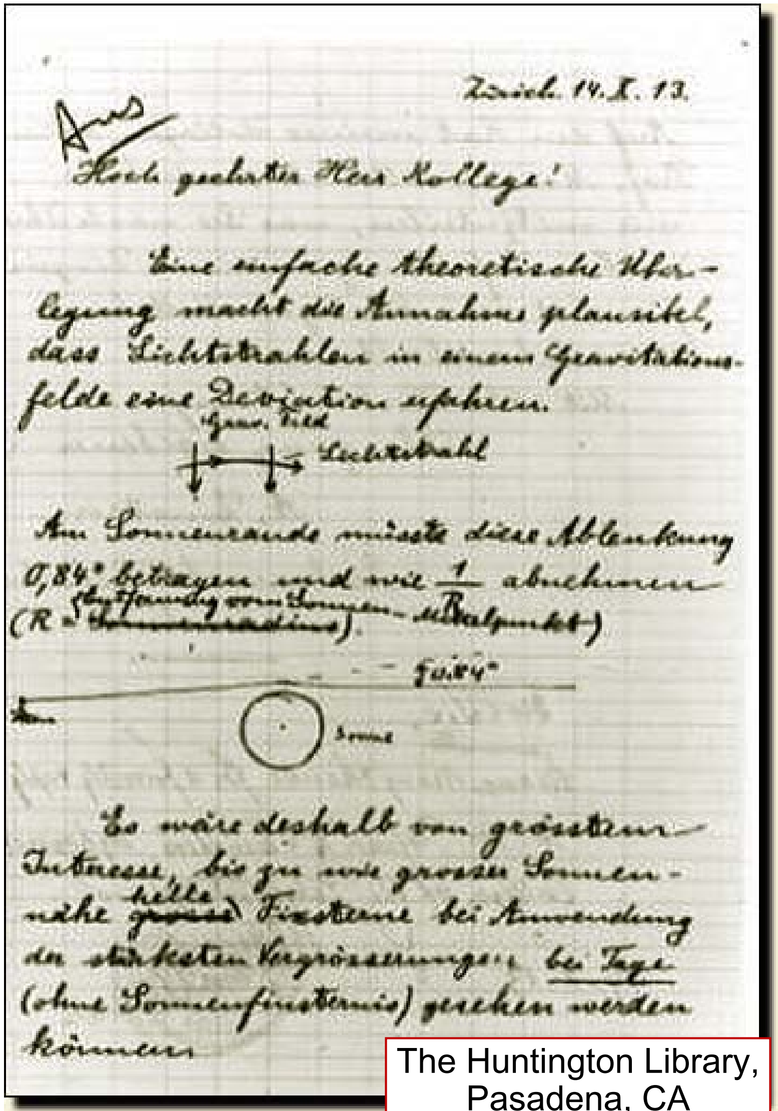
Almost in one year LeVerrier both confirmed the Newton's theory (Neptune) & cast doubt on it (Mercury's' anomaly).



Albert Einstein
(1879-1955)

Efforts before GR…

$$\alpha_{\texttt{Newton}}(b) = \frac{2GM_\odot}{c^2 b} = 0.877\left(\frac{\mathcal{R}_\odot}{b}\right)\ \mathrm{arcsec}$$



Zürich. 14. X. 13.

Ans

Hoch geehrter Herr Kollege!

Eine einfache theoretische Überlegung macht die Annahme plausibel, dass Lichtstrahlen in einem Gravitationsfelde eine Deviation erfahren.

Grav. Feld

Lichtstrahl

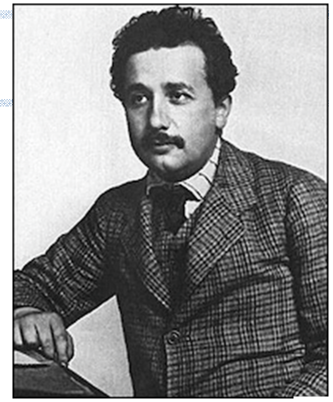
Am Sonnenrande müsste diese Ablenkung 0,84" betragen und wie $\frac{1}{R}$ abnehmen (R = Entfernung vom Sonnen-Mittelpunkt).

0,84"

Sonne

Es wäre deshalb von grösstem Interesse, bis zu wie grosser Sonnennähe helle Fixsterne bei Anwendung der stärksten Vergrösserungen bei Tage (ohne Sonnenfinsternis) gesehen werden können.

The Huntington Library, Pasadena, CA

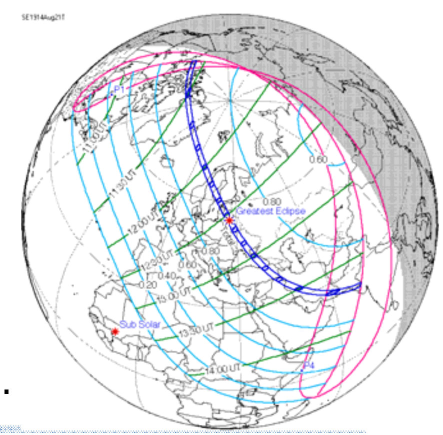


Albert Einstein
c.1913

George Ellery Hale
(1868-1938)

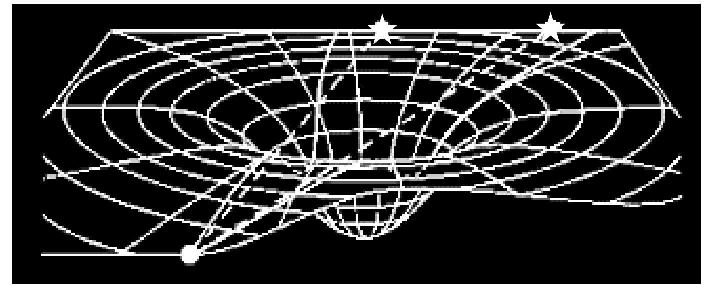
Erwin Finlay-Freundlich
(1885-1964)

- In 1913 Einstein wrote to Hale:
  - Is eclipse necessary to test this prediction?
  - Hale replied: “Yes, an eclipse is necessary, as stars near the Sun would then be visible, and the bending of light from them would show up as an apparent displacement of the stars from their normal positions.”
- In 1914, the first attempt - a German expedition
  - During a total eclipse occurring on Aug. 21, 1914 a German astronomer Finley-Freundlich led an expedition to Russia to test the Einstein's prediction.
  - However, the First World War (July 28, 1914) intervened, and no observations could be made.



Solar eclipse of Aug. 21, 1914

The First Test of General Theory of Relativity

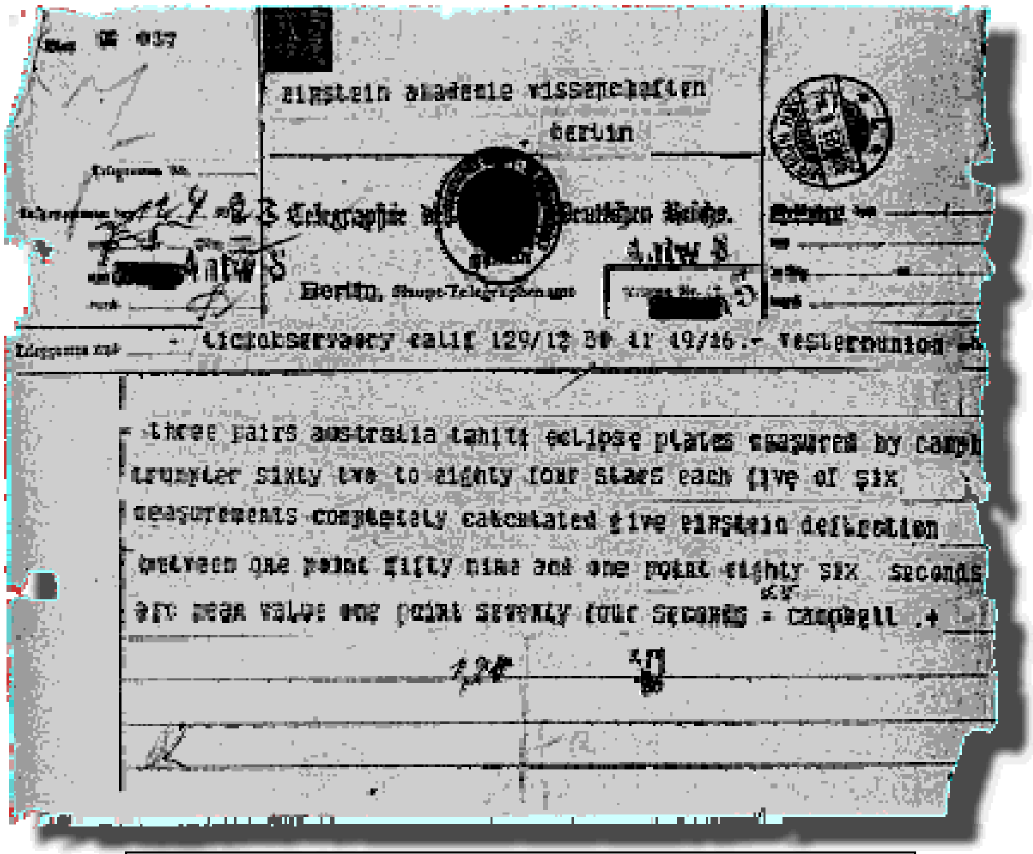


Gravitational Deflection of Light:

$$\alpha_{\mathrm{GR}}(b) = \frac{2(1+\gamma)GM_\odot}{c^2 b} \simeq 1.75\left(\frac{1+\gamma}{2}\right)\left(\frac{\mathcal{R}_\odot}{b}\right) \text{ arcsec}$$

Solar Eclipse 1919: possible outcomes

Deflection = 0;
Newton = 0.87 arcsec;
Einstein = 2 x Newton = 1.75 arcsec



einstein akademie wissenschaften
berlin

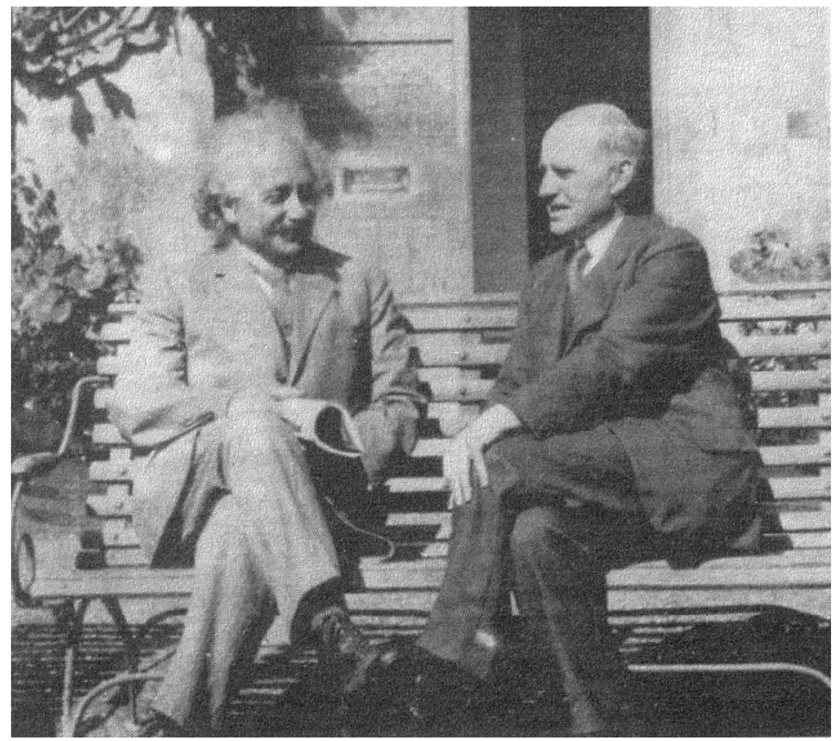
Telegraphie des Deutschen Reichs.

Berlin, Haupt-Telegraphenamt

lickobservatory calif 129/12 36 11 19/16.-

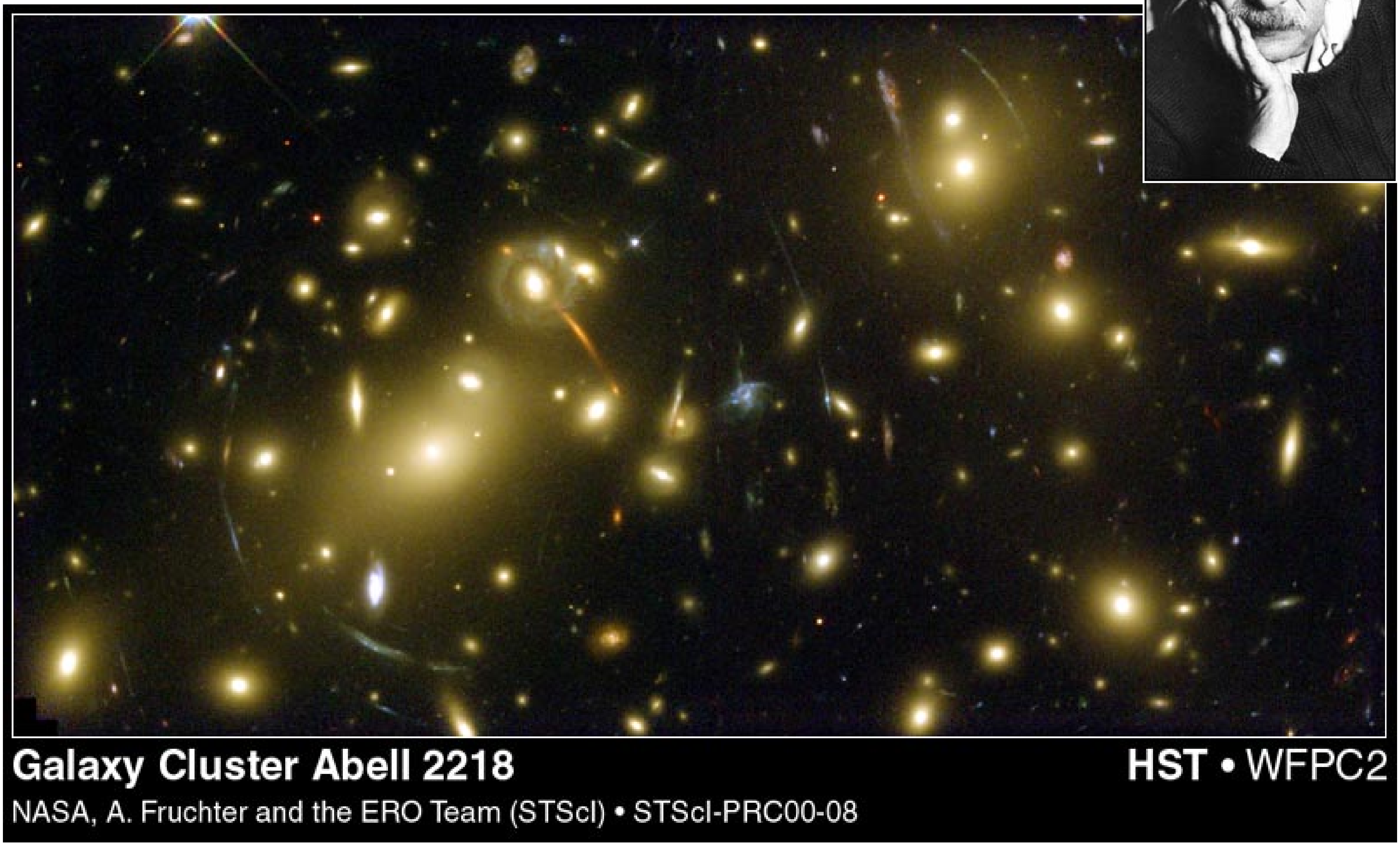
three pairs australia tahiti eclipse plates measured by campbell
trumpler sixty two to eighty four stars each five of six
measurements completely calculated give einstein deflection
between one point fifty nine and one point eighty six seconds
arc mean value one point seventy four seconds = campbell .+

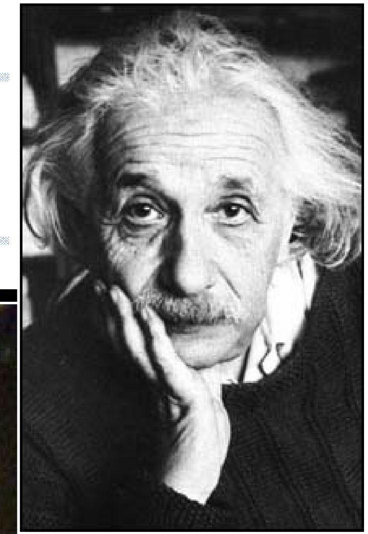
Campbell's telegram to Einstein, 1923



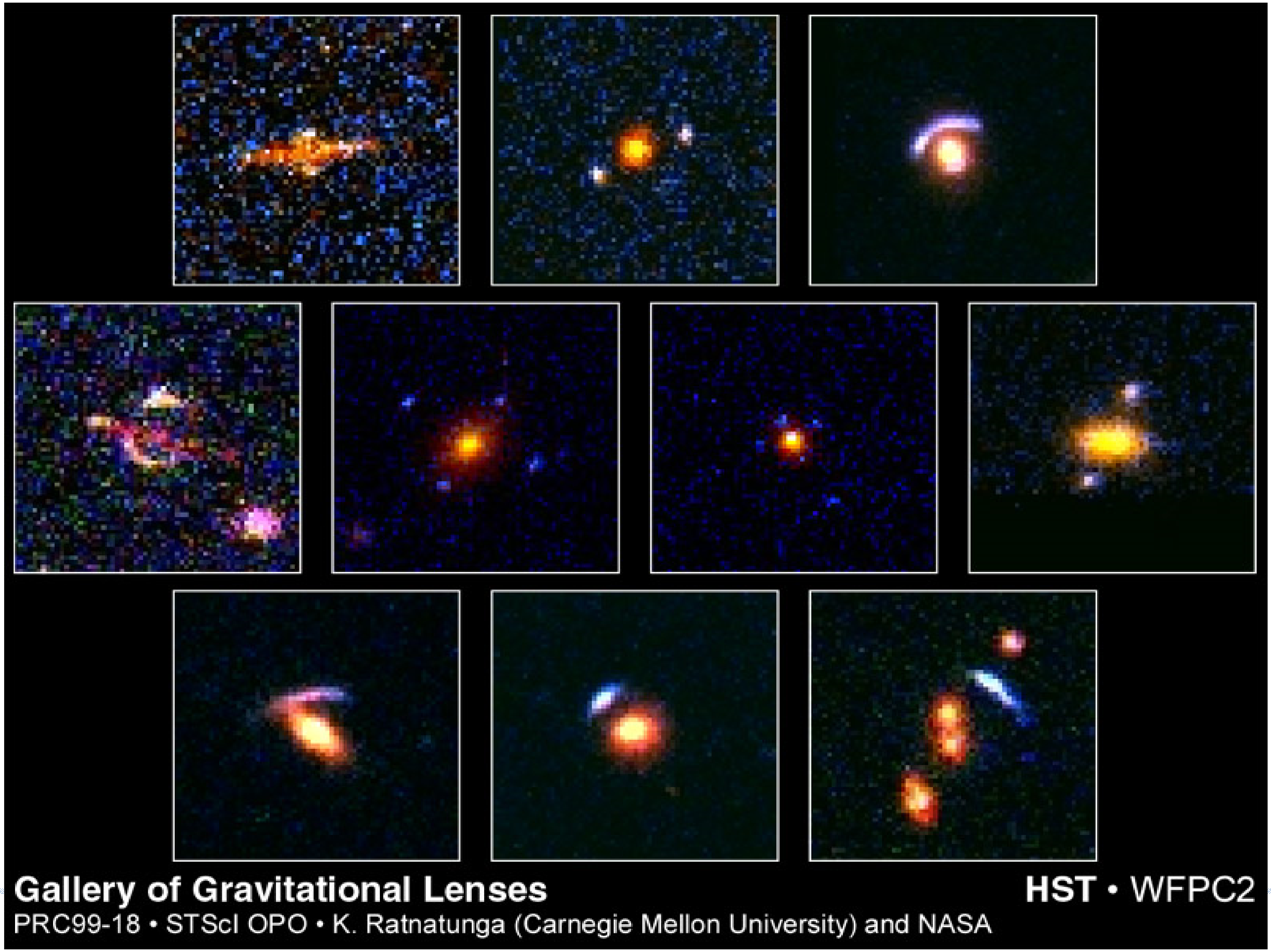
Einstein and Eddington, Cambridge, 1930

Gravitational Deflection of Light is a Well-Known Effect Today

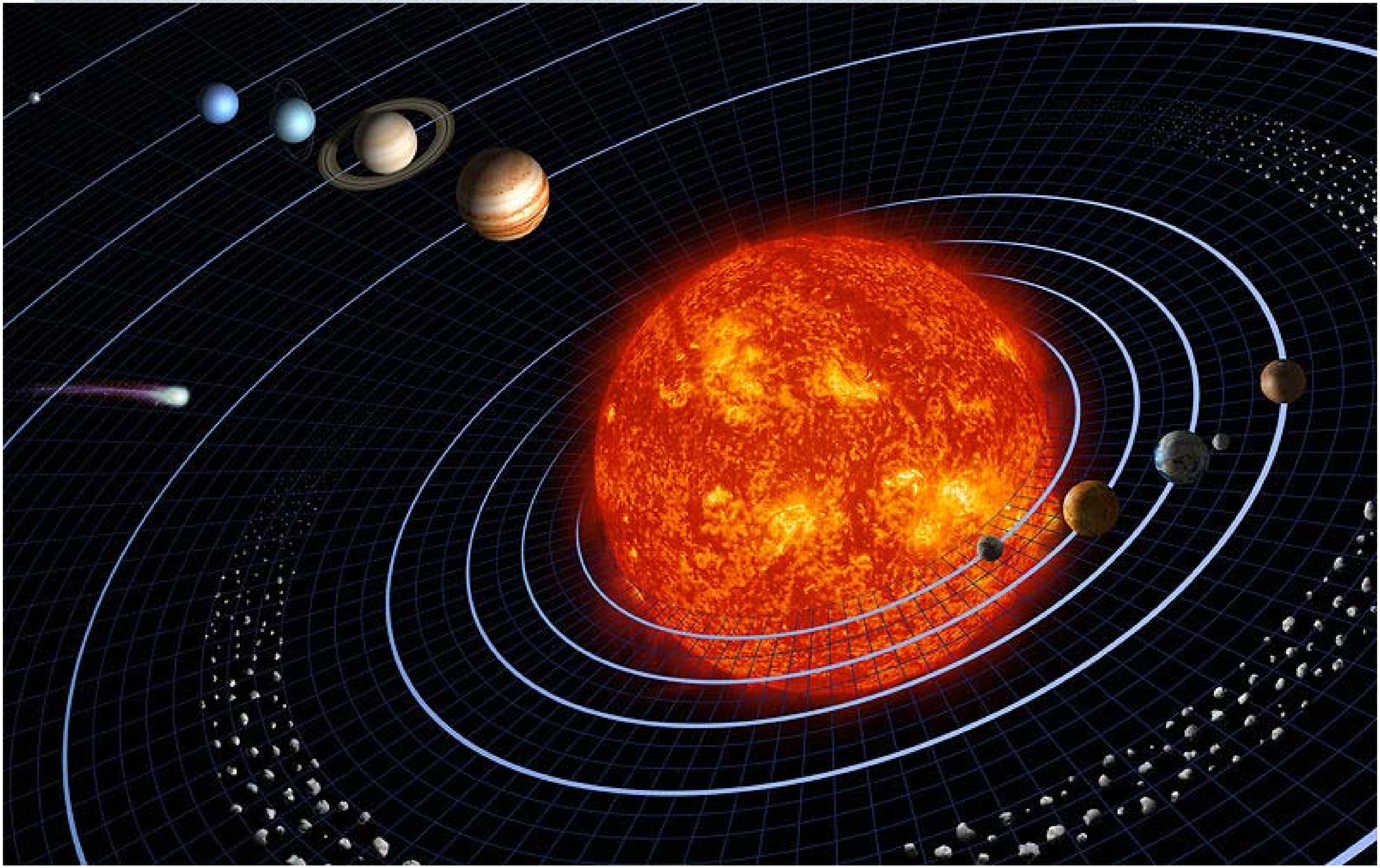




They are everywhere…



Our solar system and tests of gravity



40+ Years of Solar System Gravity Tests

Techniques for Gravity Tests:

Radar Ranging:
- Planets: Mercury, Venus, Mars
- s/c: Mariners, Vikings, Pioneers, Cassini, Mars Global Surveyor, Mars Orbiter, etc.
- VLBI, GPS, etc.

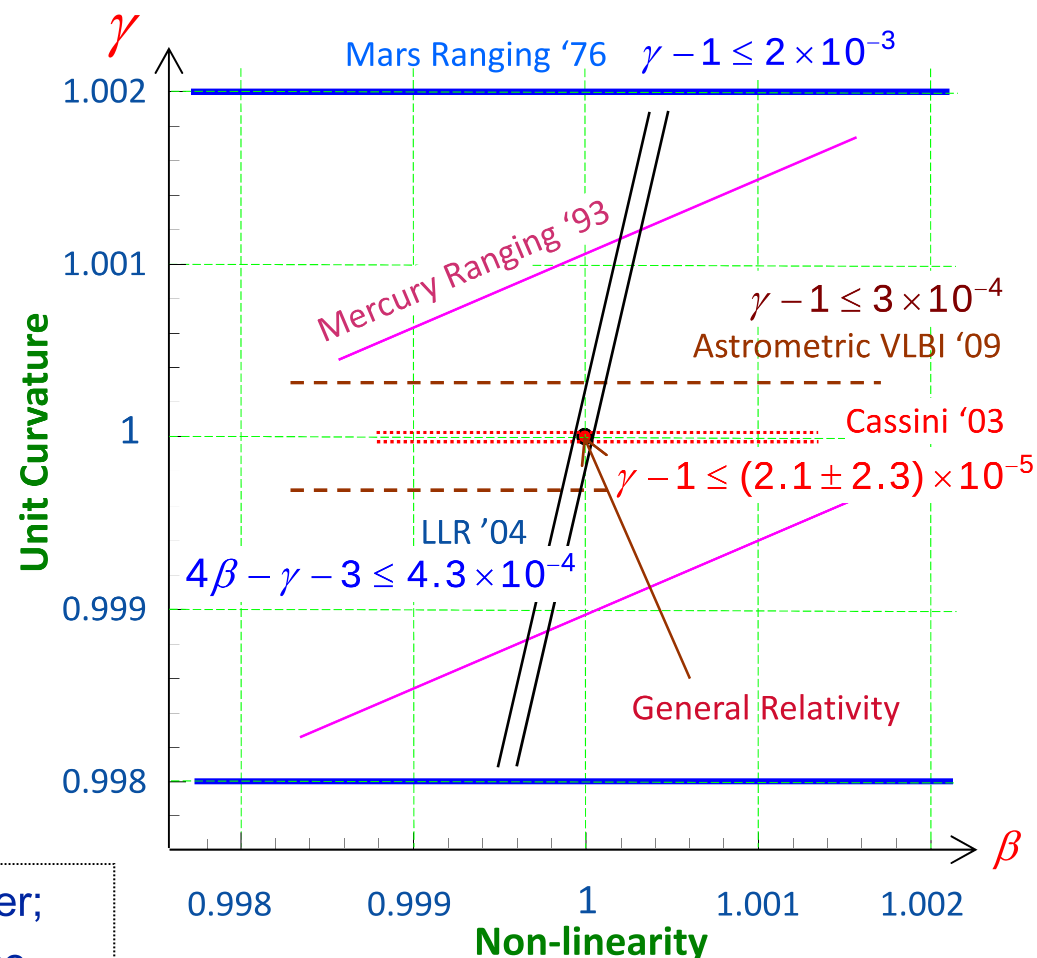
Laser:
- SLR, LLR, interplanetary, etc.

Dedicated Gravity Missions:

- **LLR (1969 - on-going!!)**
- GP-A, '76; LAGEOS, '76,'92; GP-B, '04; LARES, '12; LISA, 2030+(?)

New Engineering Discipline – Applied General Relativity:

- Daily life: GPS, geodesy, time transfer;
- Precision measurements: deep-space navigation & μas-astrometry (ESA's Gaia).



General relativity is well tested, can we use it to build something?

Mission to the Gravity Lens of the Sun

von Eshleman R., Science **205**, 1133 (1979)

Gravitational Lens of the Sun: Its Potential for Observations and Communications over Interstellar Distances

Abstract. *The gravitational field of the sun acts as a spherical lens to magnify the intensity of radiation from a distant source along a semi-infinite focal line. A spacecraft anywhere on that line in principle could observe, eavesdrop, and communicate over interstellar distances, using equipment comparable in size and power with what is now used for interplanetary distances. If one neglects coronal effects, the maximum magnification factor for coherent radiation is inversely proportional to the wavelength, being 100 million at 1 millimeter. The principal difficulties are that the nearest point on the focal half-line is about 550 times the sun-earth distance, separate spacecraft would be needed to work with each stellar system of interest, and the solar corona would severely limit the intensity of coherent radiation while also restricting operations to relatively short wavelengths.*

About 40 years ago, Einstein (*1*) published a short note in *Science* on the focusing of starlight by the gravitational field of another star. He emphasized the improbability of observing this phenomenon by the chance alignment of two stars and the earth. From concepts based on current technology and trends, however, it appears that gravitational focusing of electromagnetic radiation might be employed, by design, for highly directional observations and communications over interstellar distances.

In such use, the gravitational field of the sun could play several roles. First, it might be used to reduce fuel and time re-

$1 + \nu$, where the refractivity $\nu = g/r$ at radius r. A ray is deflected through the angle $\alpha = 2g/a$, where a is the ray impact parameter and g is the gravitational radius ($g = 2Gm/c^2$, where G is the gravitational constant, m is the mass of the central body, and c is the speed of light). It is assumed throughout that $\alpha << 1$. An observer at position z behind the lens and x from the center line, as illustrated, would see an energy density lessened by defocusing in the plane of propagation, but increased by focusing due to the curved limb normal to this plane. The relative single-ray intensity $I = F_h^2 F_v^2$, where in ray optics $F_h^2 =$

nel scales along the circumference of a circle at the ray-impact radius. Using also the wave number $k = 2\pi/\lambda$, the maximum intensification of the coherent signal is simply

$$I_{max} = 2\pi kg \qquad (2)$$

As an approximation, let the focal "spot" radius x_s be the value of x where I falls to $I_{max}/4$, so that $x_s = (2/\pi k)(z/2g)^{1/2}$. Thus the angular resolution for distinguishing two adjacent coherent sources by a corresponding change in intensity is x_s/z radians. (The first null off the center line is at $x = \pi^2 x_s/2$, and the first sidelobe is twice this distance with intensity I_{max}/π^2.) The periapsis or minimum radius of the ray relative to the center of mass is $a - g$, or essentially a, and this must be greater than r_0, the physical radius of the spherical mass. Thus $\alpha_{max} = 2g/r_0$ and the focal line begins at $z_{min} = r_0^2/2g$.

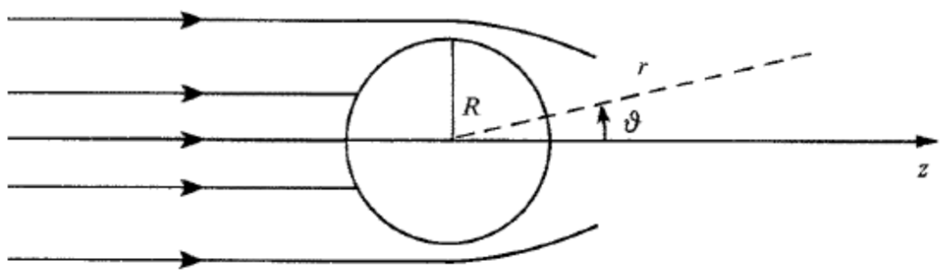
Now consider the focusing at $z > z_{min}$ of incoherent radiation from a uniformly bright, circular, extended source of radius r_p and distance $z_p >> z$. This is the problem considered by Einstein (*1*) and more completely by others, notably Liebes (*4*). The gain factor A of the gravitational lens for the intensity observed from the two individual image com-

Kraus J.D., Radio Astronomy, Cygnus-Quasar Books, Powell, Ohio, 6-115 (1986)

Many papers by C. Maccone

Turyshev & Andersson, MNRAS **341**, 577 (2003)

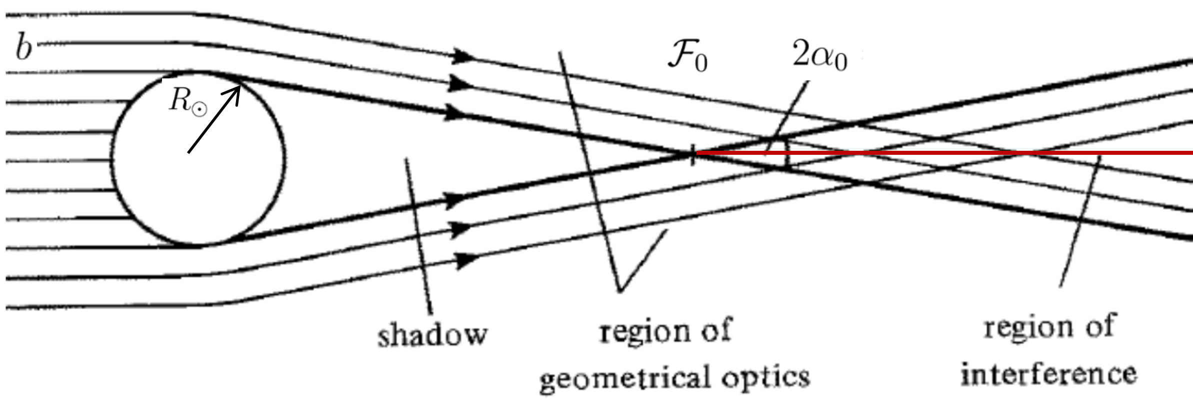
Solar Gravity Lens: wave optics treatment



The incident plane wave

$$\alpha_0 = \frac{2r_g}{R_\odot} \approx 8.5\ \mu\mathrm{rad} \quad \rightarrow \quad \alpha(b) = \alpha_0 \frac{R_\odot}{b}$$

$$\mathcal{F}_0 = \frac{R_\odot}{\alpha_0} = \frac{R^2_\odot}{2r_g} \approx 547\ \mathrm{AU} \quad \rightarrow \quad \mathcal{F}(b) = \mathcal{F}_0 \frac{b^2}{R^2_\odot}$$



The three different regions of space

- Wave-theoretical treatment of the gravitational lens:
  - Consider the scattering of a plane EM wave by the Sun
  - Solve Maxwell's equations with the symmetry of a plane wave
  - The Sun is assumed to be spherical and, also,

$$\lambda \ll r_g = \frac{2GM_\odot}{c^2} \approx 2.95\ \mathrm{km} \ll R_\odot$$

  - Consider the image of a distant star and analyze the relevant components of Pointing vector

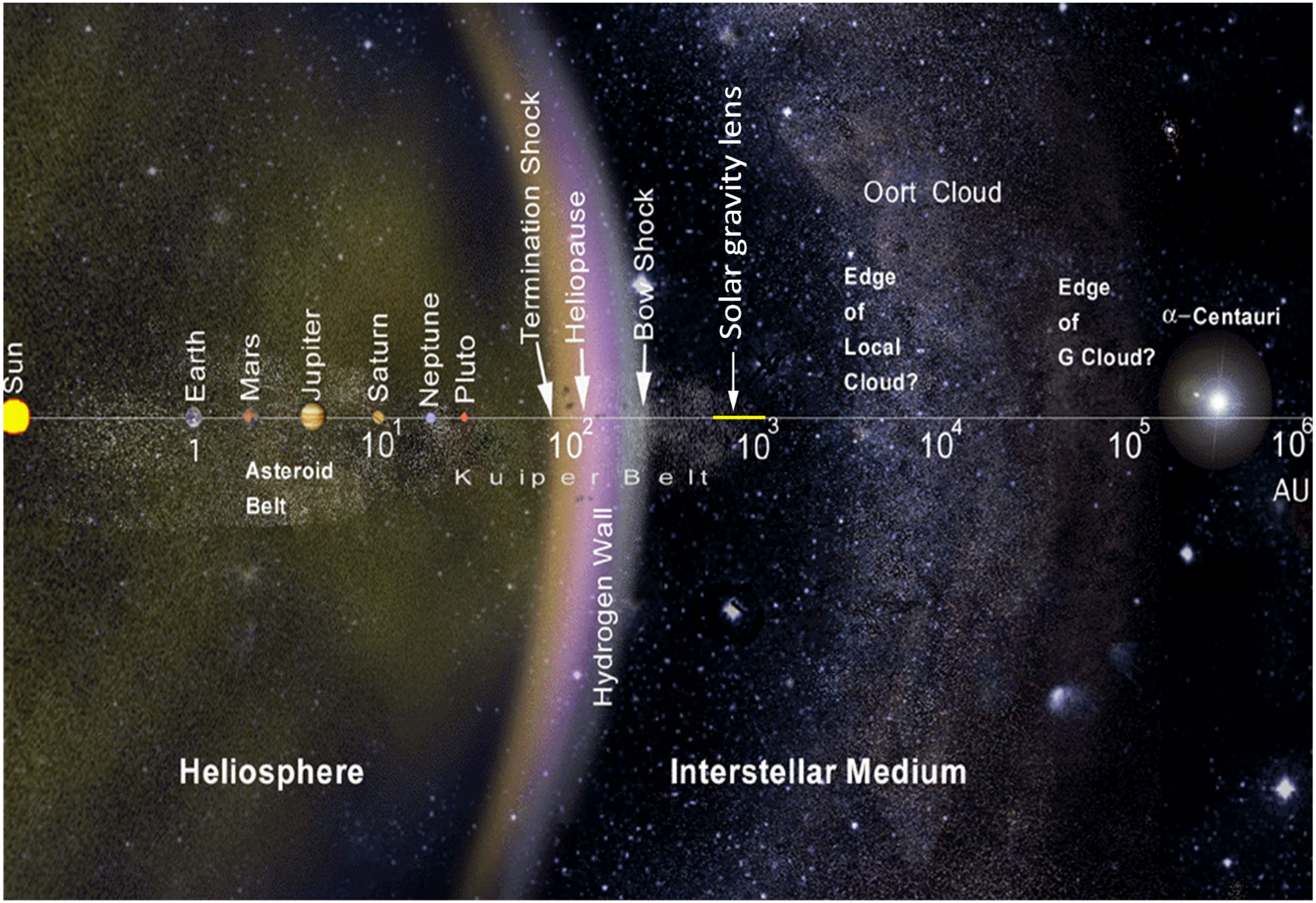
Points of stationary phase provide the boundaries between the regions:

$$\sin\theta \approx \frac{b}{r} - \frac{2r_g}{b}, \qquad r \le \frac{b^2}{2r_g}$$

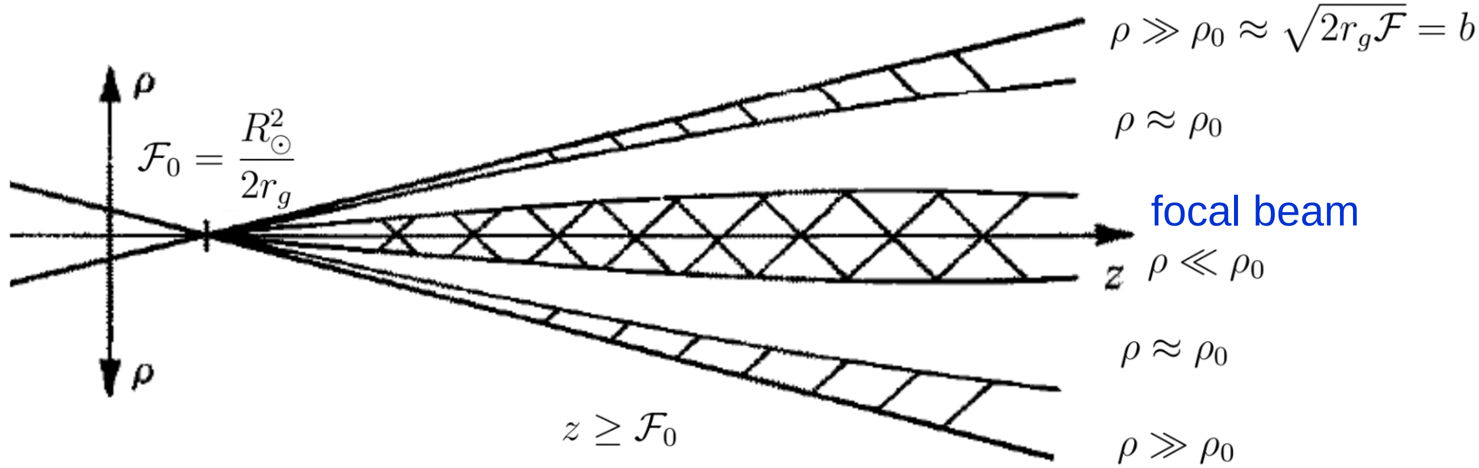
$$\sin\theta \approx \frac{2r_g}{b} - \frac{b}{r}, \qquad r \ge \frac{b^2}{2r_g}$$

Herlt & Stephani, IJTP **15**, 45 (1976); Turyshev & Andersson, MNRAS **341**, 577 (2003)

Distance scale

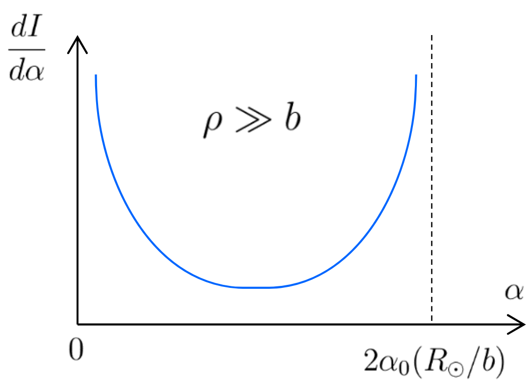


Solar Gravity Lens: region of interference



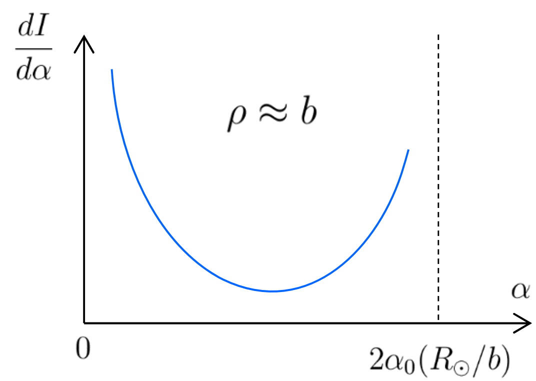
Double images of equal brightness

– luminosities are very close: $\mathcal{L}_1 \simeq \mathcal{L}_2$



Double images of unequal brightness

– luminosities may differ up to $\mathcal{L}_1/\mathcal{L}_2 \simeq 10^2$



– In both cases, the two stars are located at $\alpha_1 = 0$ (no deflection) and at $\alpha_2 = 2\alpha_0(R_\odot/b)$, with a weak bridge in between.

Herlt & Stephani, IJTP **15**, 45 (1976); Turyshev & Andersson, MNRAS **341**, 577 (2003)

Image of a moving source

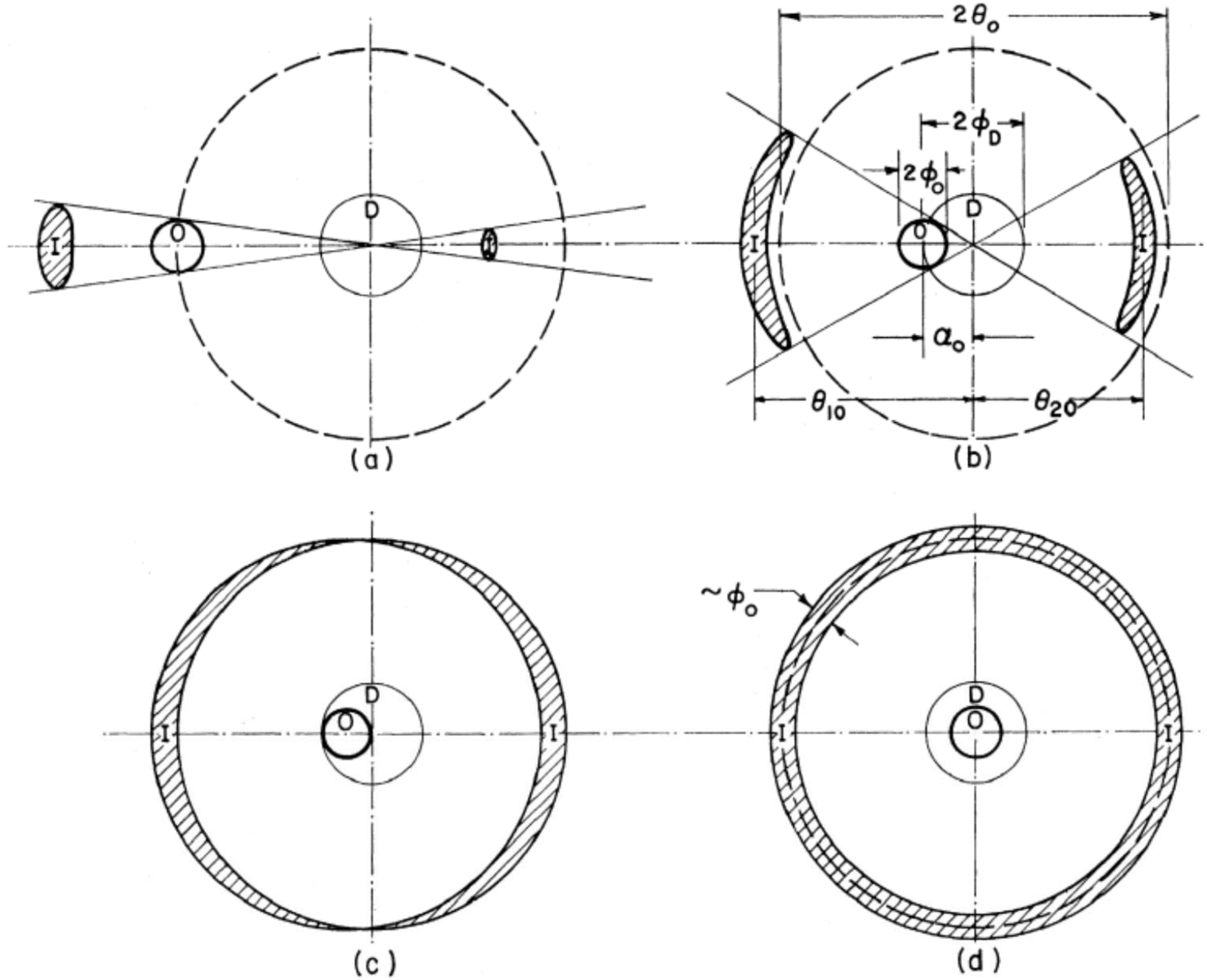
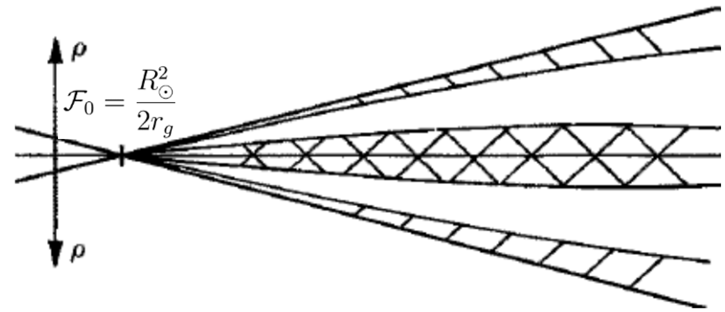


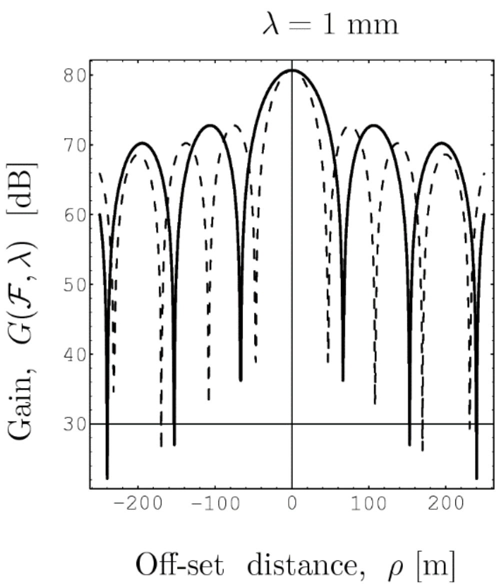
FIG. 2. Transformation of the image configuration I of the object O as the object moves from left to right behind the deflector D.

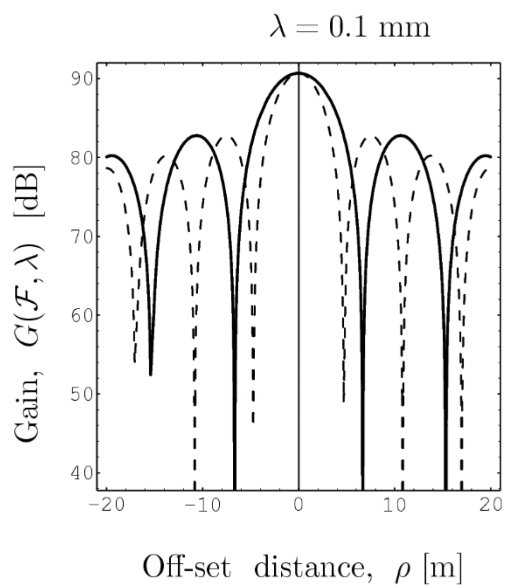
Focal beam of extreme intensity

- Satisfies conditions: $\frac{\lambda}{2\pi b} \ll \theta \ll \frac{2r_g}{b}$ and $z \geq \mathcal{F}_0$
  - For small departures from the optical axis $\rho \ll \rho_0 \approx b$, the gain of the solar gravitational lens is:

$$\boxed{G(\rho,\lambda) \cong 4\pi^2 \frac{r_g}{\lambda} J_0^2\left(\frac{2\pi\rho}{\lambda}\sqrt{\frac{2r_g}{\mathcal{F}}}\right)}$$







Gain of the solar gravity lens as seen in the image plane as a function of possible observational wavelength λ and the optical distance $\mathcal{F}$: $\mathcal{F} = 600$ AU and $\mathcal{F} = 1200$ AU

- Max value of $G(\rho,\lambda)$ is on axis:

$$G_{\tt max}(0,\lambda) = 4\pi^2 \frac{r_g}{\lambda}$$

  - if ρ increases, gain decreases (while oscillating), reaching

$$G(\rho_1,\lambda) \cong \left(\frac{8\pi r_g}{\lambda}\right)^{\frac{1}{2}} \text{ for } \rho_1 = \left(\frac{\lambda\mathcal{F}}{\pi}\right)^{\frac{1}{2}}$$

  - going further down to its mean value $G = 1$ for $\rho \gg \rho_0 \approx b$.

As expected, the gravity lens is very sensitive to a tangential motions (*e.q.*, in the image plane).

Turyshev & Andersson, MNRAS **341**, 577 (2003)

Important properties of the Lens

- High angular resolution:
  - We use the angle, $\epsilon_{10\mathrm{dB}}$, that corresponds to the distance, $\rho_{10\mathrm{dB}} \cong \frac{\lambda}{\pi^2}\sqrt{\frac{\mathcal{F}}{2r_g}}$ from the optical axis to the point where gain decreases by 10 dB:

$$\epsilon_{10dB}(b,\lambda) = \frac{\rho_{10dB}}{\mathcal{F}} \cong \frac{\lambda}{\pi^2 b},$$

for λ = 1 mm

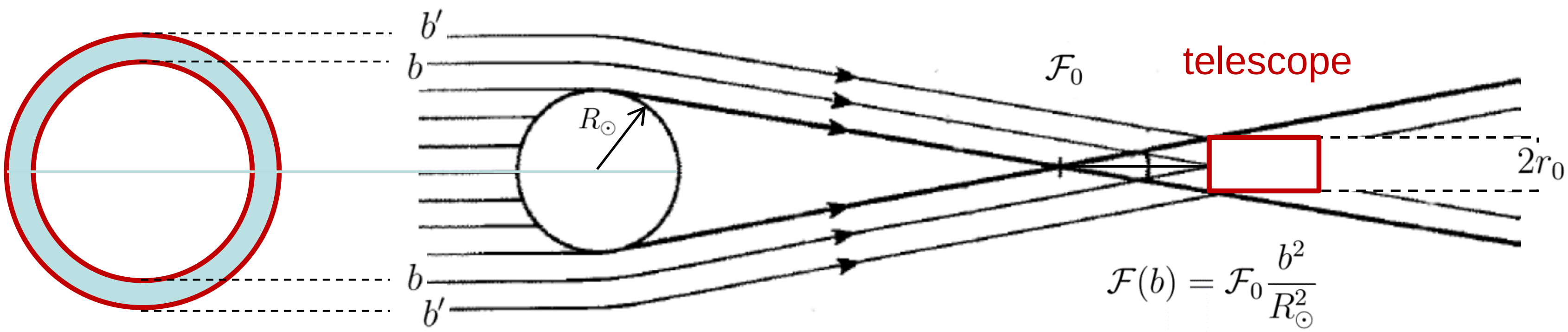
$$\epsilon_{10dB}(R_\odot, 1\ \mathrm{mm}) \simeq 3.0\times 10^{-8}\ \mathrm{arcsec}$$

$$\rho_{10\mathrm{dB}}(R_\odot, 1\ \mathrm{mm}) \simeq 11.9\ \mathrm{m}$$

- Major brightness increase:
  - Since for the Sun the frequency can have values between 10^4 Hz (radio waves) and 10^{10} Hz (visible light) or even 10^{14} Hz (γ-rays), a gravitational lens can enlarge the brightness of a star by the same remarkable factor.
- Extremely narrow “pencil” beam:
  - The plane wave is focused into a very narrow beam of extreme intensity; the radius of which is $\rho = \sqrt{\lambda\mathcal{F}/\pi}$, thus, $\rho(1\ \mathrm{mm}, \mathcal{F}_0) \approx 114.1\ \mathrm{km}$
- Rapid spatial oscillations:
  - The components of the Poynting vector and the intensity are oscillating with a spatial period $\delta\rho = \lambda\sqrt{\mathcal{F}/8r_g}$ thus, $\delta\rho(1\ \mathrm{mm}, \mathcal{F}_0) \approx 60\ \mathrm{m}$
- Angular distribution of intensity: $$\frac{d\mathcal{I}}{d\theta} = \left(\frac{\mathcal{F}}{2\rho}\right)^{\frac{3}{2}} \frac{\lambda\rho\ d\theta d\phi}{\pi\sqrt{\theta}}$$
  - Observer would eventually see a significantly magnified source at $\theta \cong 0$.

Turyshev & Andersson, MNRAS **341**, 577 (2003)

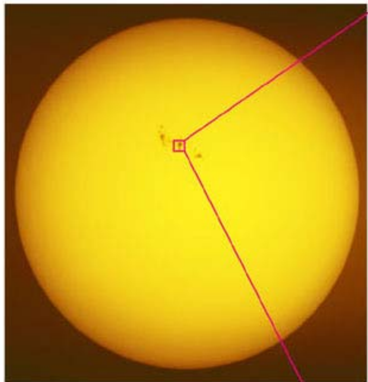
Geometry of the lens

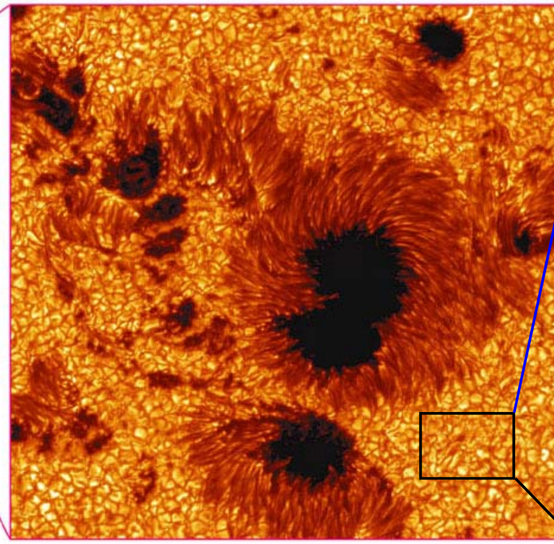


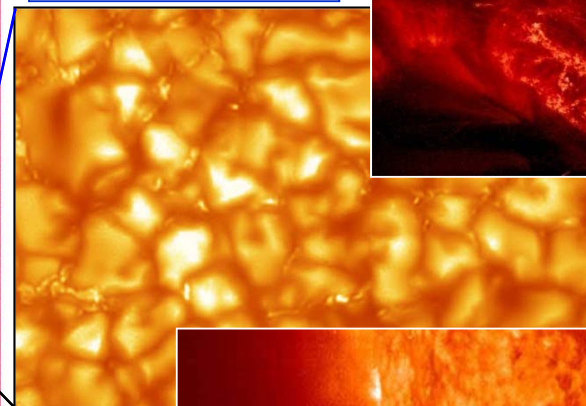
- The width of the Lens for a given telescope diameter:
  - At a distance $\mathcal{F}(b)$ from the Lens, a telescope with diameter $2r_0$ also collects the light from the rays with impact parameter $b' \approx b + \frac{r_0}{2}$, therefore, the width is $\delta b = r_0/2$
  - The distance between $\mathcal{F}(b')$ and $\mathcal{F}(b)$ is $\delta\mathcal{F}(r_0) = \mathcal{F}_0 \frac{r_0}{R_\odot}$, for 1m telescope this distance is $\delta\mathcal{F}(0.5\ \mathrm{m}) = 59\ \mathrm{km}$
- Collecting area:
  - At this distance, the total collecting area of the solar gravitational lens is

    $\mathcal{S}_{\rm GL} = \mathcal{S}(b') - \mathcal{S}(b) \approx \pi b r_0 = \mathcal{S}_{\rm GL0}(r_0)\frac{b}{R_\odot}$ where $\mathcal{S}_{\rm GL}(r_0) = \pi R_\odot r_0$
  - For a 1-m telescope, the total collecting area of the solar lens is: $1.1\times10^9\ \mathrm{m}^2$, which is equivalent to a telescope with a diameter of ~37.3 km…

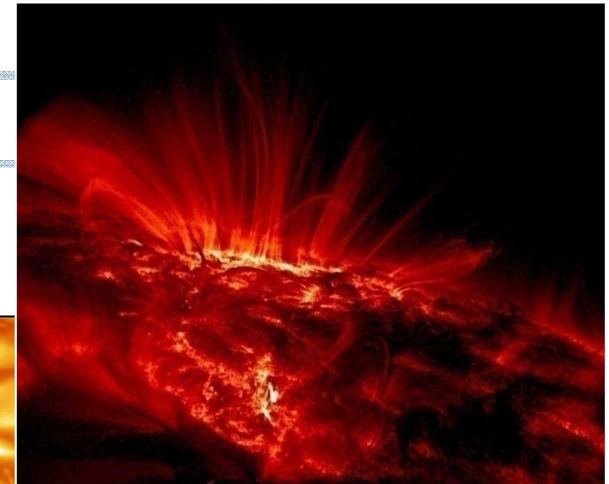
The Solar Boundary



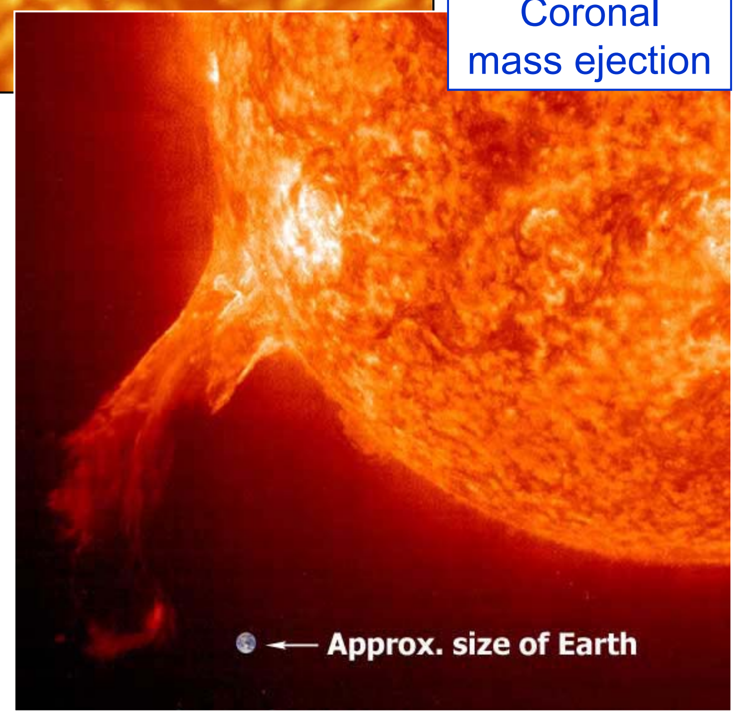




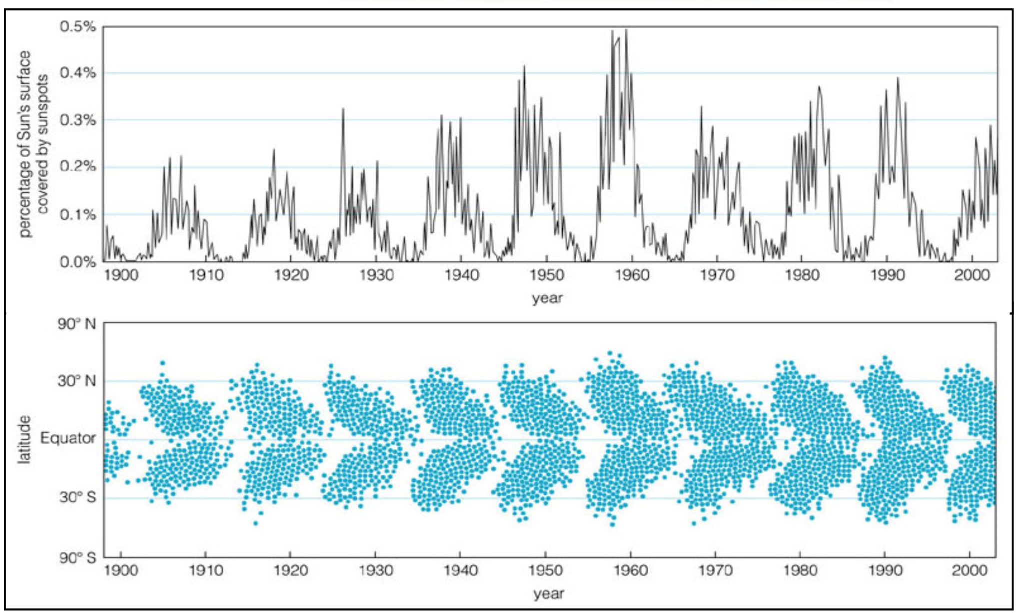
Granulation of solar surface



A solar flare



Coronal mass ejection



Solar boundary is complex – how to define the limb of the Sun at 0.1 picorad (or ~1.5 cm)?


NASA
Deep Space Network
Goldstone, California


Goldstone, California


Canberra, Australia


Madrid, Spain

The effects of the solar atmosphere

- Complications:
  - A free electron gas responds to the variable electric field of a passing EM wave and, for low enough frequencies (< 100 MHz), will absorb it.
  - With refractive index <1, the anisotropy of the atmosphere causes refractive bending and a divergence in the propagating rays.
  - Hence, the focus for a given impact parameter will be shifted to larger distances than expected if the Sun did not have an atmosphere.
- Modeling the solar corona:
  - The plasma contribution to the total deflection angle is related to the change in the optical path $$\Delta\ell = \frac{e^2}{2\pi m_e \nu^2}\int_{\rm path} n_e(t,\vec{r})d\ell$$
  - We decompose the electron density n_e in static, spherically symmetric part $\overline{n}_e(r)$ plus a fluctuation δn_e i.e. $n_e(t,\mathbf{r}) = \overline{n}(r) + \delta n_e(t,\mathbf{r})$
  - We use the, so-called, "Cassini model": $$\overline{n}_e(r) = \Big[\Big(\frac{2.99}{\eta^{16}} + \frac{1.55}{\eta^6}\Big)\times 10^8 + \frac{3.44\times 10^5}{\eta^2}\Big], \quad {\rm cm}^{-3}$$ where $\eta = r/\mathcal{R}_\odot$
  - Deflection due to solar plasma: $$\alpha_{\tt pl}(b,\nu) = \Big(\frac{\nu_0}{\nu}\Big)^2\Big[2.683\times 10^3\Big(\frac{\mathcal{R}_\odot}{b}\Big)^{16} + 2.07\times 10^2\Big(\frac{\mathcal{R}_\odot}{b}\Big)^6 + \Big(\frac{\mathcal{R}_\odot}{b}\Big)^2\Big]$$, with $\nu_0 = 6.62$ MHz

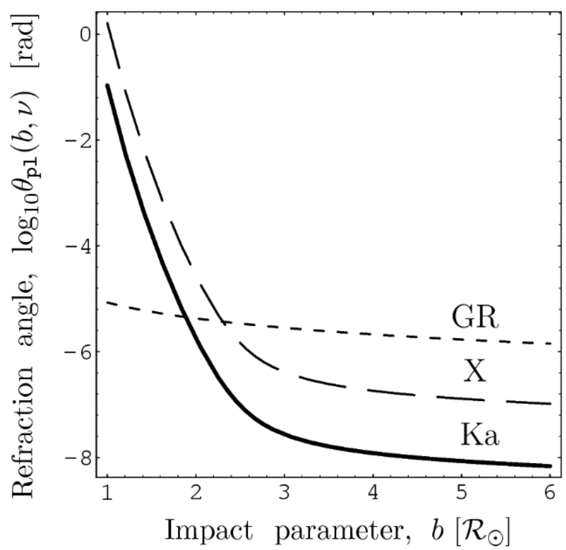
Turyshev & Andersson, MNRAS **341**, 577 (2003)

Effects of plasma on the Solar Lens

- Total deflection: gravity & plasma

$$\alpha_{\rm tot}=\alpha_{\rm GR}-\alpha_{\rm pl}=\frac{2r_g}{b}-\alpha_{\rm pl}$$

  – Note the opposite sign. For observer: gravity bends the ray outwards, plasma inwards, and the different dependence on b, plasma being steeper.



Refraction of radio-waves in the solar atmosphere: the steady-state model, and X- and K-bands radio freqs. The absolute value for GR is also shown.

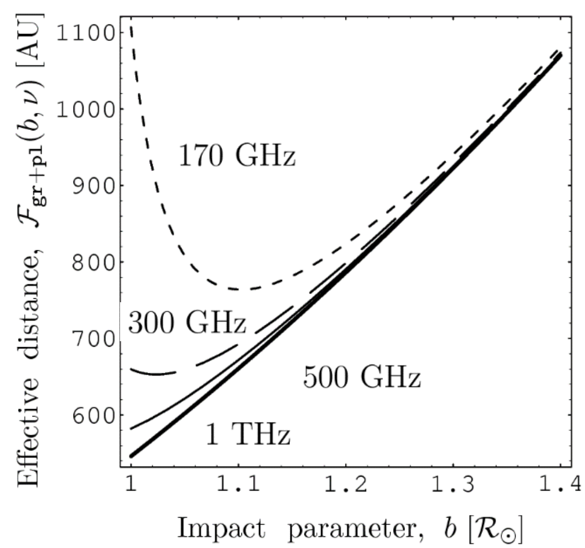
- Moving the interference zone out:

$$\frac{b}{\mathcal{F}_{\tt gr+pl}}=\alpha_{\tt tot}=\frac{2r_g}{b}-\alpha_{\tt pl}$$

  – For impact parameters $b/\mathcal{R}_\odot\in\Big[1.05,\ 1.35\Big]$

$$\mathcal{F}_{\tt gr+pl}(b,\nu)=546\left(\frac{b}{\mathcal{R}_\odot}\right)^2\left[1-\frac{\nu_{0\,\tt crit}^2}{\nu^2}\left(\frac{\mathcal{R}_\odot}{b}\right)^{15}\right]^{-1}\ \mathrm{AU}$$

$$\nu_{0\,\tt crit}\approx 120\ \mathrm{GHz}$$



Effective optical distances for different freqs and impact parameters. From top to bottom: 170 GHz, 300 GHz, 500 GHz, and last 1 THz.

Practical considerations

- Observational considerations:
  - Turbulence in solar corona: is likely of the same order as its average: shortening the observational wavelength. $\delta n_e(t, b) \approx \langle n_e \rangle(b).$
  - To minimize the focal distances, one must consider only frequencies >1 THz.
  - Solar radiation will dominate the detector at most wavelengths, requiring careful selection of the optimum observing frequency and observing strategy.
  - At 550 AU the Sun subtends ~3.5''. At 1mm, the diffraction limited size of an antenna will be large compared to the size of the solar disk: a 10m antenna has a beam size of ~50''. However, a 1m telescope at 1um is ~1.3''.
  - Unless the source to be observed is strong, observations in the mm-wave range will have to be differential in wavelength space and have to require that the source has a different spectrum than the Sun.
- Instrumental impact:
  - This observing strategy imposes severe requirements on the stability of the detector system, to achieve a reliable subtraction of the solar flux.
  - Coronografic observations can be envisioned at short (IR, vis) wavelengths, however, in the IR and optical, emission and/or scattering from the Zodiacal dust will present a challenge for such observations.

Possible science:

- Constraints:
  - The Solar Gravity Lens presents a set of unique challenges
  - We are forced to consider only frequencies >1 THz (0.3 mm).
  - Both the magnification and the plate scale - are very large.
  - Since controlled tangential motion is unlikely, the science has to be restricted to those goals that can be accomplished utilizing the trajectory given by the initial ejection from the solar system.

- Two possible scenarios:
  - Either pre-selected specific targets and require navigation to place the spacecraft at the focus corresponding to those specific target or utilize the large amplification provided by the gravity lensing to observe "random" parts of the sky to great depth.
  - The source selection for this telescope is limited to sources that are small, but with interesting smaller-scale variations which can be explored with one-dimensional mapping.

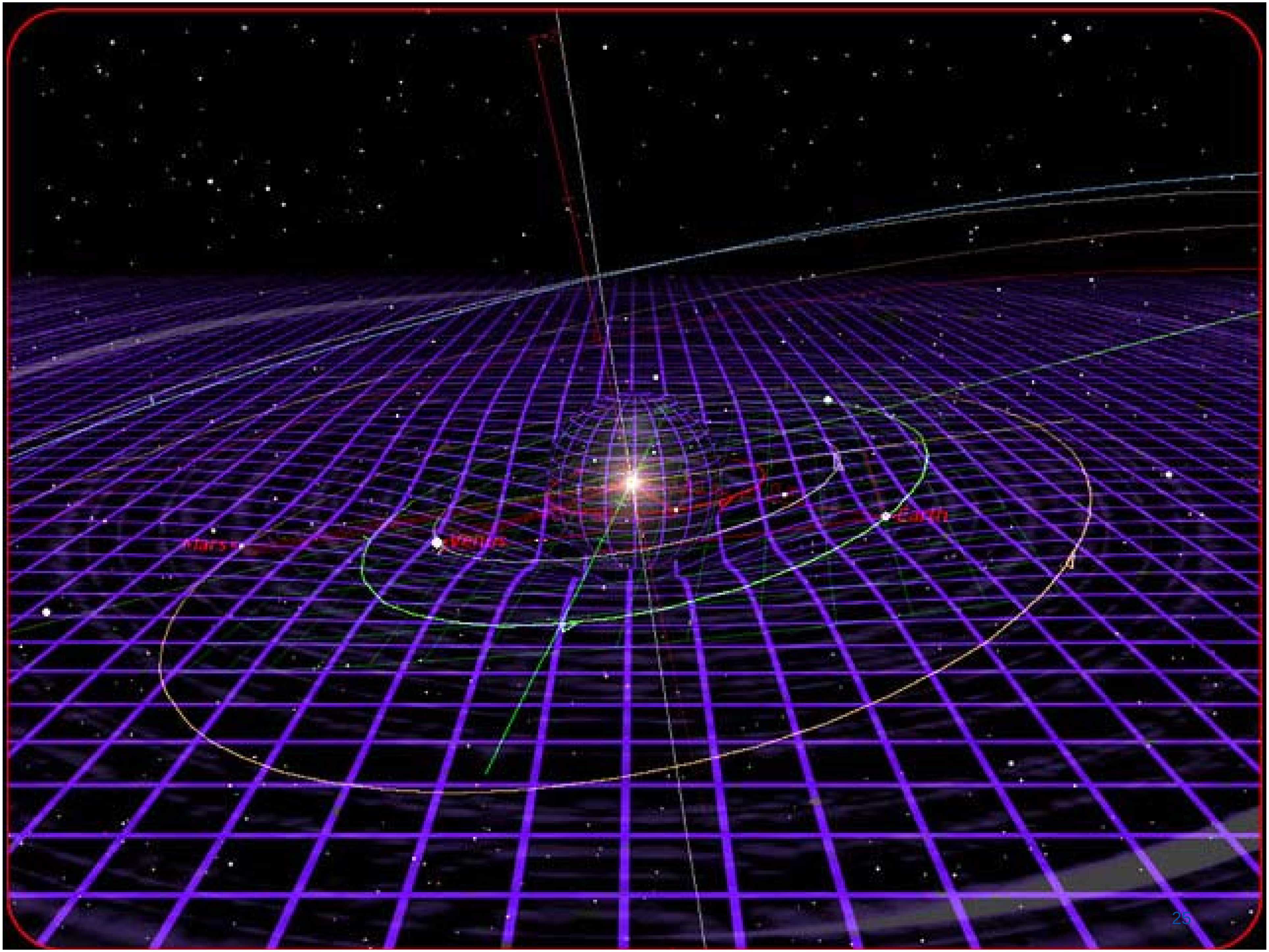
Possible science:

- Potential targets may include
  - Pre-selected targets: imaging of exoplanets (license plate numbers…), accretion disks around BH, compact stars, the cores of AGNs, etc.
  - An observing scenario is to allow the tangential component of the trajectory velocity to sweep the s/c across the image of the source to gather a one dimensional map. The number of targets in this mode is small as acquisition of secondary targets will require large s/c motions.
  - Sweeping offers a promising approach given that suitable high surface density structure/targets can be identified. Utilizing the high signal amplification deep surveys of small scale structure could be performed.
  - Target classes have to be uniformly distributed over the sky contain small scale structure and be of low enough luminosity. Most likely targets are cosmology or galaxy cluster studies.
  - A possibility might include structures in the emission from the cosmic web. As the majority of the baryonic mass changes between the present time and $z>1$ from gas contained in a hot phase to warm gas. For a flat universe and $H0=60$ corresponds to an angular size of 1", a scale well suited for mapping with Solar Gravitational Lens.

Summary: Gravitational Lens of the Sun

- The inner Solar Gravitational Lens: 25 AU outwards
  - Most of the solar mass is concentrated in its core (radius is 0.25 Rsun). Over 95% of the solar mass is concentrated in the solar core, which is inaccessible to photons; however, it can be reached by extrasolar neutrinos.
  - Foci is 16 times shorter than for photons ~ 25 AU
  - We do not have much statistics on extrasolar neutrinos
  - Detector technology need improvements…. but worth thinking about this.
  - Should there be a well-known neutrino source, such as AGNs, supernovae, etc., this may be of interest to a wider scientific community

- The outer Gravitational Lens: 550 AU outwards
  - Challenging, but exciting possibilities relying on Gain of 110dB (at 1 um)
  - Even a confirmation would be great
  - Pre-selected, well-defined in advance specific targets and require navigation to place the spacecraft at the foci corresponding to those targets (exoplanets);
  - Choose to observe "random" parts of the sky to great depth. Potential targets may include accretion disks around black holes, compact stars or the cores of AGNs, small-scale structure, deep fields...



Mars
Venus
Earth