Linking Tidal, Rotational, and Orbital Evolution

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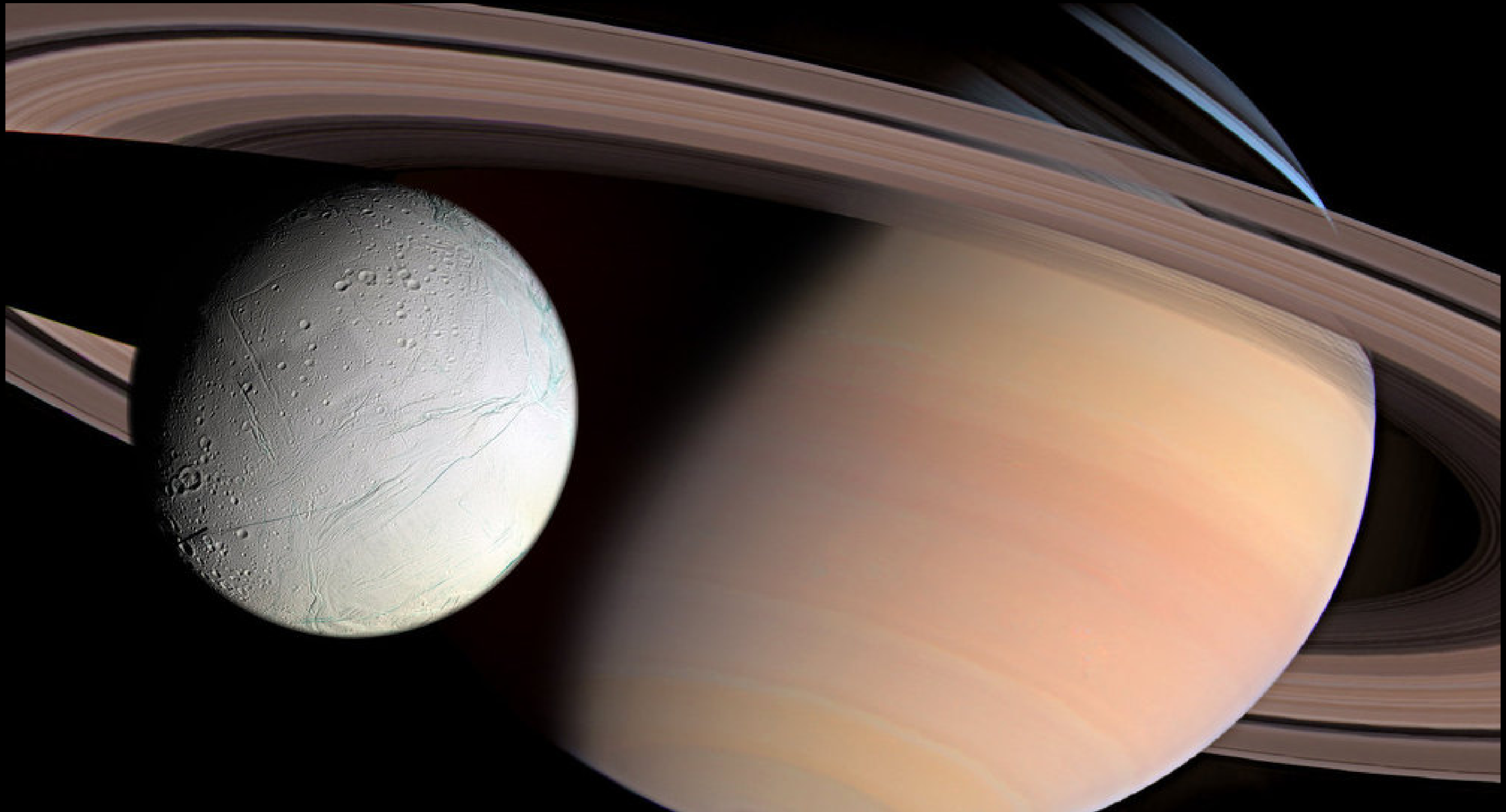
Caltech



Pop Quiz!

What is the energy source for Enceladus’ geysers?

- Saturn’s rotational energy
- Enceladus’s orbital energy
- Elastic energy stored in Enceladus’s tidal deformation
- Thermal energy in Enceladus’ ocean

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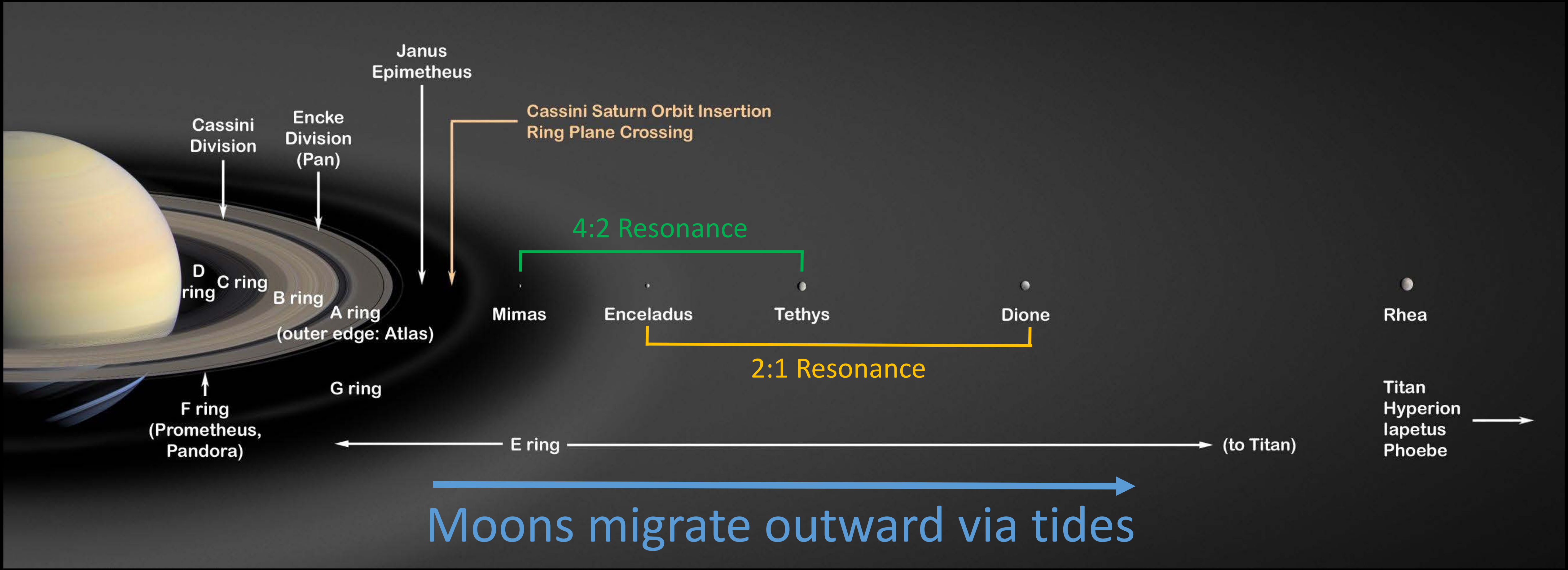
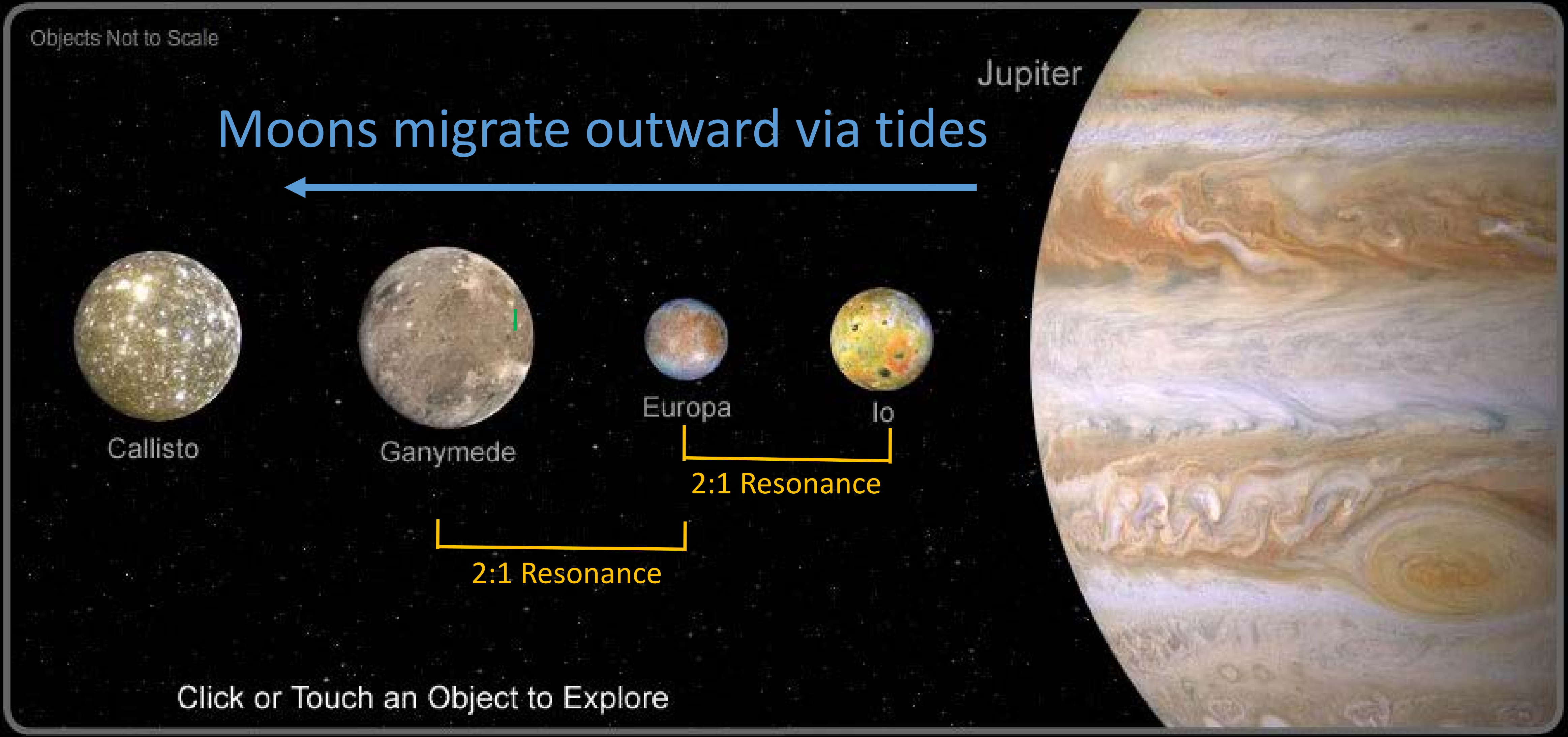
- All of the above

Orbital Architecture of Saturn System



Orbital Architecture of Jovian System



Tidal Basics

- Tidal Potential is

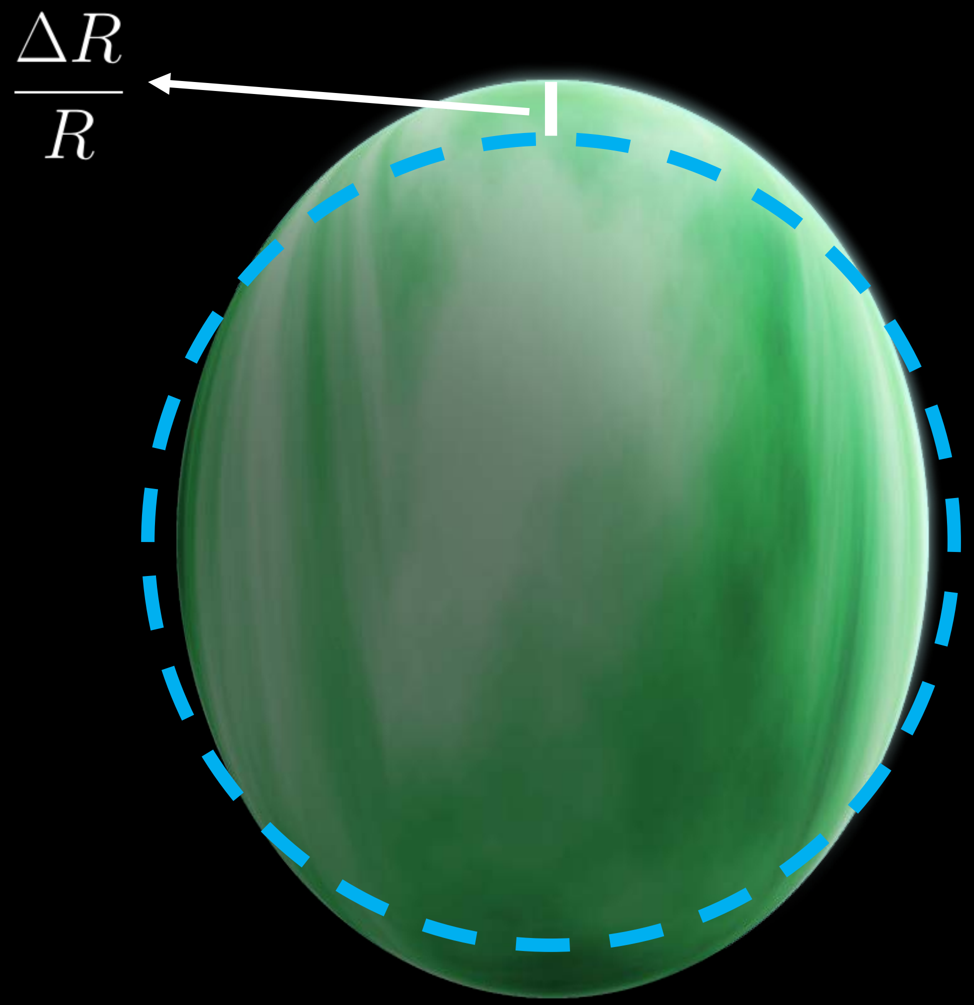
$$U_{\text{tide}} \approx \frac{GM}{R}\frac{M'}{M}\left(\frac{R}{a}\right)^3$$

- Tidal Displacement is

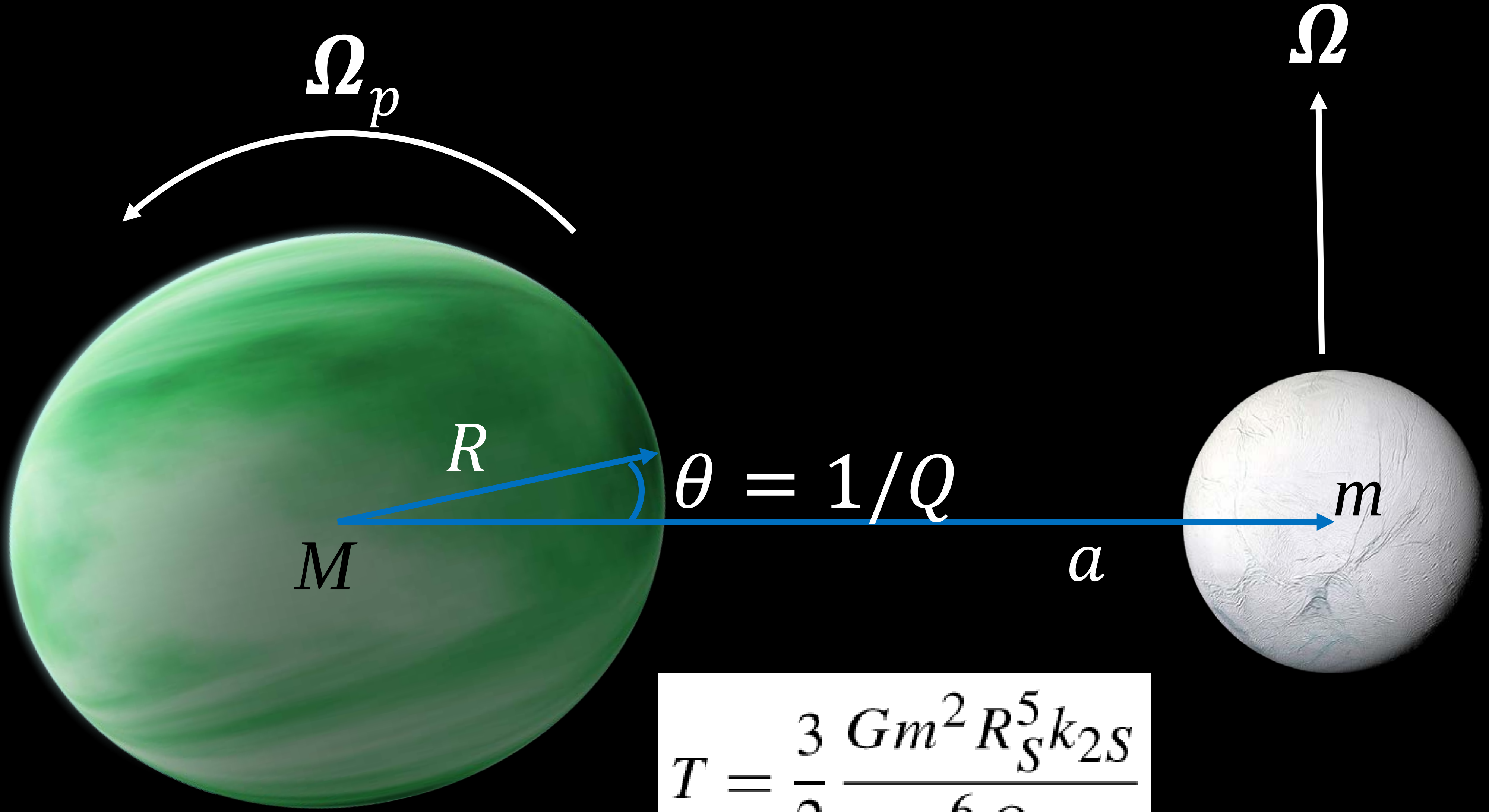
$$\frac{\Delta R}{R} \approx \frac{U_{\text{tide}}}{gR} \approx \frac{M'}{M}\left(\frac{R}{a}\right)^3$$

- Energy in tidal bulge is

$$E_{\text{tide}} \approx \frac{U_{\text{tide}} M \Delta R}{R} \approx \frac{GM^2}{R}\left(\frac{M'}{M}\right)^2\left(\frac{R}{a}\right)^6$$



Classical Tidal Theory



$$T = \frac{3}{2}\frac{Gm^2 R_S^5 k_{2S}}{a^6 Q_S}$$

Tidal Torques

Mind your T's and Q's

- Define migration time scale

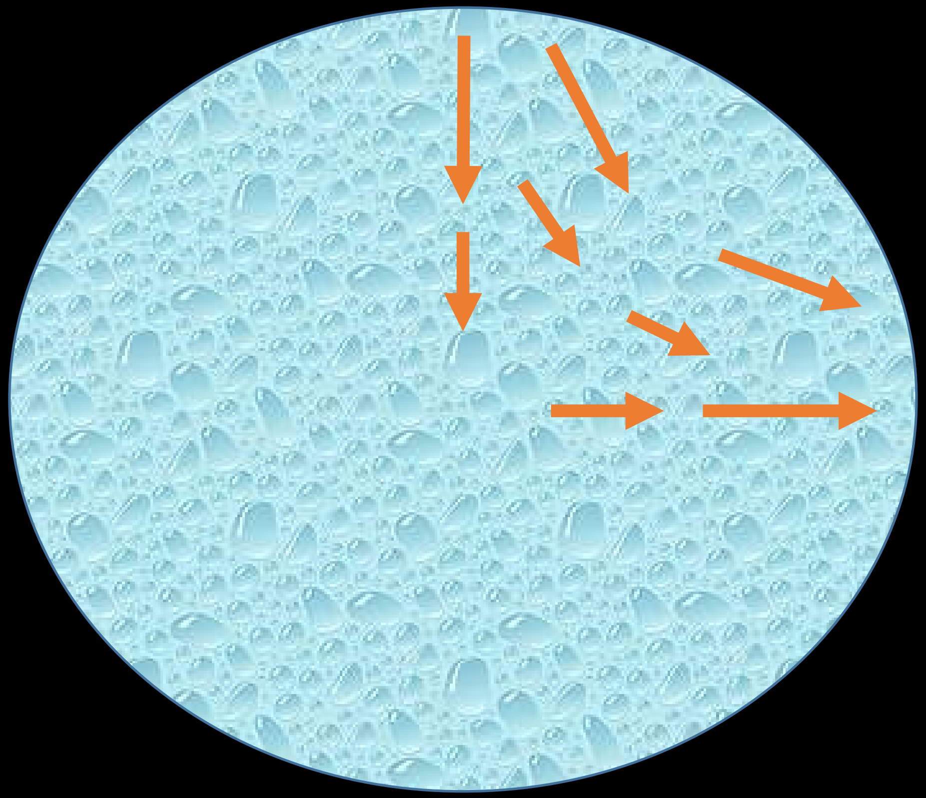
$$t_{\rm tide} = -\frac{E_{\rm orb}}{\dot{E}_{\rm tide}} = \frac{a}{\dot{a}_{\rm tide}}$$

- And effective tidal *Q* can be defined as

$$Q \equiv 3k_2 \frac{M_{\rm m}}{M_{\rm p}} \left(\frac{R_{\rm p}}{a_{\rm m}}\right)^5 \Omega_{\rm m} t_{\rm tide}$$

Equilibrium Tides

- Tidal bulge raised by gravity of companion
- Friction acts on shear associated with bulge
  - Turbulent viscosity in convective envelope
  - Viscoelasticity of solid core
- Energy is dissipated, producing tidal torque

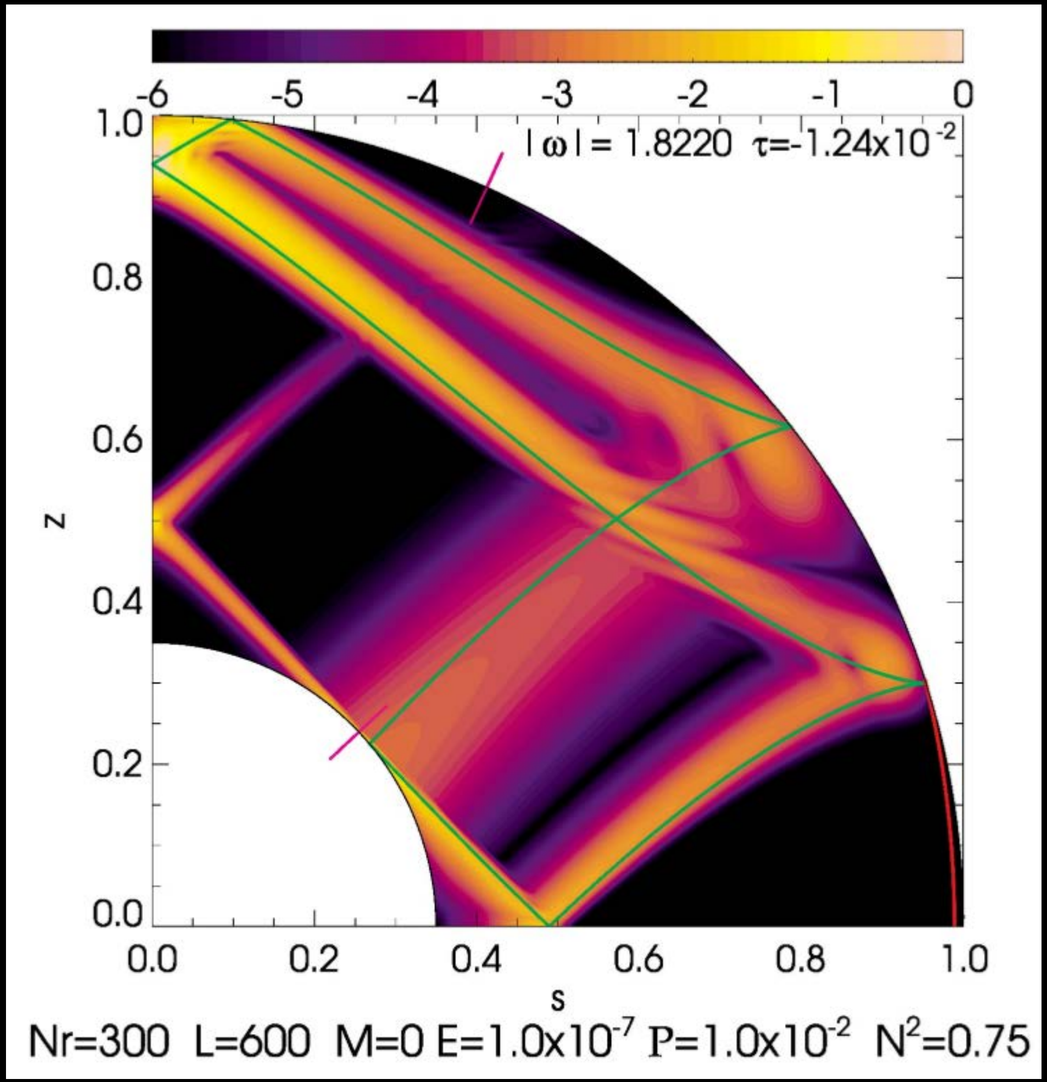


$$\dot{E}_{\rm eq} \simeq \frac{\nu_{\rm turb}}{R^2} E_{\rm tide,eq}$$

Dynamical Tides

- Dynamical Tide
  - Waves excited by gravitational forcing of companion
  - Friction dissipates waves
  - Energy is dissipated, producing tidal torque

$$\dot{E}_{\rm dyn} \simeq k^2 \nu_{\rm eff} E_{\rm tide,dyn}$$

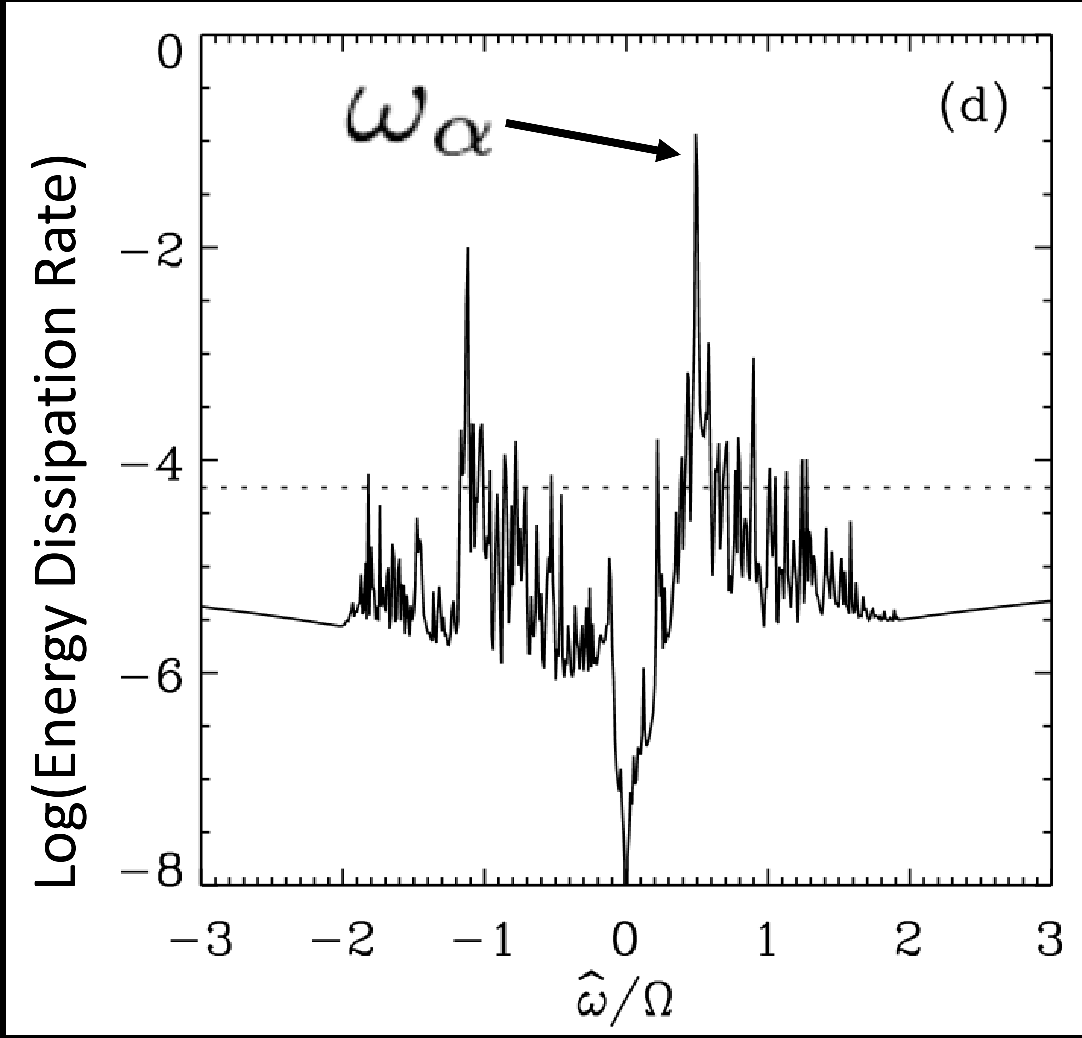


Mirouh et al. 2015

Dynamical Tides

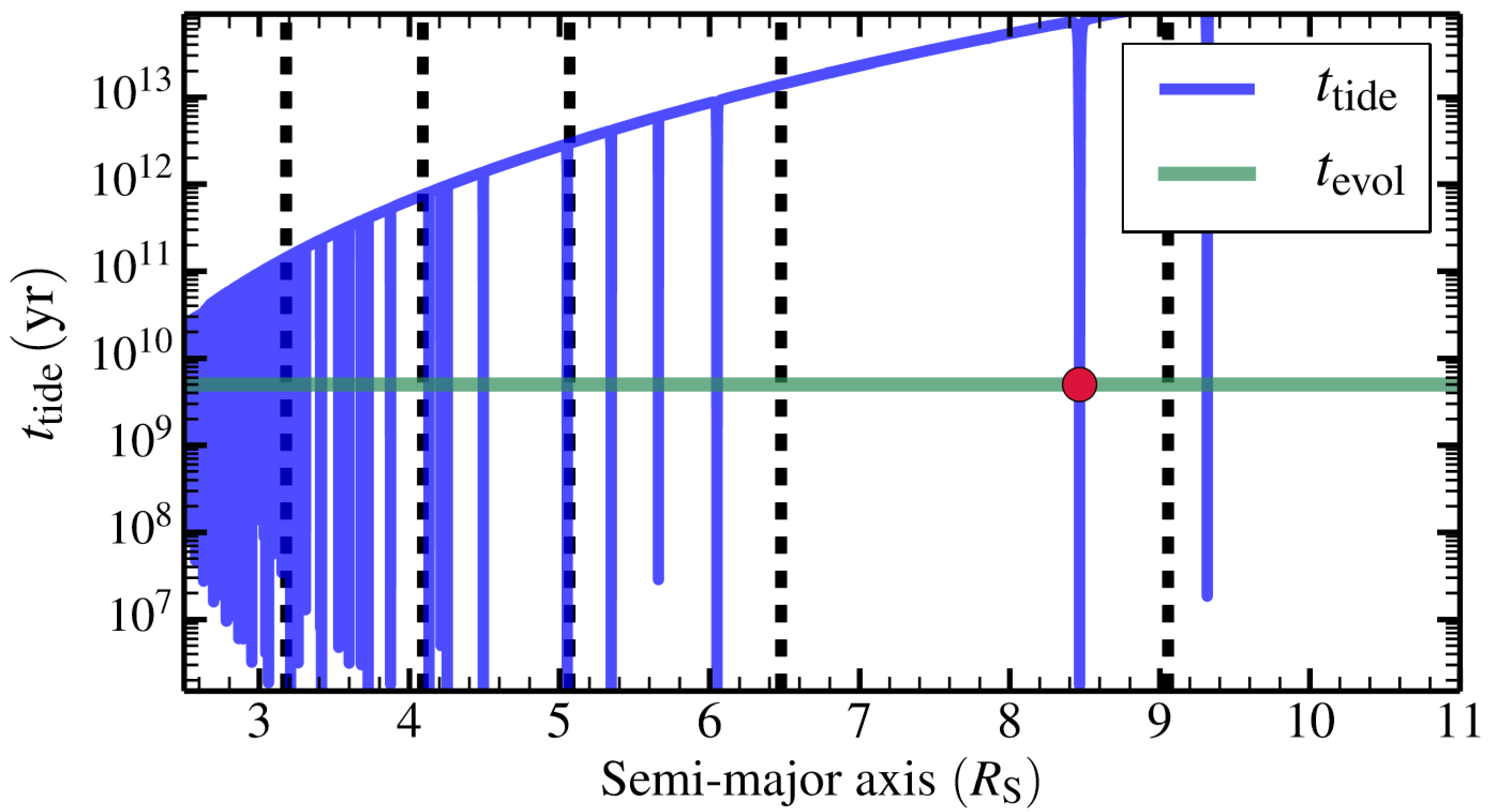
- Waves and or oscillations in the planet
- Energy dissipation rate varies strongly with forcing frequency
- Tidal dissipation greatly enhanced around resonant peaks where

$$\omega_\alpha \simeq \omega_{\mathrm{f}} = m(\Omega_{\mathrm{p}} - \Omega_{\mathrm{m}})$$

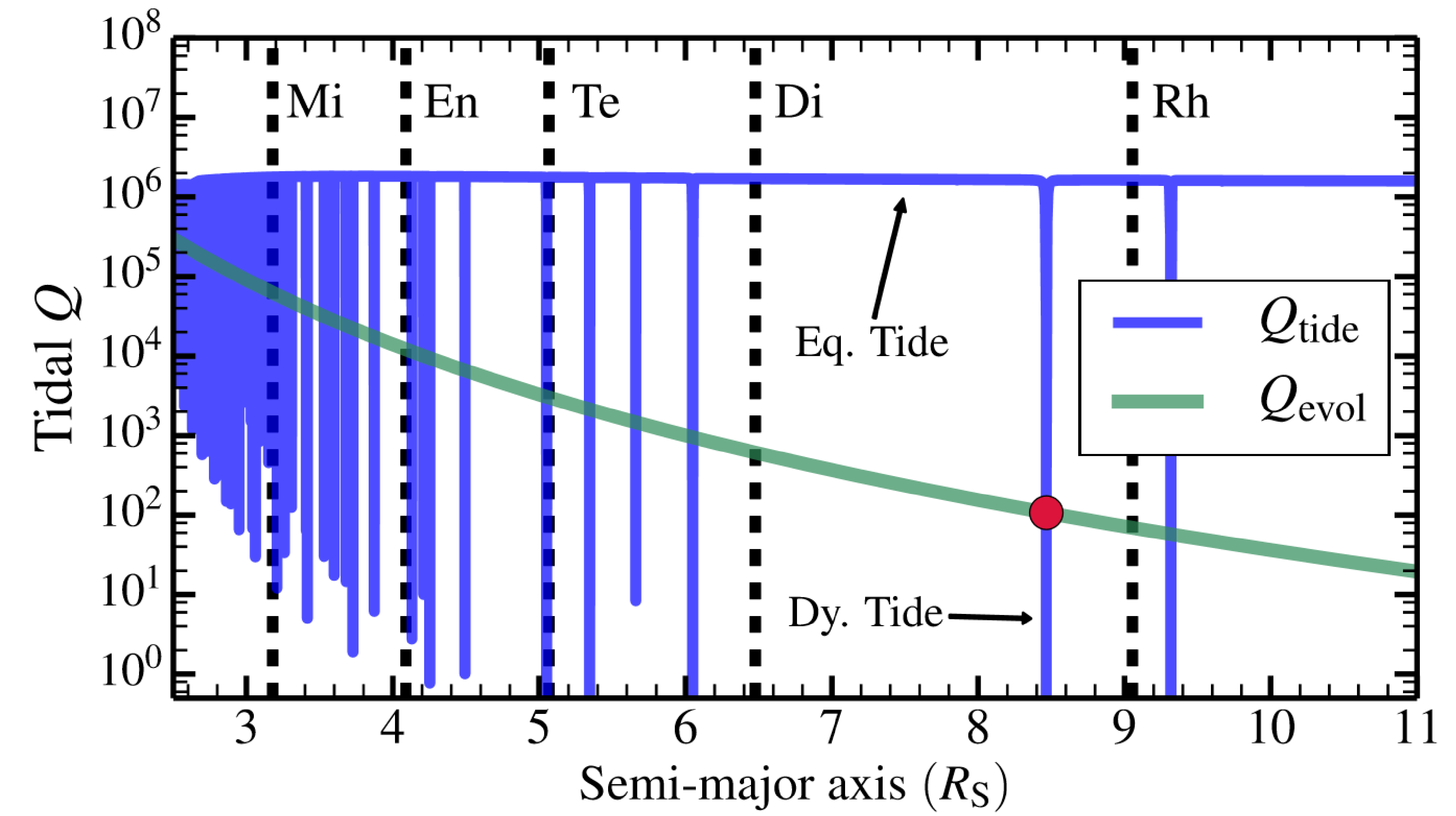


Ogilvie & Lin 2004

Modeling Saturnian Tides



Modeling Saturnian Tides

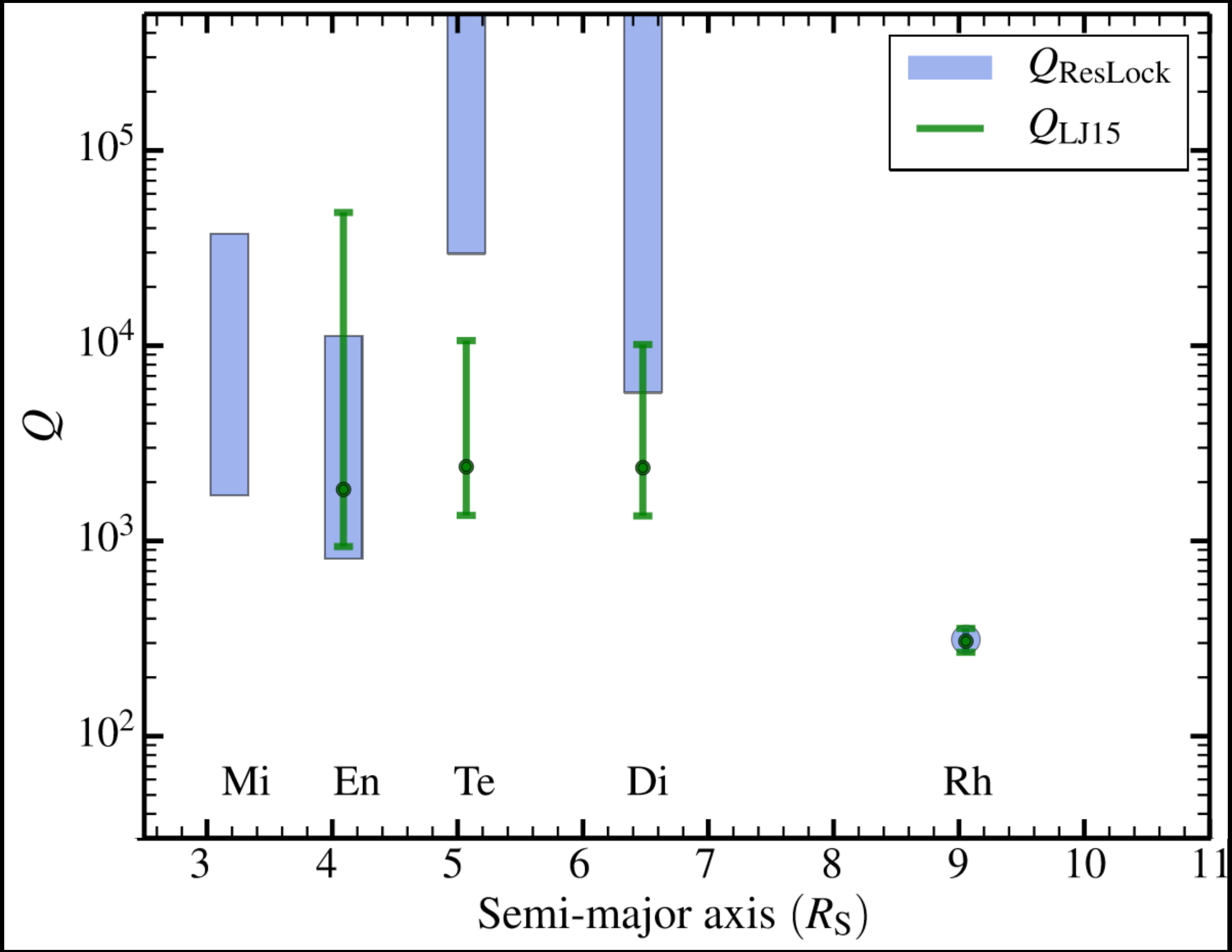


Measurements

- Outward migration rates measured by Lainey et al. (2009,2012,2015) using astrometric data

- Measured effective Q values are different from one another and smaller than expected

- Inconsistent with equilibrium tides



Resonance Locking

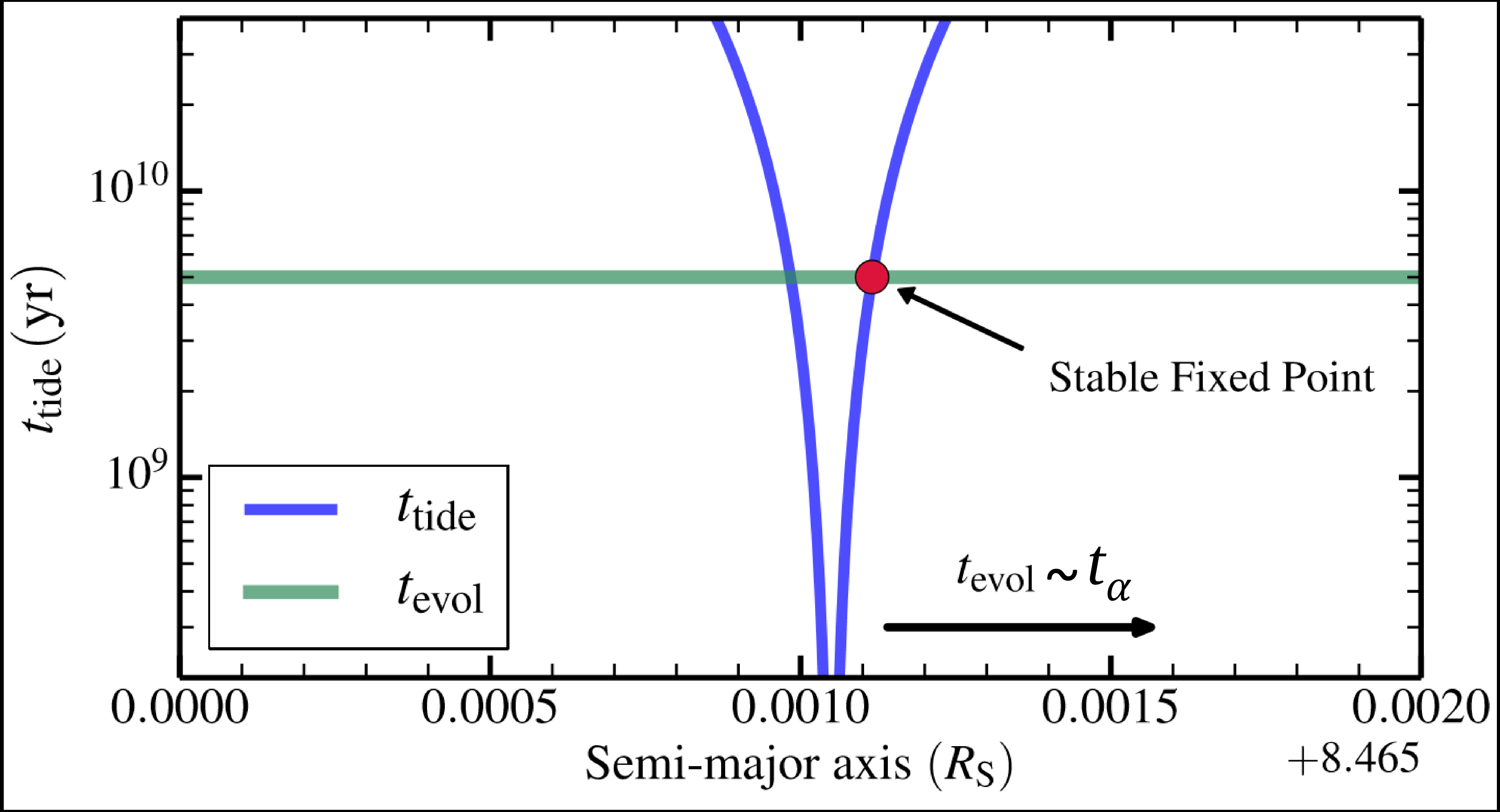
- Frequencies of resonant peaks are dependent on planet's internal structure

- Planet's internal structures gradually evolve
  - Cooling
  - Compositional settling, e.g., Helium rain

$$T_{\mathrm{Sa}} = \frac{GM_{\mathrm{Sa}}^2}{R_{\mathrm{Sa}} L_{\mathrm{Sa}}} \approx 100\,\mathrm{Gyr}$$

- Frequencies of resonant peaks evolve on planetary evolution timescale

$$\dot{\omega}_\alpha = \frac{\omega_\alpha}{t_\alpha}$$



10^{10}
10^{9}
$t_{\rm tide}$ (yr)
$t_{\rm tide}$
$t_{\rm evol}$
Stable Fixed Point
$t_{\rm evol} \sim t_\alpha$
0.0000
0.0005
0.0010
0.0015
0.0020
+8.465
Semi-major axis ($R_{\rm S}$)

Tidal Dynamics in Resonance Lock

- Migration rate is simply

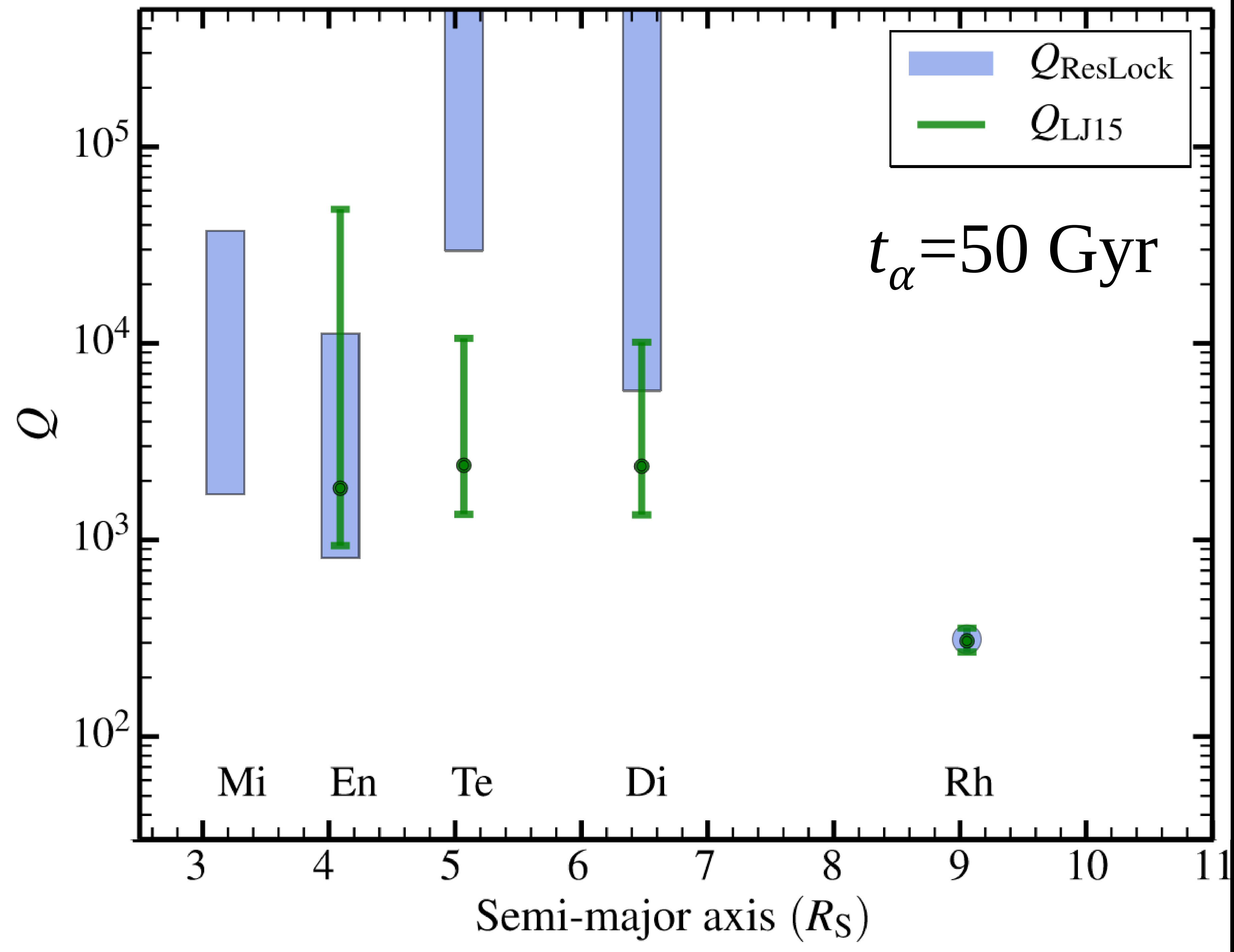
$$\frac{\dot{a}_{\rm m}}{a_{\rm m}} = \frac{2}{3}\left[\frac{\omega_\alpha}{m\Omega_{\rm m}t_\alpha} - \frac{\Omega_{\rm p}}{\Omega_{\rm m}t_{\rm p}}\right]$$

- Tidal timescale is closely related to planetary evolution timescale

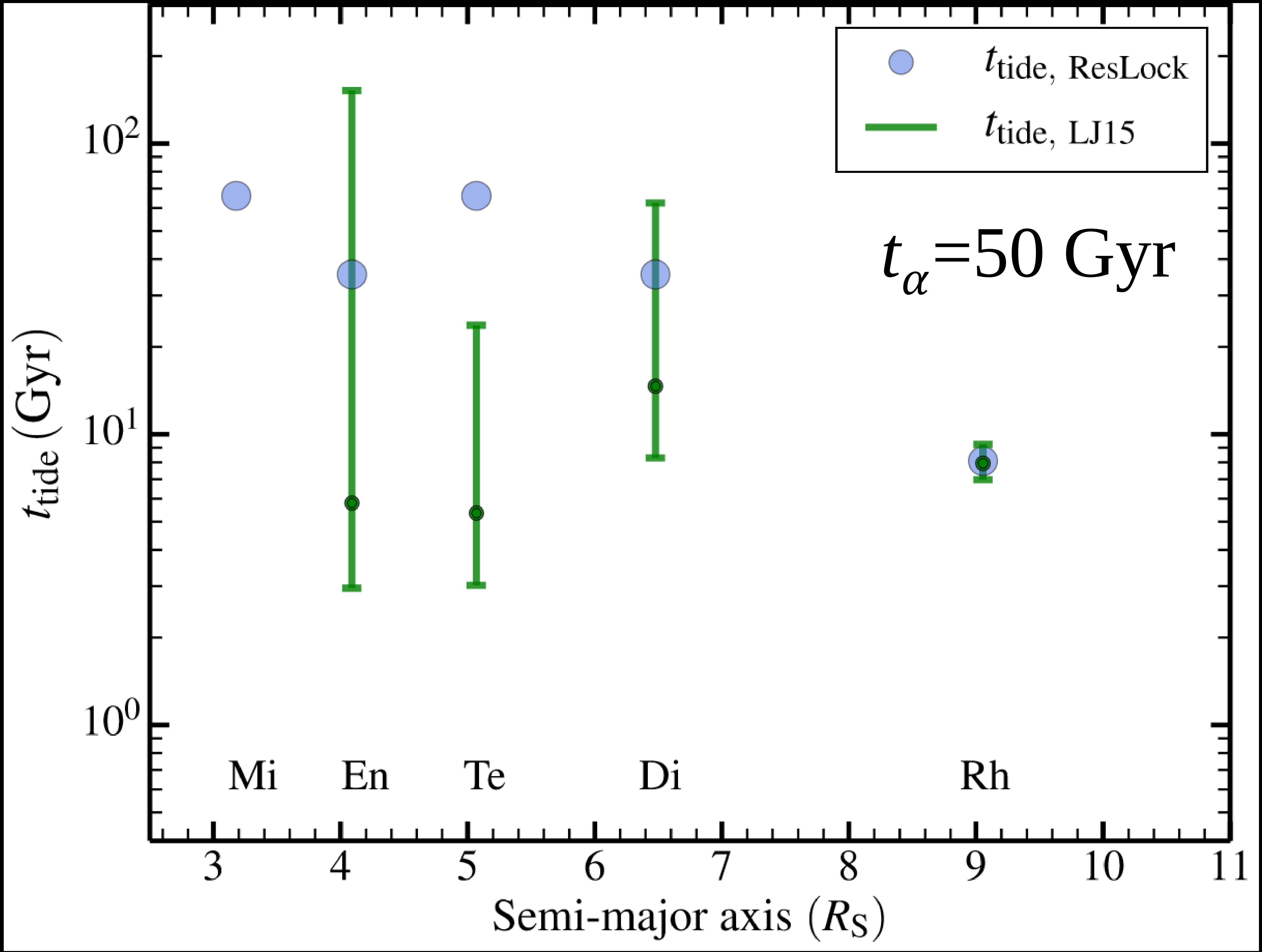
$$t_{\rm tide} \approx \frac{3}{2}\frac{\Omega_{\rm m}}{\Omega_{\rm p} - \Omega_{\rm m}} t_\alpha$$

- Effective tidal *Q* is usually much smaller than frequency-averaged *Q*

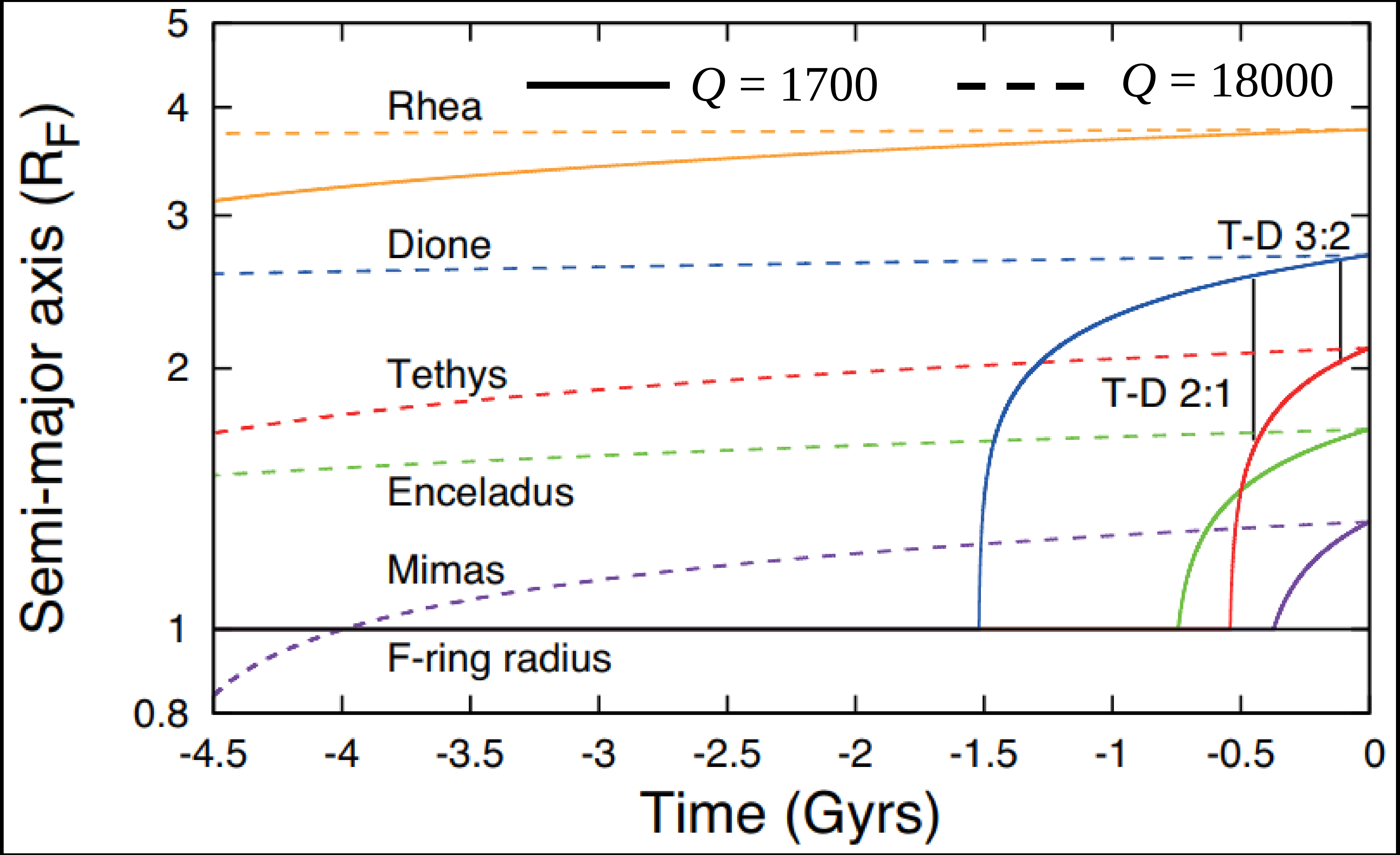
$$Q_{\rm ResLock} = \frac{9k_2}{2}\frac{M_{\rm m}}{M_{\rm p}}\left(\frac{R}{a}\right)^5\left[\frac{\omega_\alpha}{m\Omega_{\rm m}^2 t_\alpha} - \frac{\Omega_{\rm p}}{\Omega_{\rm m}^2 t_{\rm p}}\right]^{-1}$$



Q_{ResLock}
Q_{LJ15}
t_α=50 Gyr
10^5
10^4
10^3
10^2
Q
Mi
En
Te
Di
Rh
3
4
5
6
7
8
9
10
11
Semi-major axis (R_S)



$t_{\rm tide,\ ResLock}$
$t_{\rm tide,\ LJ15}$
t_α=50 Gyr
$t_{\rm tide}$ (Gyr)
10^2
10^1
10^0
Mi
En
Te
Di
Rh
3
4
5
6
7
8
9
10
11
Semi-major axis ($R_{\rm S}$)



Nakajima et al. 2018

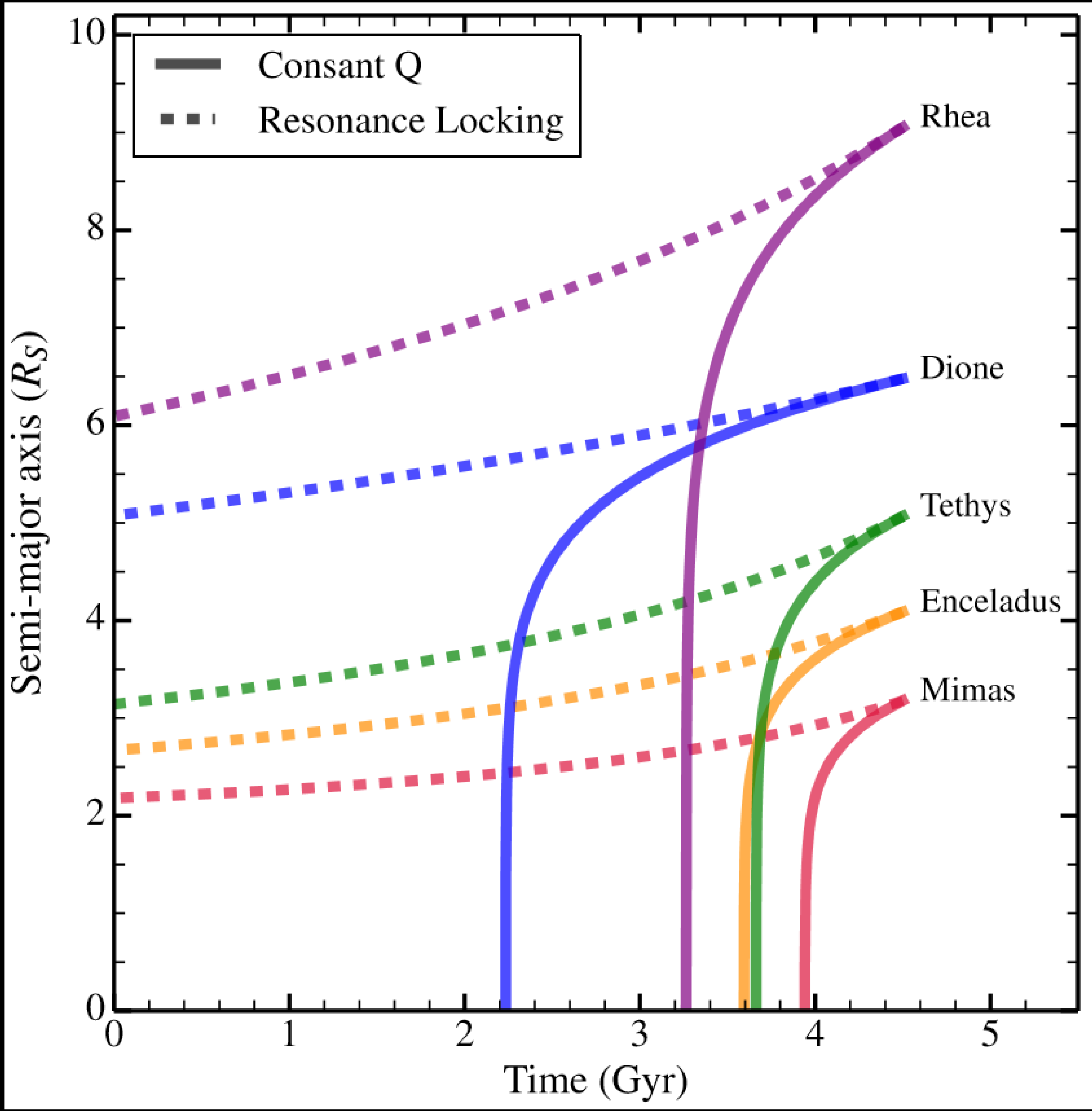
Evolutionary History

- In equilibrium tidal theory, migration rate strongly dependent on semi-major axis

$$Q \equiv 3k_2 \frac{M_{\rm m}}{M_{\rm p}} \left(\frac{R_{\rm p}}{a_{\rm m}} \right)^5 \Omega_{\rm m} t_{\rm tide}$$

- According to resonance locking, tidal migration rate is only weakly dependent on semi-major axis

$$t_{\rm tide} \approx \frac{3}{2} \frac{\Omega_{\rm m}}{\Omega_{\rm p} - \Omega_{\rm m}} t_\alpha$$



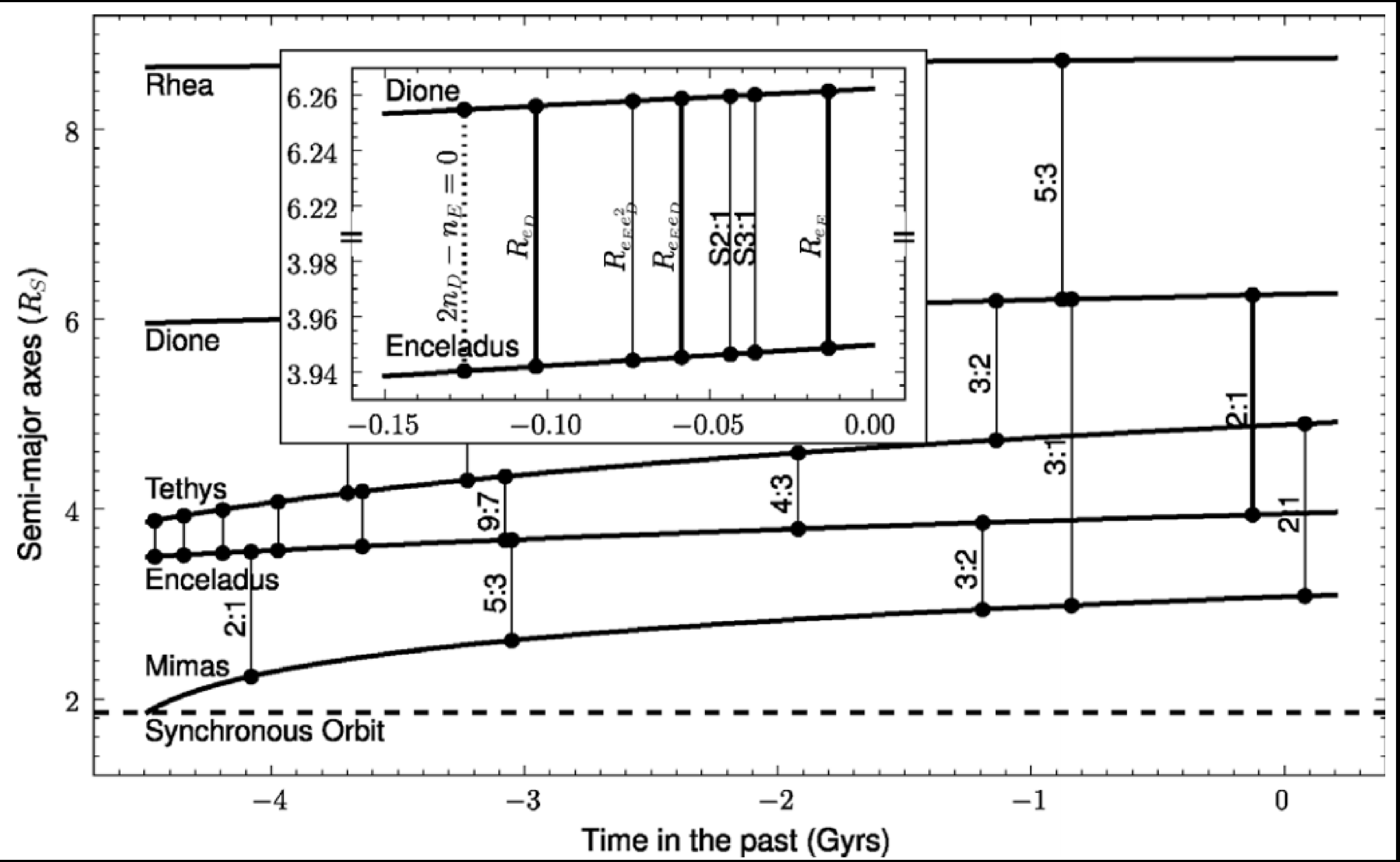
Orbital Resonances

- Mean motion resonances occur when

$$(j+1)\Omega_2 - j\Omega_1 \simeq 0$$

- For pair of moons tidally driven outward by inner moon and locked in MMR, angular momentum deficit develops, and eccentricity/inclination must increase

- Due to non-spherical gravitational potential, resonances are split into multiple components
  - Eccentricity type resonances excite eccentricity $$2\lambda' - \lambda - \varpi$$ $$2\lambda' - \lambda - \varpi'$$
  - Inclination type resonances excite inclination $$4\lambda' - 2\lambda - \Omega - \Omega'$$



Zhang & Nimmo 2009

Tidal Heating

Tidal Heating

- Tidal heating rate via eccentricity tides is

$$\dot{E}_{\rm heat} = \frac{21}{2}\frac{k_1}{Q_1}\frac{GM_{\rm p}^2 R_1^5}{a_1^6}\Omega_1 e_1^2$$

- Eccentricity is boosted by mean motion resonance but damped by tidal heating. At equilibrium eccentricity, these effects balance

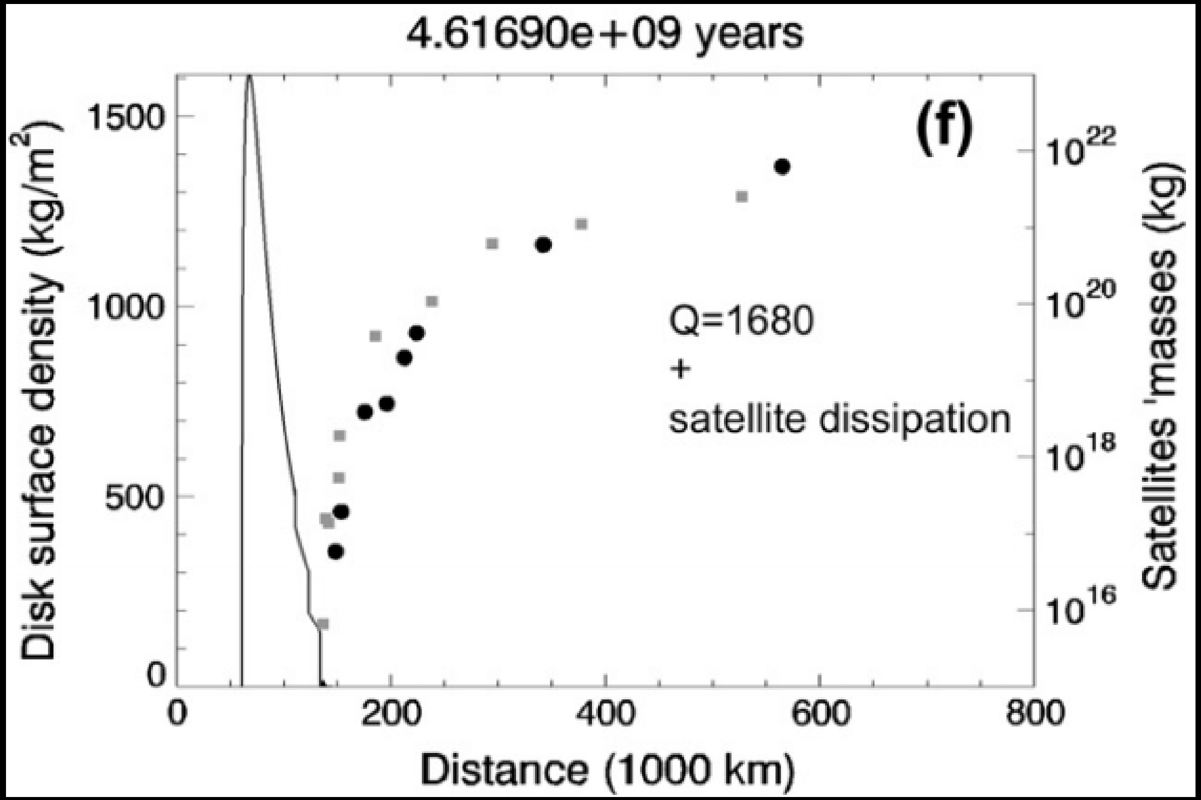
$$e_{\rm eq}^2 = \frac{1}{7(j-1)}\frac{M_1 M_2}{M^2}\left(\frac{R_{\rm p}}{R_1}\right)^5 \frac{Q_1}{Q_{\rm p,1}}\frac{k_{\rm p}}{k_1}$$

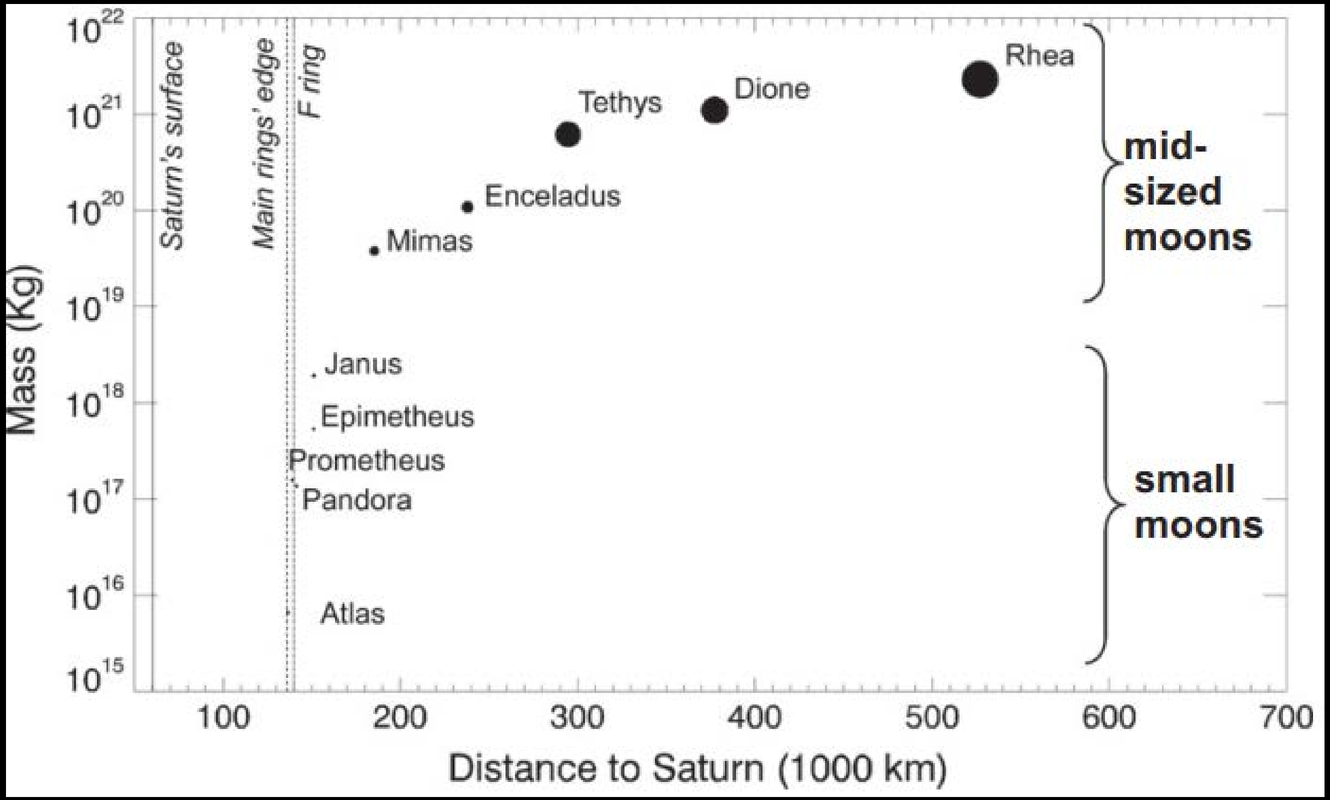
- If inner moon pushes outer moon outward via mean motion resonance, tidal heating rate of inner moon is

$$\dot{E}_{\rm heat,1} \simeq \frac{1}{j-1}\frac{|E_{2,\rm orb}|}{t_{\rm tide}}$$

For Enceladus: $\dot{E}_{\rm heat} \approx 50\,{\rm GW}$

Spawning Moons from Rings





Charnoz et al. 2011

Standard Lore

- Tides drive moons outward, MMRs encountered, tidal heating ensues

New Paths Forward

- Effective tidal Q is different for each moon, and varies with time (dynamical tides)
- Tidal evolution occurs on planetary evolution timescale (resonance locking)
- Substantial migration of outer moons (e.g., Titan & Callisto)
- Rapid migration, ring-driven migration, late formation of inner moons

Thanks!