How do we study a planetary interior with gravity and topography?

- We study the interior but looking at its response to various forcings such as:
  - Rotation
  - Tides
  - Surface loads
  - Subsurface loads

Hydrostatic equilibrium

- In hydrostatic equilibrium
  - Surfaces of constant density, pressure and potential coincide
  - No shear stresses

Hydrostatic equilibrium

- In hydrostatic equilibrium

Hydrostatic equilibrium

- In hydrostatic equilibrium

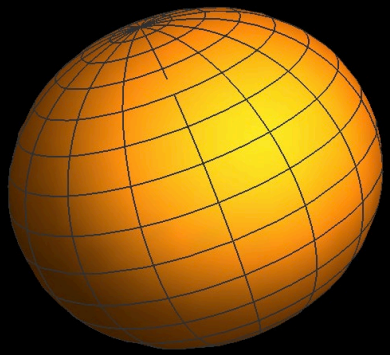
$\rho = \rho(r)$, ω

Hydrostatic equilibrium

- **In** hydrostatic equilibrium

easy

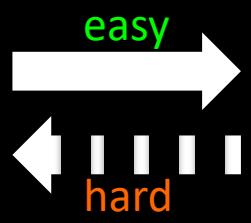
$\rho = \rho(r)$, ω

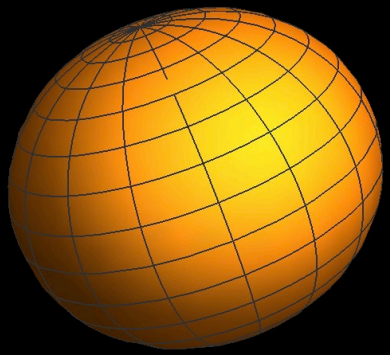


Hydrostatic equilibrium

- In hydrostatic equilibrium

$\rho = \rho(r)$, ω

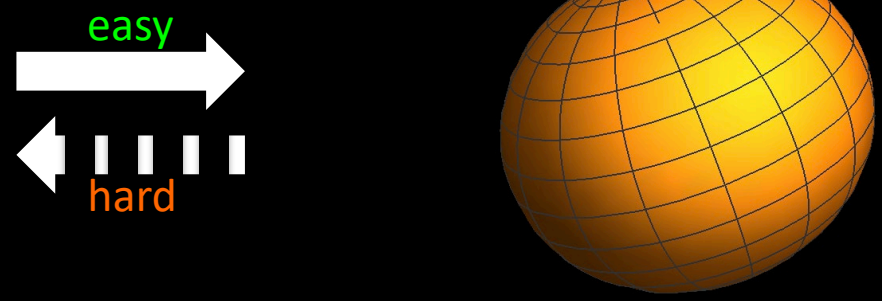




Hydrostatic equilibrium

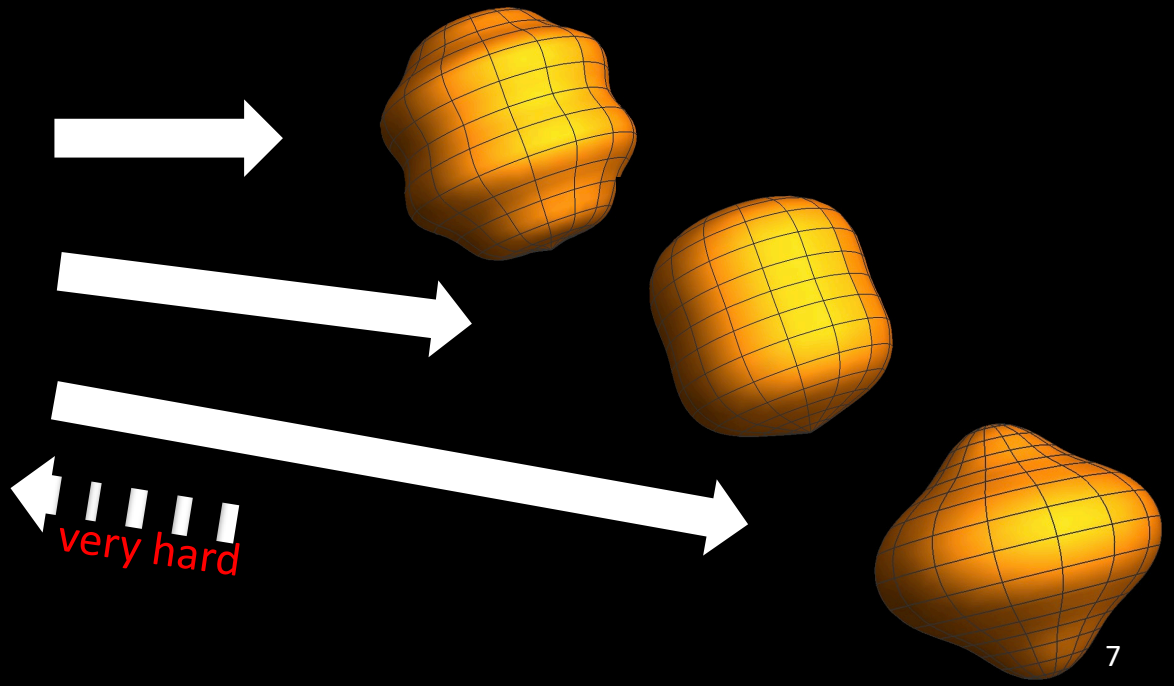
- **In** hydrostatic equilibrium

$\rho = \rho(r), \omega$



- **Not in** hydrostatic equilibrium

$\rho = \rho(r), \omega$

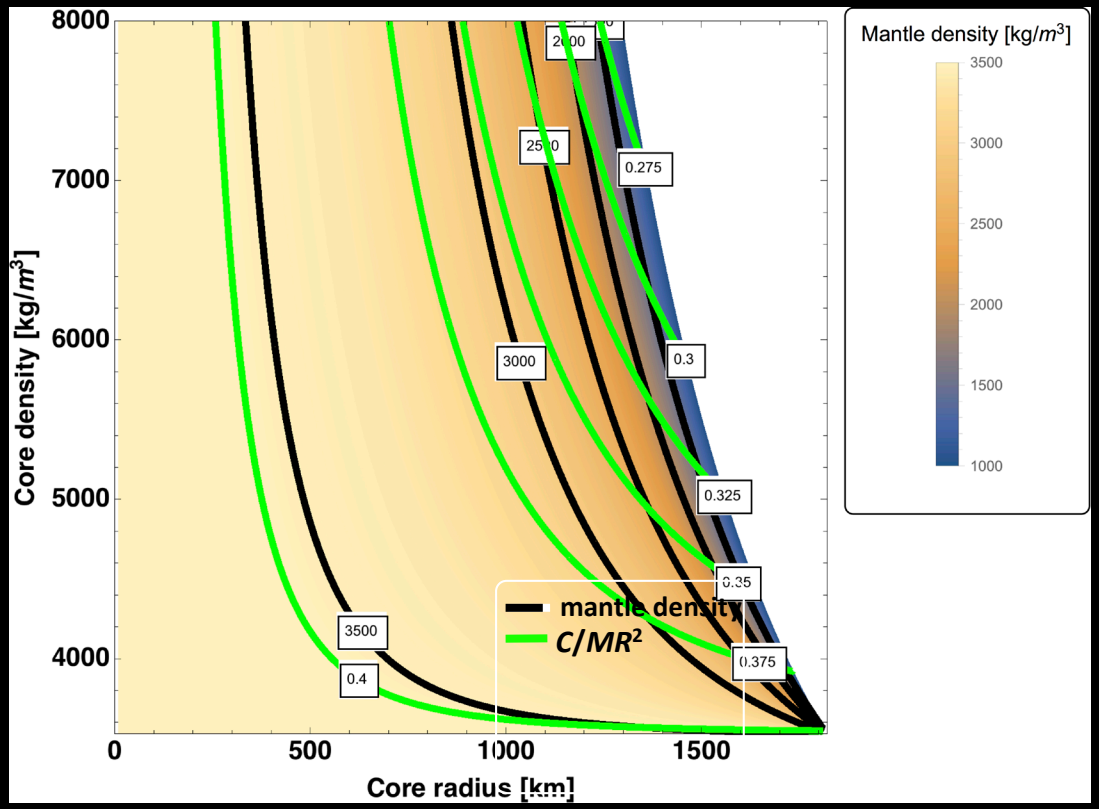


Is hydrostatic equilibrium a good assumption?

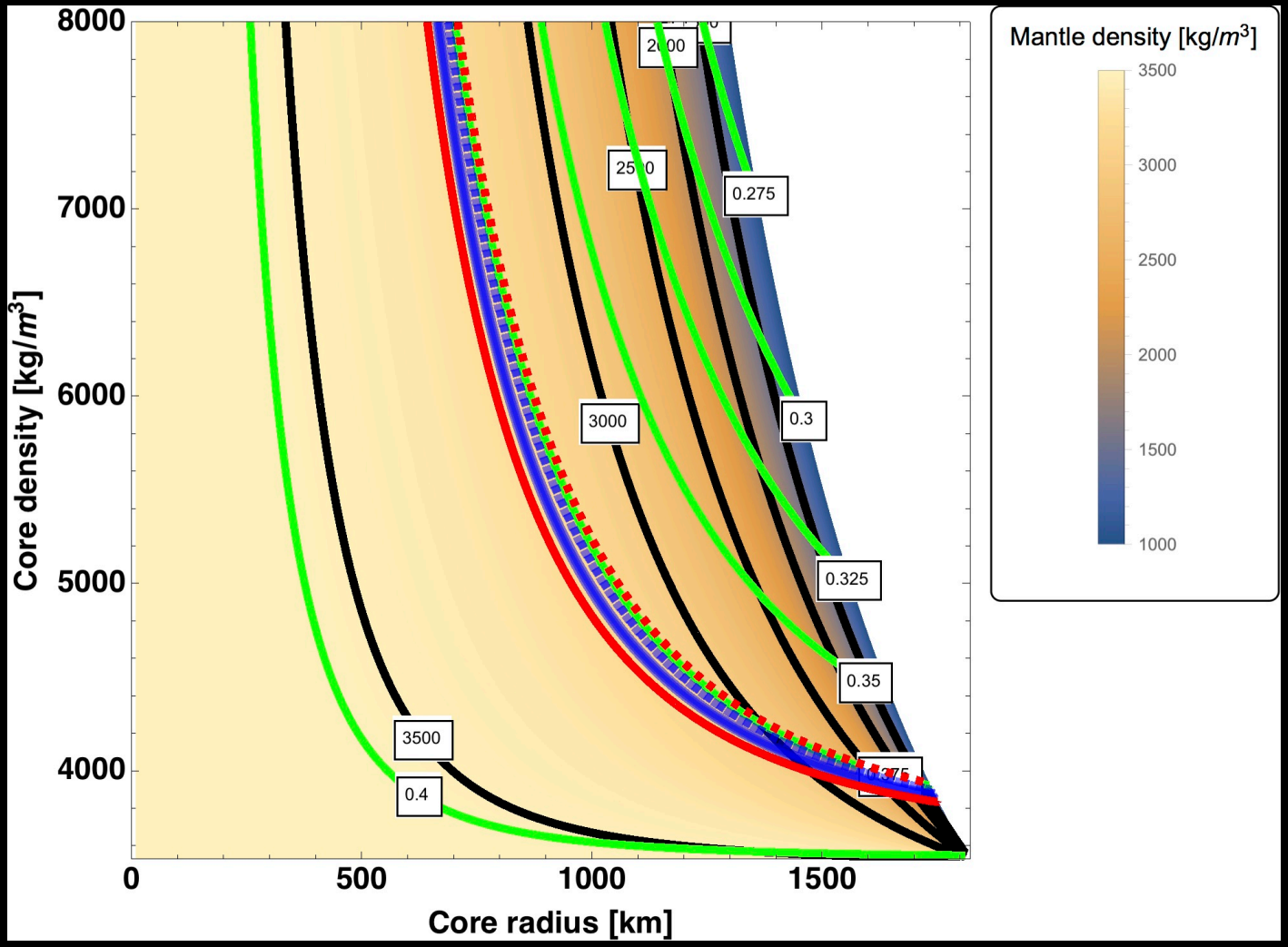
- **Need to estimate the magnitude of non-hydrostatic *effects* to the magnitude of hydrostatic effects.**
- **To separate non-hydrostatic effects, typically need to assume an internal structure.**
- **If internal structure is not known.**
- **Measuring gravity-topography admittance can provide an additional constraint.**

Two-layer model

- Gravity and shape provide two solutions (solution families) on the internal structure
- In hydrostatic equilibrium, the two solutions should be identical.
  - Should give same moment of inertia

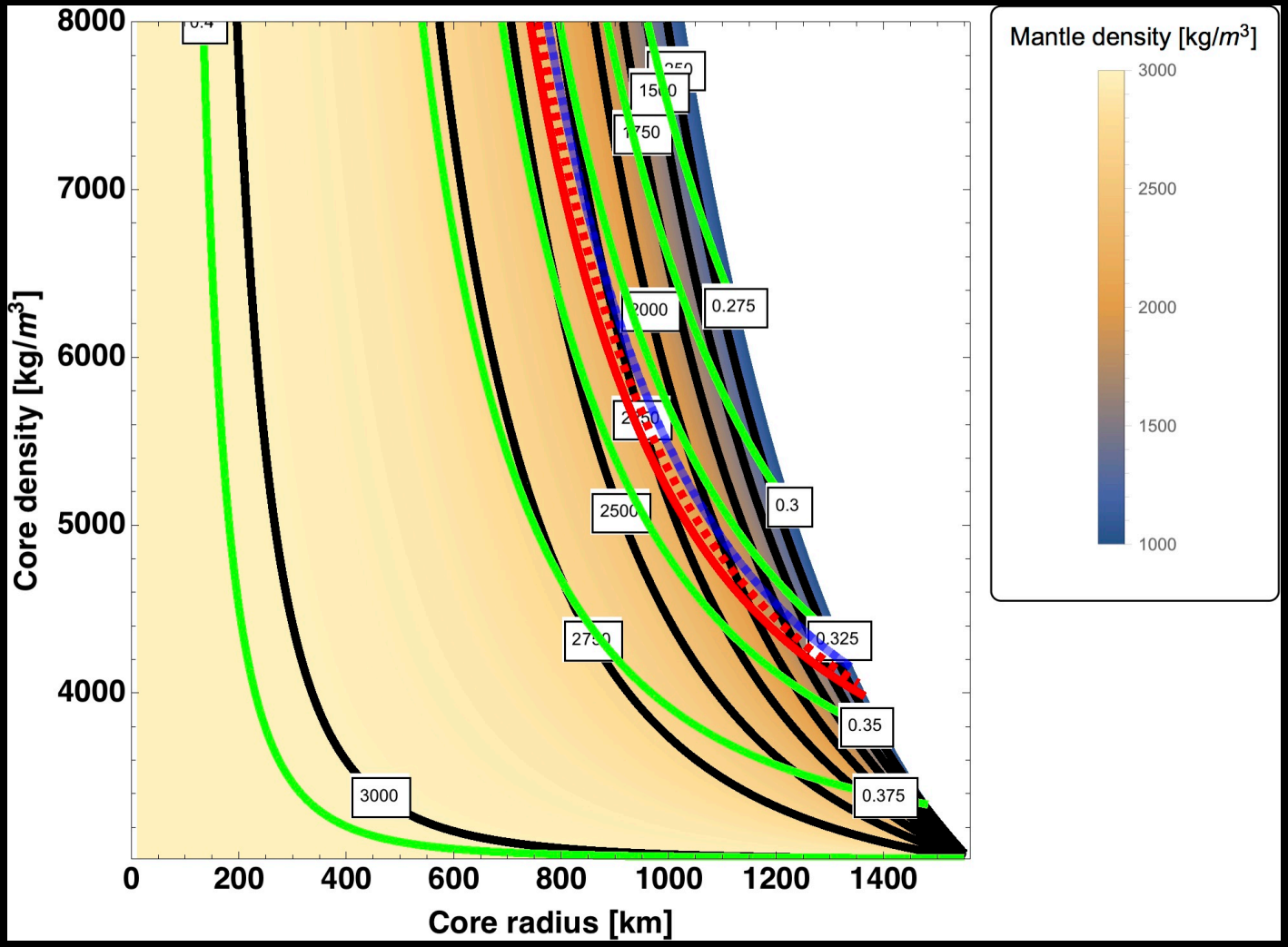


Io

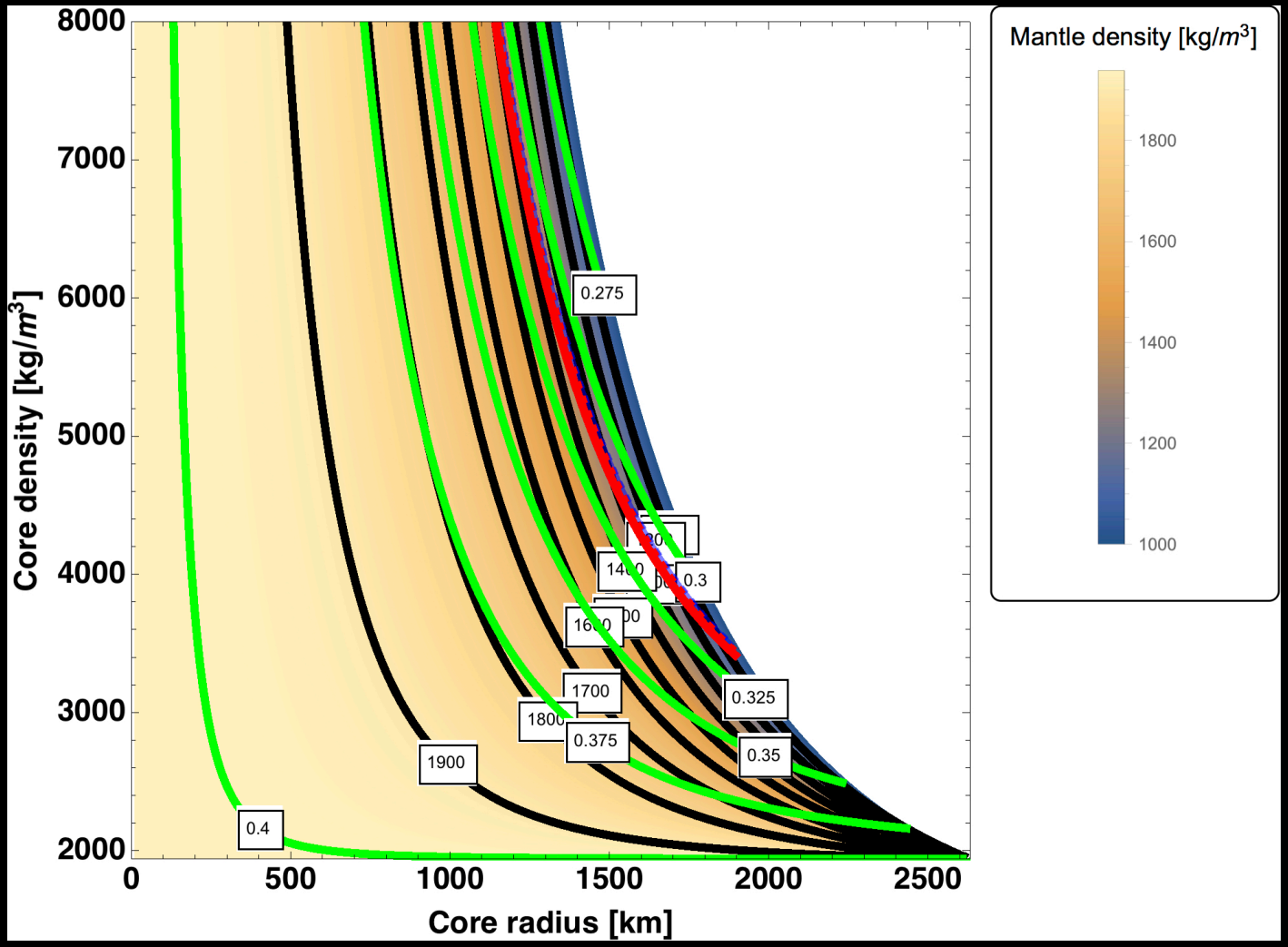


Anderson et al., 1996; Thomas et al., 1998

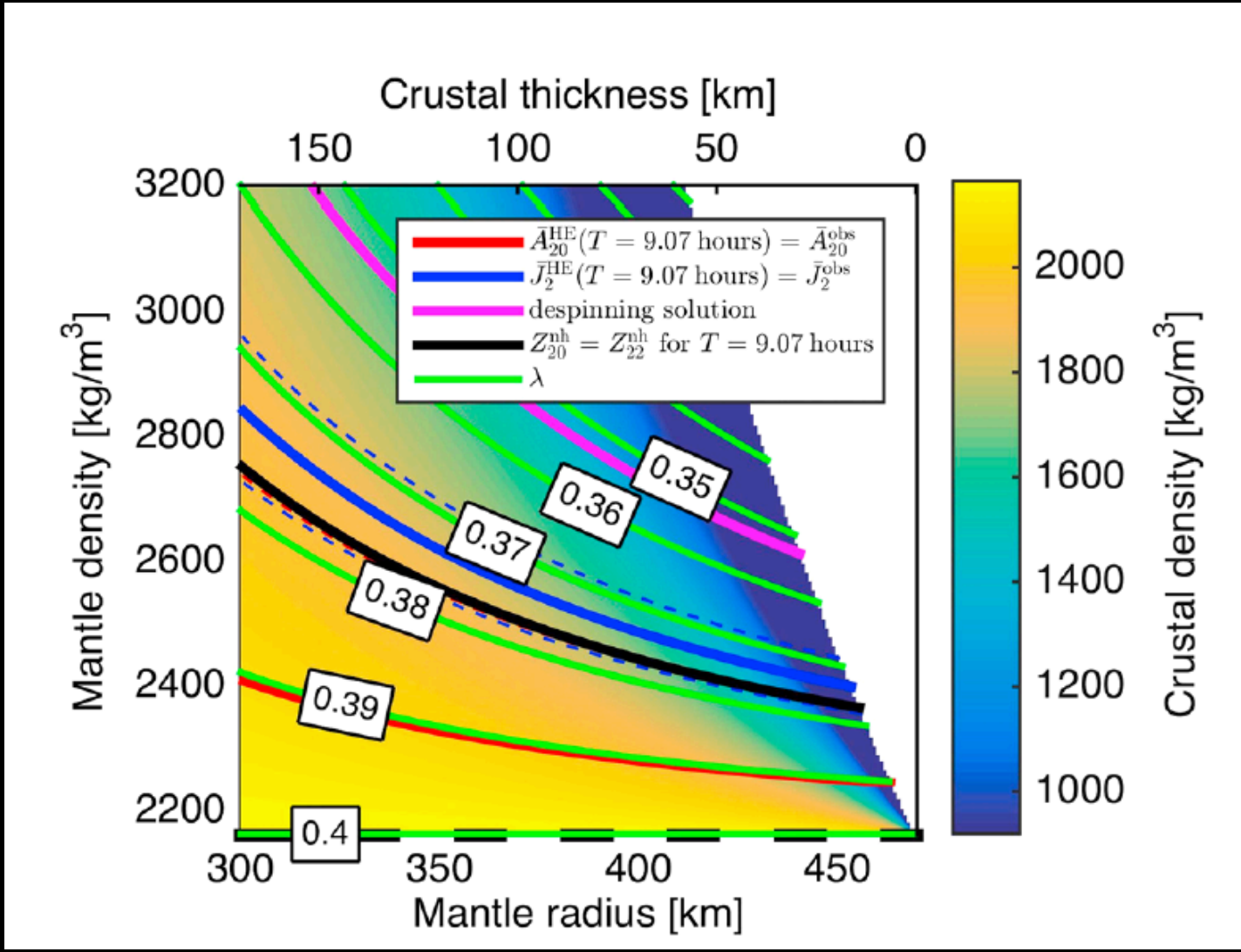
Europa


Mantle density [kg/m^3]
3000
2500
2000
1500
1000
Core density [kg/m^3]
8000
7000
6000
5000
4000
0
200
400
600
800
1000
1200
1400
Core radius [km]
0.275
0.3
0.325
0.35
0.375
3000

Ganymede


Mantle density [kg/m^3]
1800
1600
1400
1200
1000
Core density [kg/m^3]
8000
7000
6000
5000
4000
3000
2000
0
500
1000
1500
2000
2500
Core radius [km]
0.275
0.3
0.325
0.35
0.375
0.4
1700
1900

Ceres



Ermakov et al., 2017

Libration

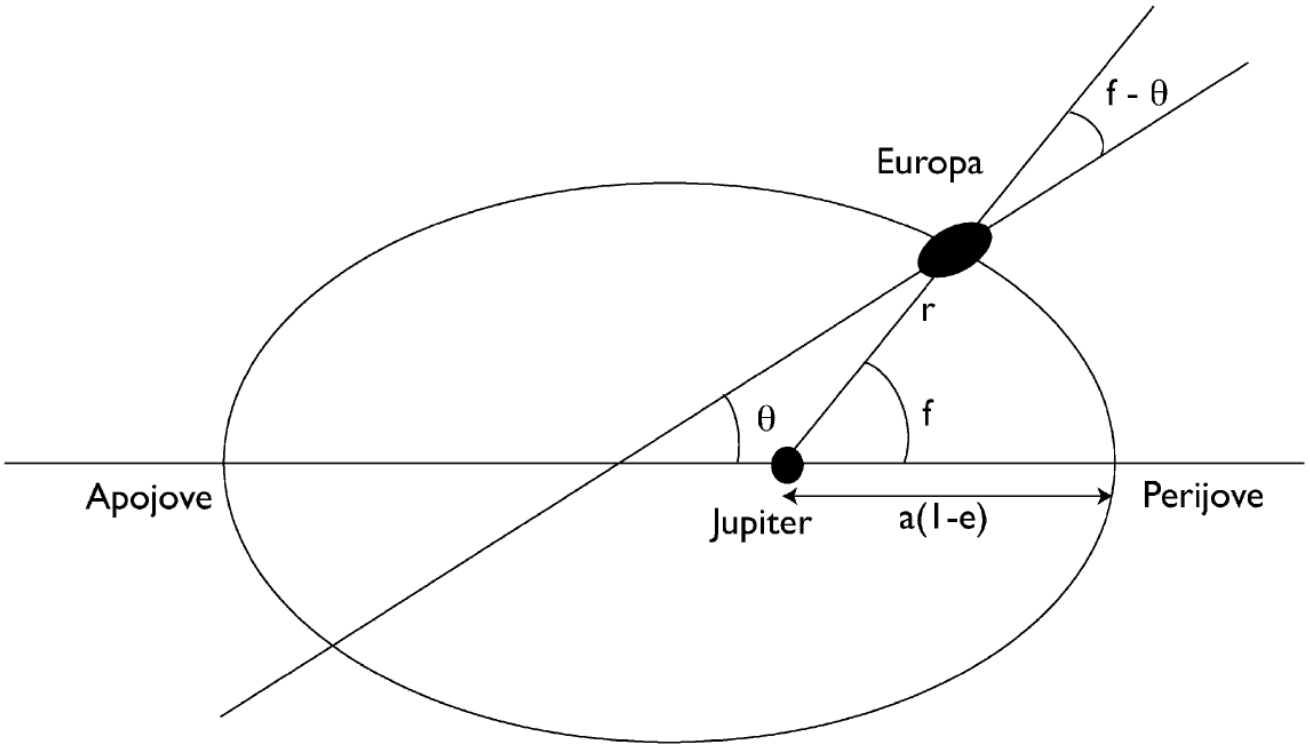


Fig. 1. Geometry of Europa's libration. The long axis of Europa makes an angle θ with the major axis of the orbit and f is the true anomaly.

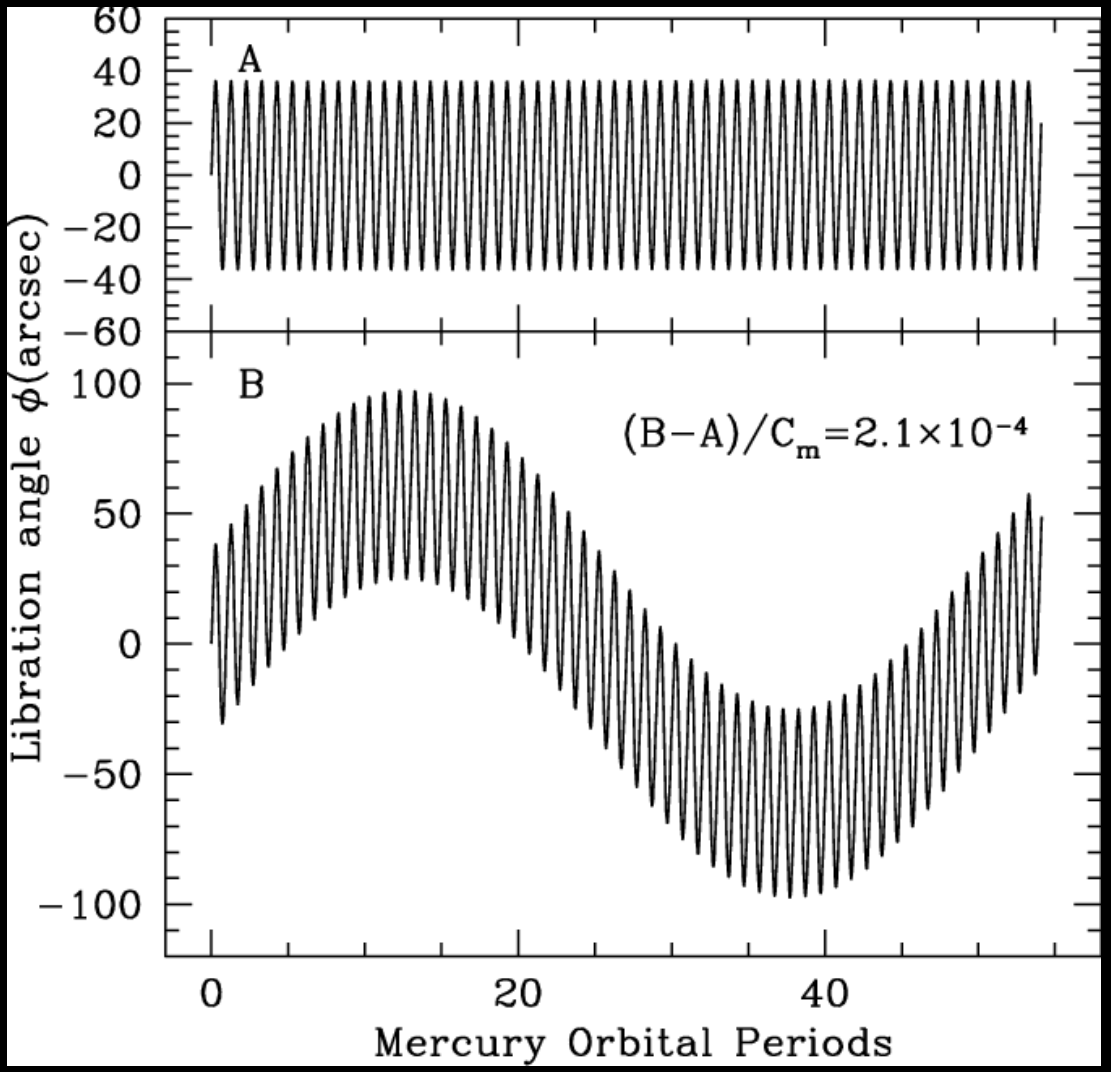
Van Hoolst et al., 2008

Libration

- **Optical**
- **Free**
- **Physical**

Libration

- Optical
- **Free**
- Physical



Zuber et al., 2007

Libration

- Optical

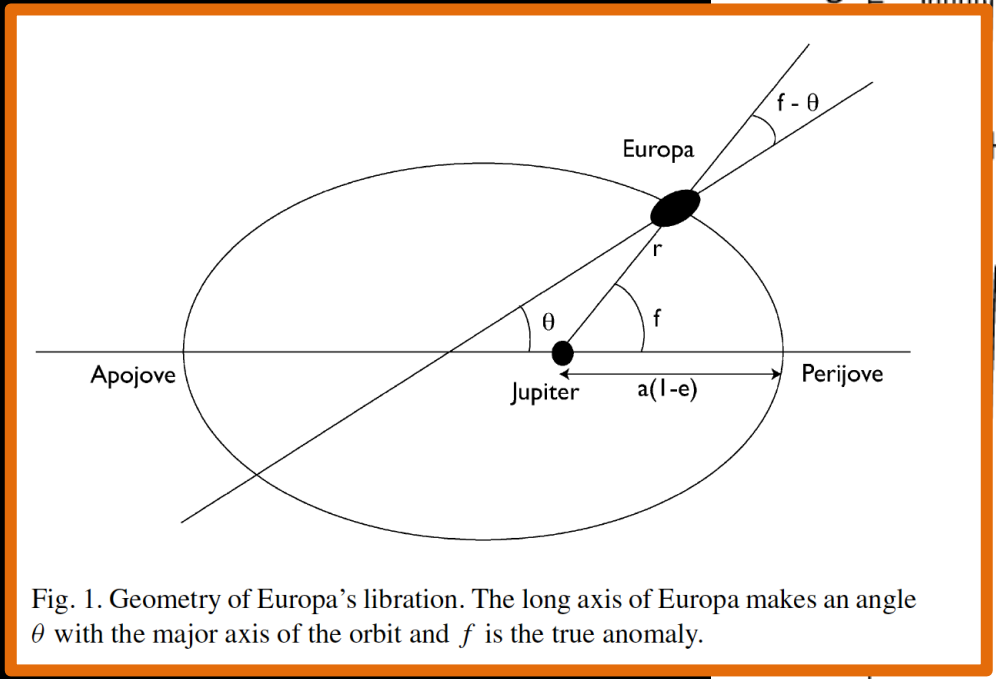
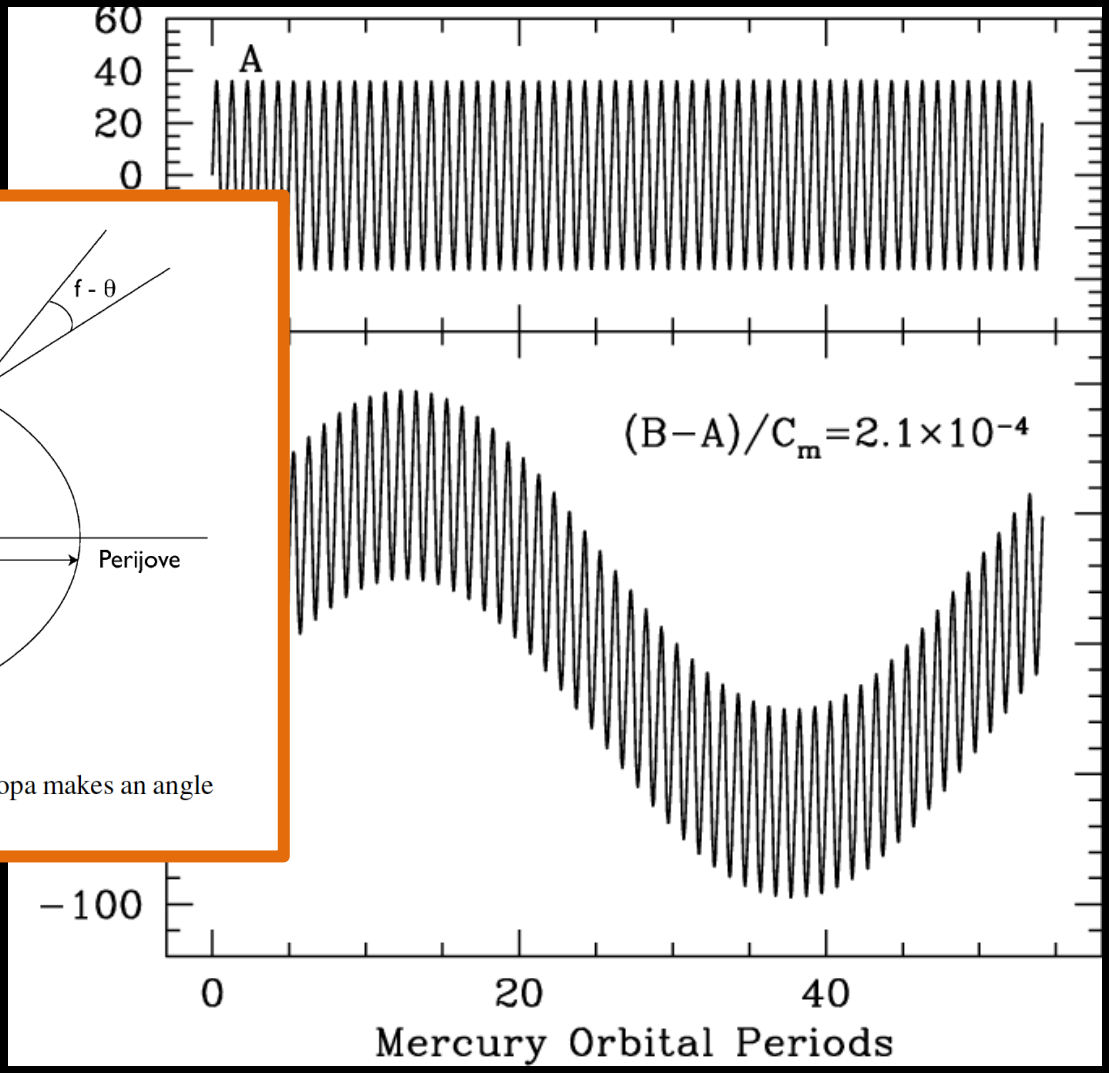


Fig. 1. Geometry of Europa's libration. The long axis of Europa makes an angle θ with the major axis of the orbit and f is the true anomaly.



Zuber et al., 2007

Physical libration: effect of liquid layers

- When the shell is out of alignment with the solid interior, it exerts a torque on the solid interior.
- The ocean also exerts a torque on the solid interior.
- Both core and shell have their own free libration frequencies

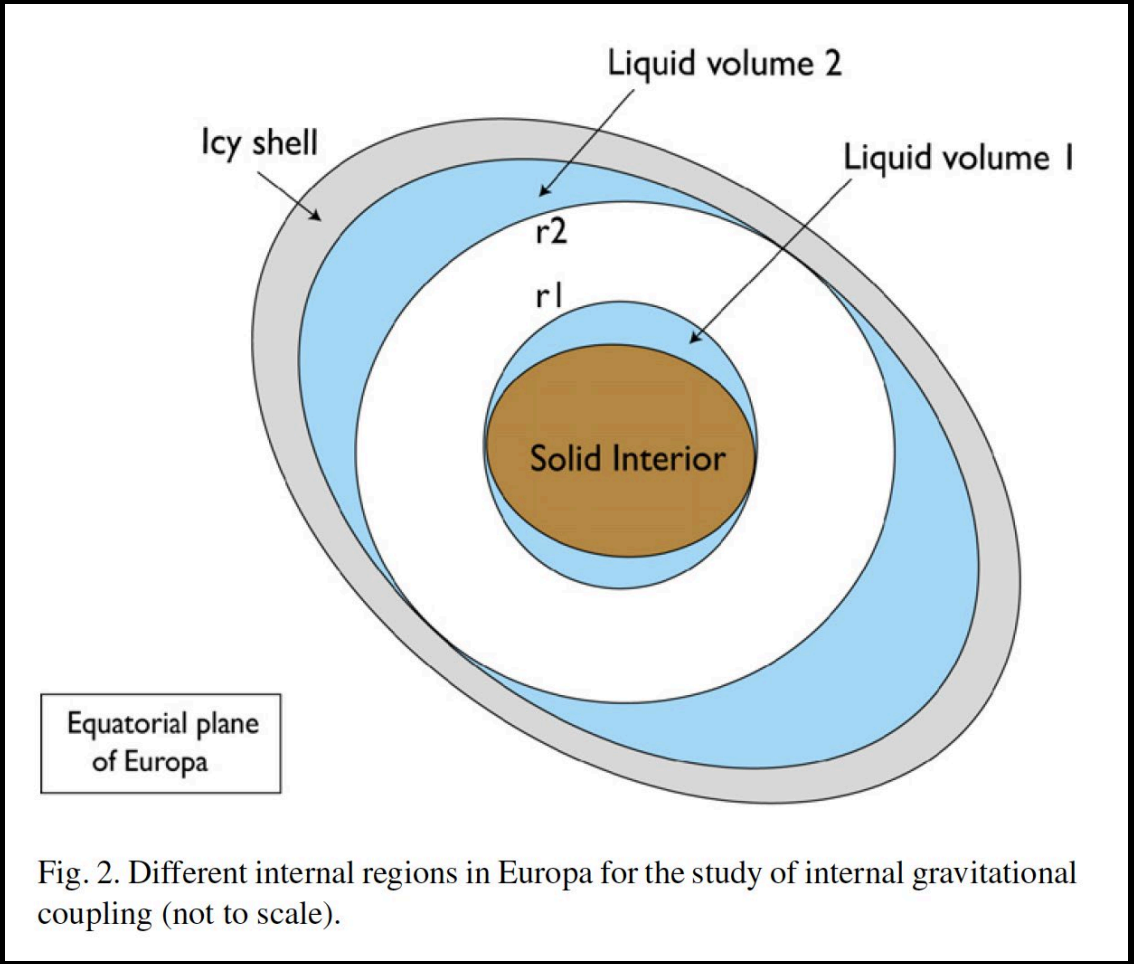


Fig. 2. Different internal regions in Europa for the study of internal gravitational coupling (not to scale).

Van Hoolst et al., 2008

Physical libration: effect of liquid layers

Rigid body

$$A_\theta = -\frac{2\omega_f^2 e}{n^2 - \omega_f^2} = -6\frac{(B-A)}{C}\frac{n^2 e}{n^2 - \omega_f^2}.$$

$$\omega_f = n\sqrt{\frac{3(B-A)}{C}}$$

decoupled shell

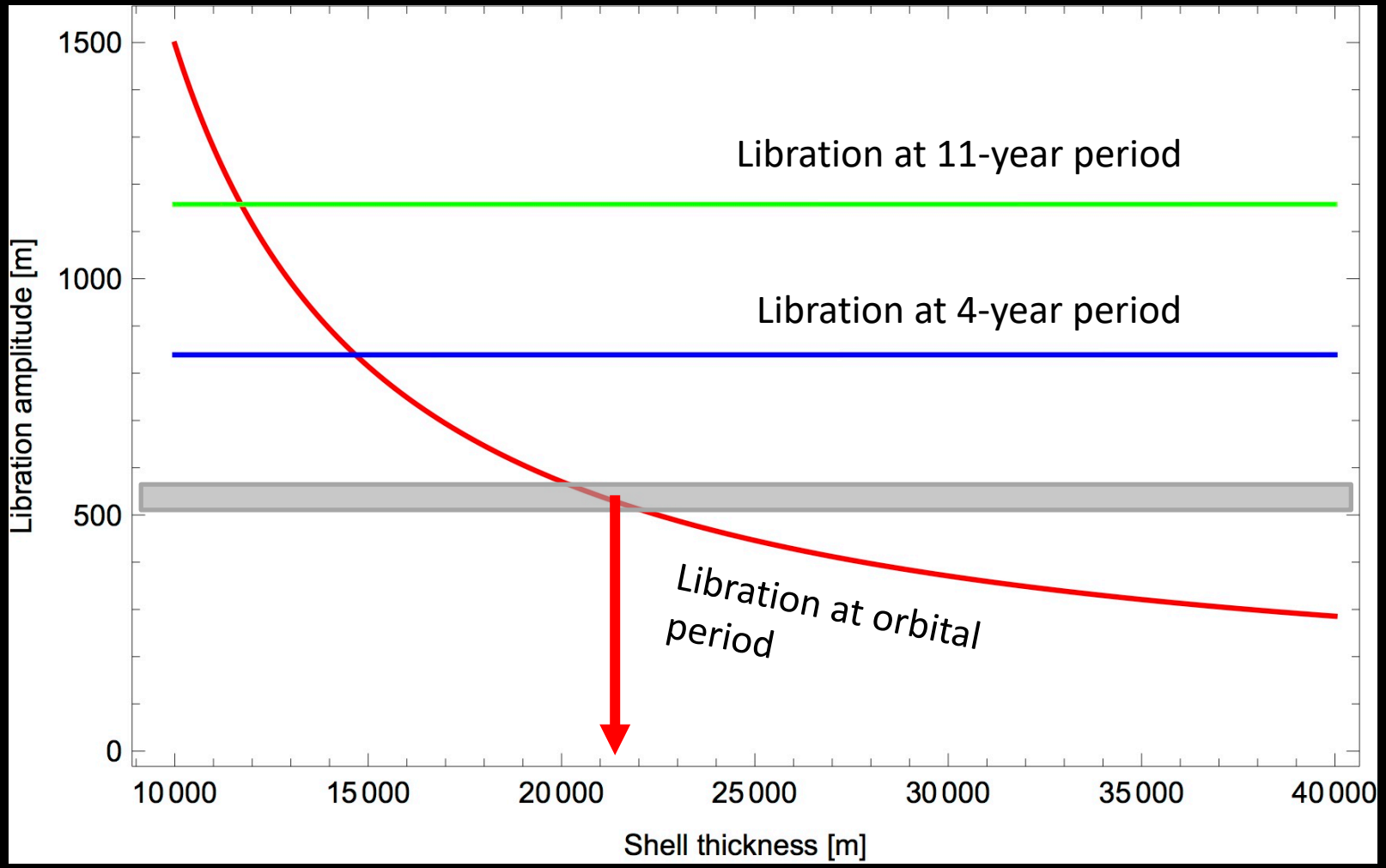
$$A_{\theta_s} = -\frac{2\omega_{fs}^2 e}{n^2 - \omega_{fs}^2} = -6\frac{(B-A)}{C_s}\frac{n^2 e}{n^2 - \omega_{fs}^2},$$

where the free shell libration frequency ω_{fs} is given by

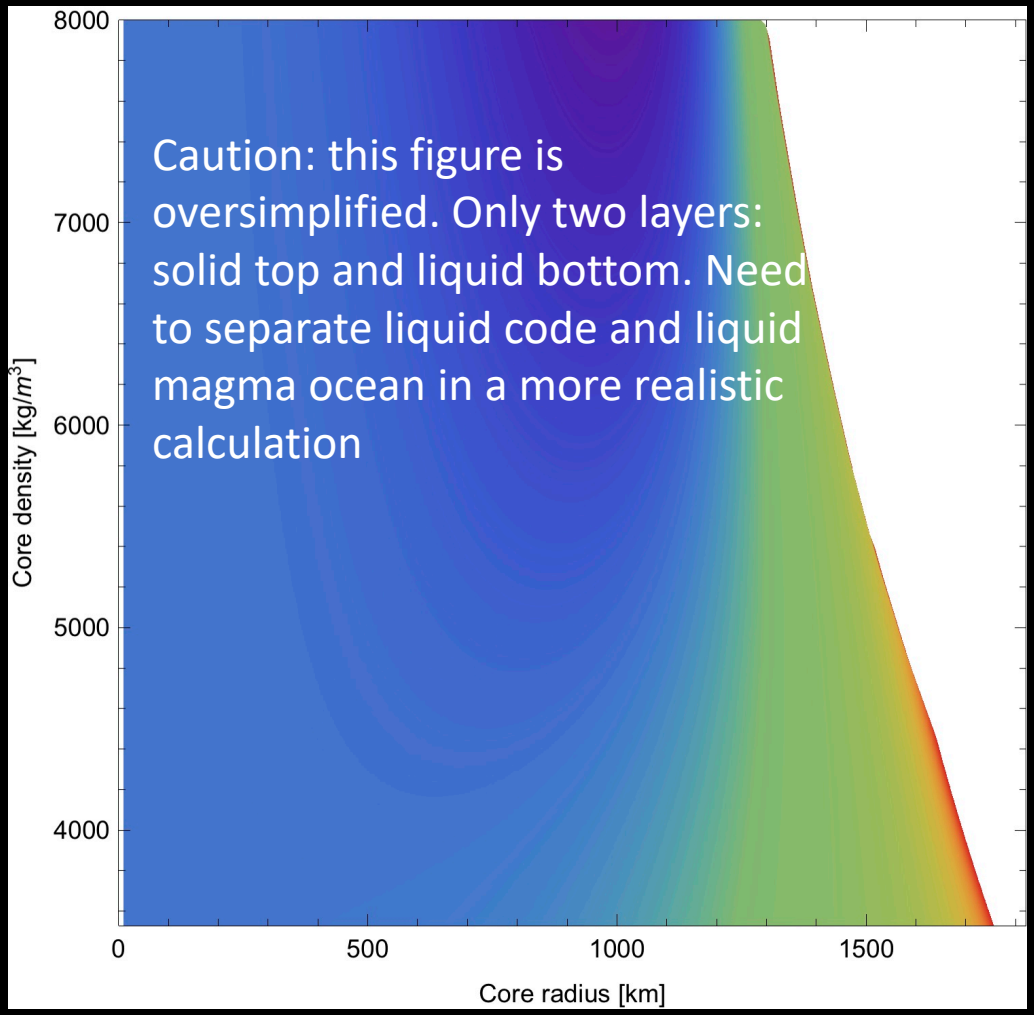
$$\omega_{fs} = n\sqrt{\frac{3(B-A)}{C_s}} = \omega_f\sqrt{\frac{C}{C_s}}.$$

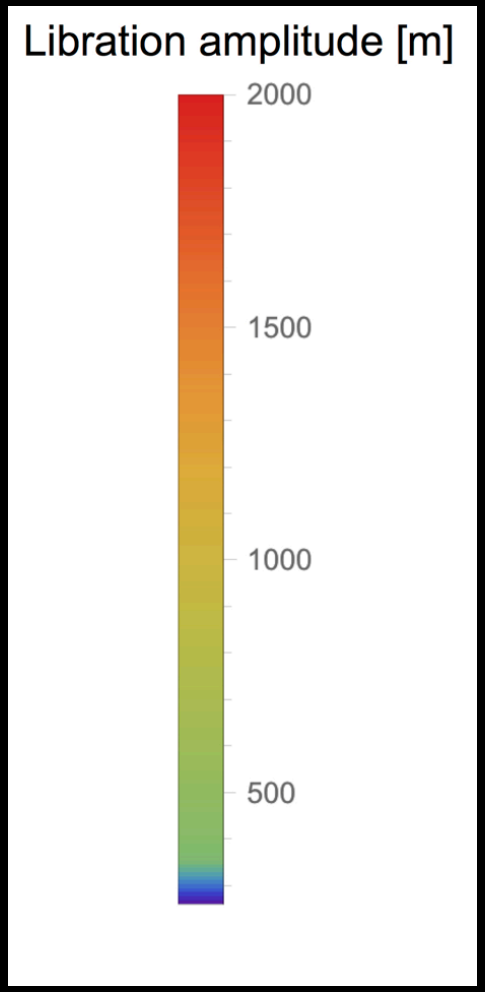
Van Hoolst et al., 2008

Physical libration of Enceladus



Physical libration of Io (with liquid core and solid shell)





Conclusions

- **Hydrostatic equilibrium provides a constraint for modeling internal structure.**
- **Libration is diagnostic for decoupled shells.**

Caveats

- **Libration is coupled to tides. There is a torque acting on the tidal bulge. Libration becomes dependent on the Love numbers. Need to solve for tides and libration simultaneously.**
- **Typically, hydrostatic equilibrium is assumed to compute the shape of the internal surfaces. Non-hydrostatic effects can affect libration.**