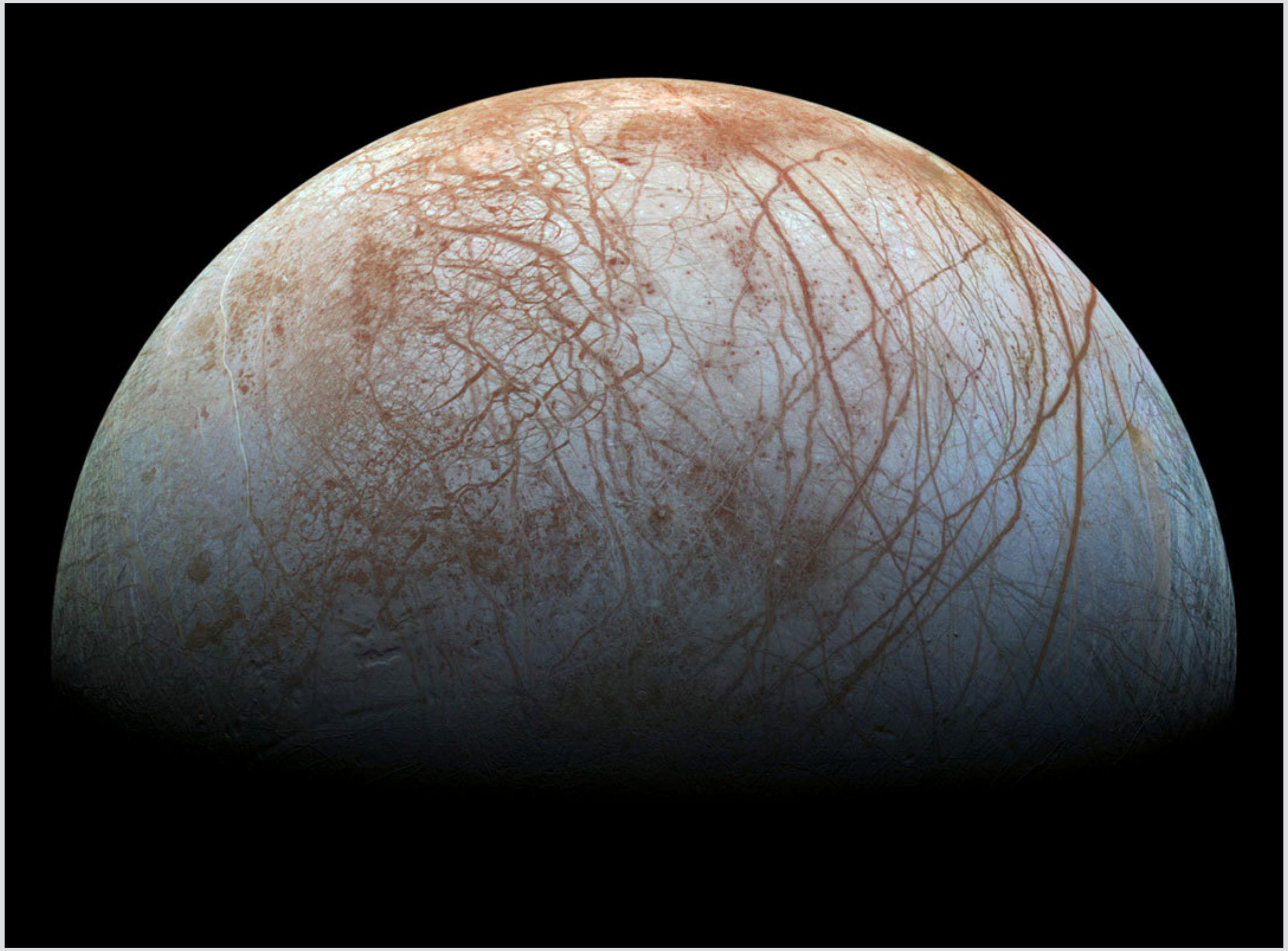
Tidal Heating of Ocean Worlds

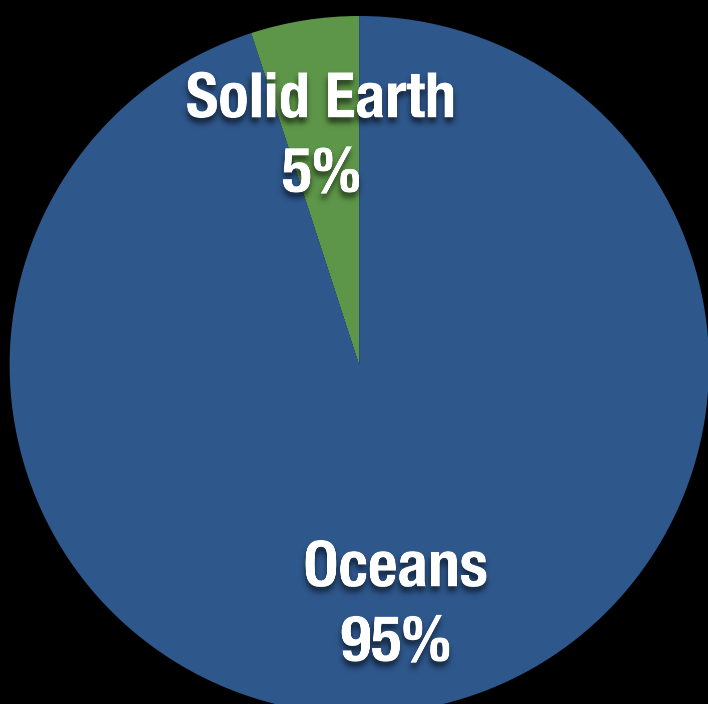


Isamu Matsuyama

Lunar and Planetary Laboratory

Tidal energy dissipation

Earth's tidal energy dissipation



Solid Earth: ~ 100 GW (Ray et al. 2001)
Oceans: ~ 2 TW (Egbert & Ray 2000)

- Tidal energy dissipation affects the thermal, rotational, and orbital evolution
- Subsurface oceans are common on icy satellites
- Earth: most of the tidal energy is dissipated in the oceans
- Does ocean dissipation also dominate tidal energy dissipation in icy satellites?
- What is the the spatial and temporal variation of ocean tidal heating?

Tides due to orbital eccentricity

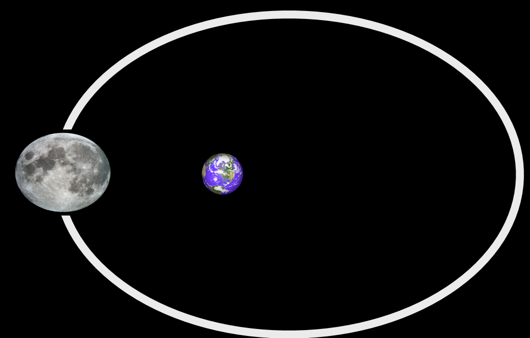


Fig. 1. Top view, eccentric orbit

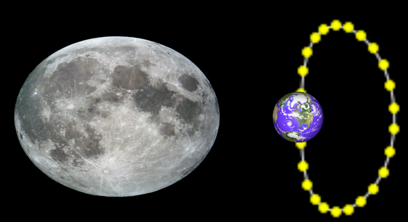


Fig. 2. Top view, reference frame rotating with the satellite in synchronous rotation

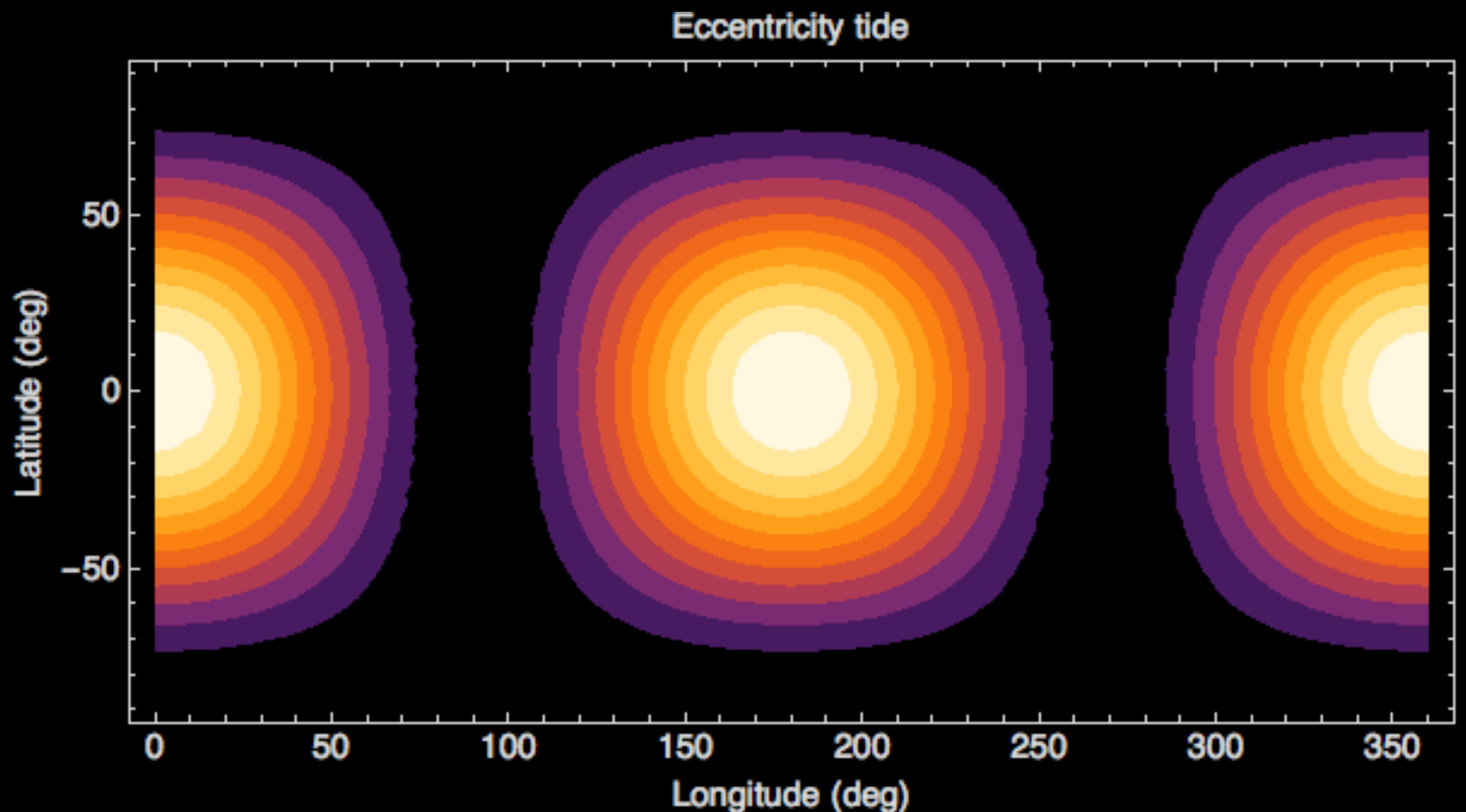


Fig. 3. Tidal potential, reference frame rotating with the satellite in synchronous rotation

Tides due to obliquity

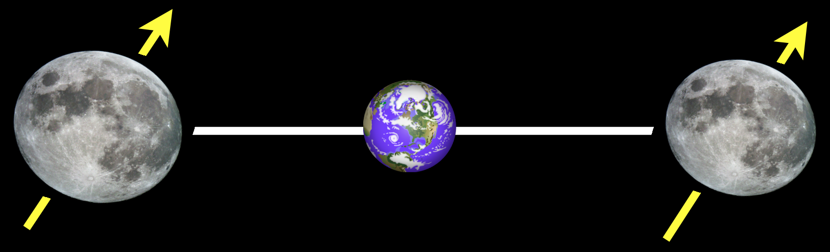


Fig. 1. Side view, satellite with non-zero obliquity

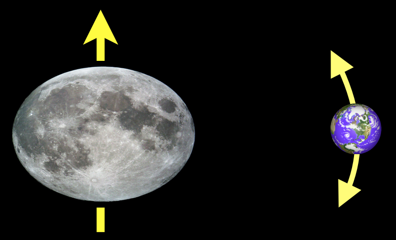


Fig. 2. Side view, reference frame rotating with the satellite in synchronous rotation

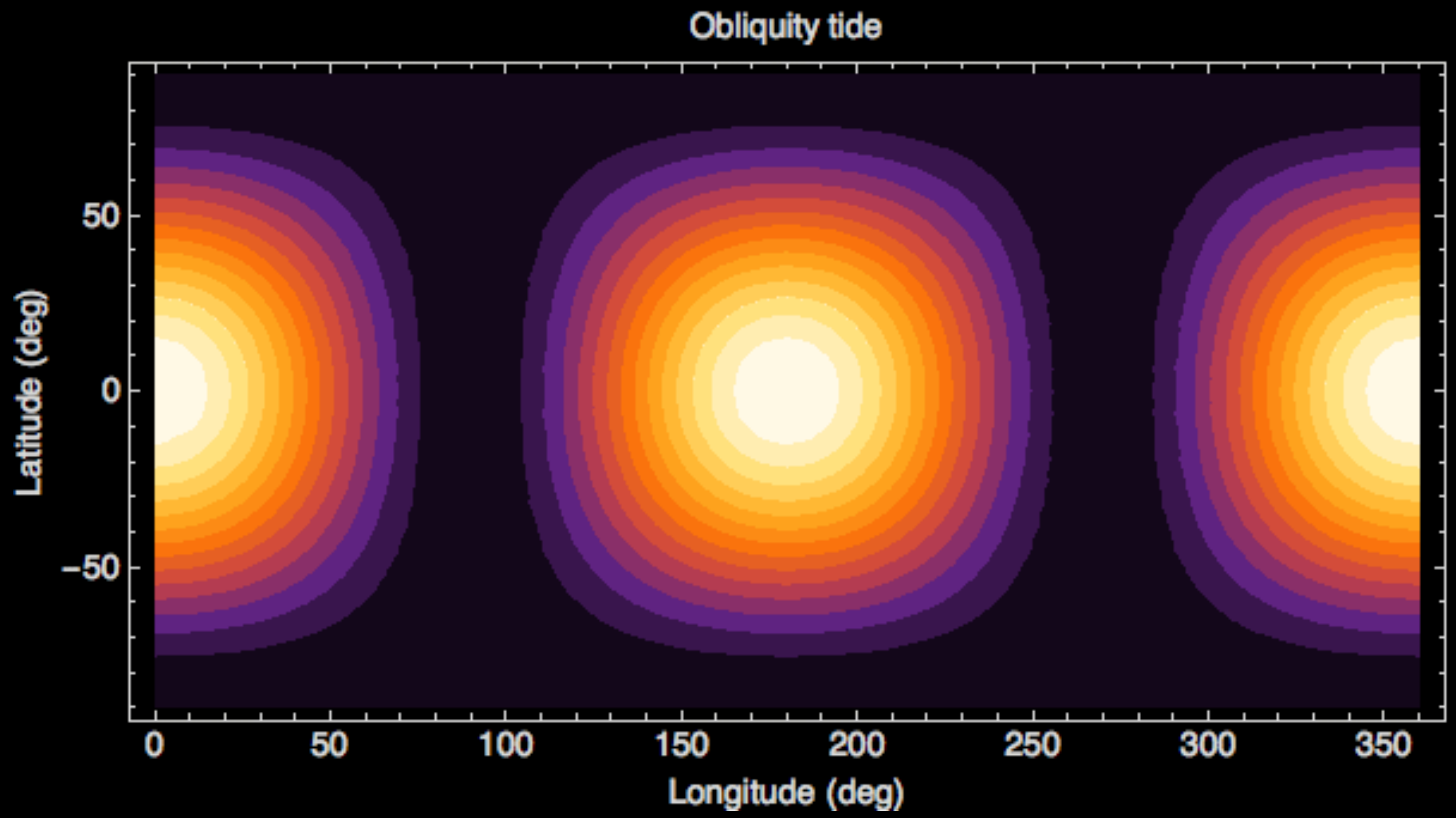
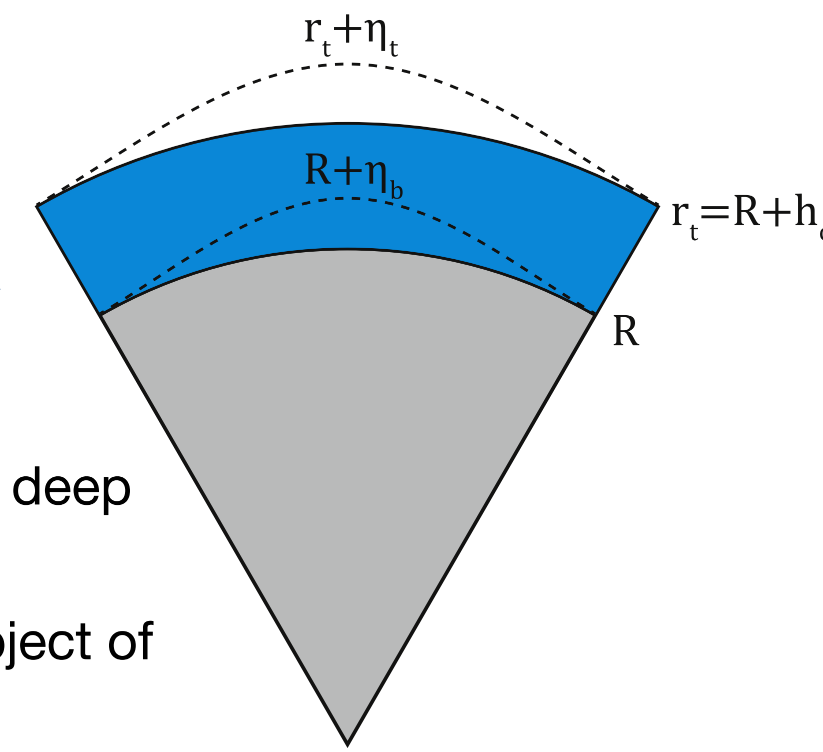


Fig. 3. Tidal potential, reference frame rotating with the satellite in synchronous rotation

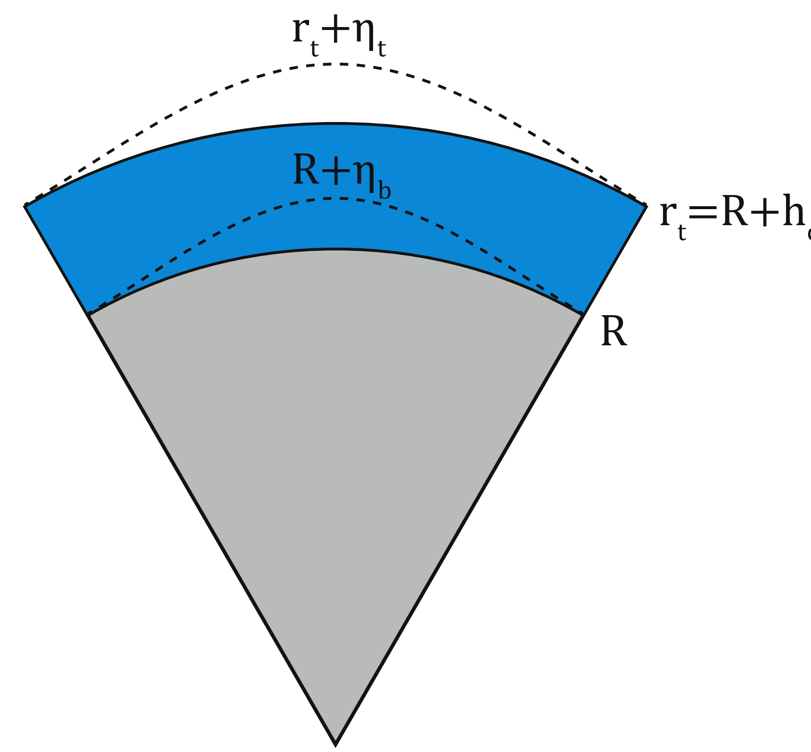
Surface Oceans



- Laplace tidal equations describing ocean tides in a thin, incompressible surface ocean
  - Mass conservation: $\partial_t h_o + \nabla \cdot (h_o \mathbf{u}) = 0$
  - Momentum conservation: $\partial_t \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} + \underbrace{\boxed{\alpha \mathbf{u} + \frac{c_D}{h_o} |\mathbf{u}| \mathbf{u}}}_{\text{dissipation}} = -\frac{1}{\rho_o} \nabla P + \nabla U$
    - Linear drag: internal wave generation over rough topography in the deep ocean
      - Earth: α~10^{-5} s^{-1} (Webb, 1980; Egbert and Ray, 2000, 2001), subject of ongoing research (Green and Nycander, 2013).
      - Possible to obtain semi-analytic solutions (Tyler, 2011; Chen et al., 2014; Matsuyama, 2014; Beuthe, 2016)
    - Non-linear bottom drag: friction in a turbulent boundary layer at the solid fluid-interface
      - A nominal value c_D~$2x10^{-3}$ (has been assumed to model tidal heating on
        - Earth (Lambeck, 1980; Jayne and Laurent, 2001; Egbert and Ray, 2001, Green and Nycander, 2013)
        - Titan (Sagan & Dermott, 1982; Sohl et al. 1995)
        - Jovian planets (Goldreich & Soter, 1966)
      - Non-linear nature requires numerical solutions (Sears 1995; Chen & Nimmo, 2014; Hay & Matsuyama 2016)

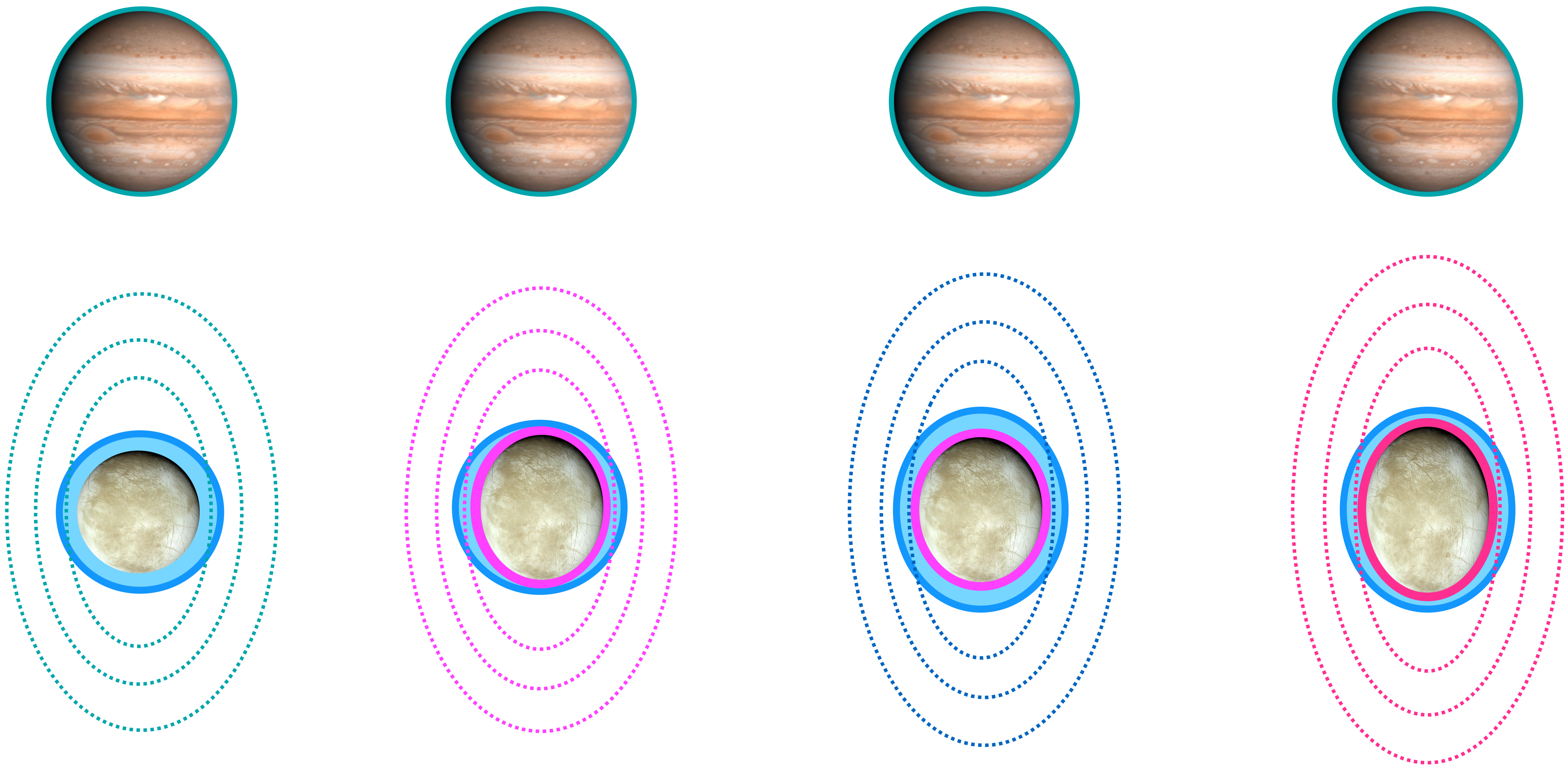
Surface Oceans

- Laplace tidal equations describing ocean tides in a thin, incompressible surface ocean
  - Mass conservation: $\partial_t h_o + \nabla \cdot (h_o \mathbf{u}) = 0$
  - Momentum conservation: $\partial_t \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} + \underbrace{\boxed{\alpha \mathbf{u} + \frac{c_D}{h_o} |\mathbf{u}| \mathbf{u}}}_{\text{dissipation}} = -\frac{1}{\rho_o} \nabla P + \nabla U$



- Linear drag, non-linear bottom drag
- Energy dissipation per unit time and surface area: $F_{diss} = \alpha \rho_o h_o \mathbf{u} \cdot \mathbf{u} + c_D \rho_o (\mathbf{u} \cdot \mathbf{u})^{3/2}$
- Pressure: $P = \rho_o g (r_t + \eta_t - r)$
- Forcing potential: U

Surface oceans: Forcing Potential



Tide raising primary

$$U = U^T$$

+deformation of the satellite in response to U^T

$$U = (1 + k^T)U^T$$
$$\delta R = h^T U^T / g$$

+additional potential due to ocean tides U^L

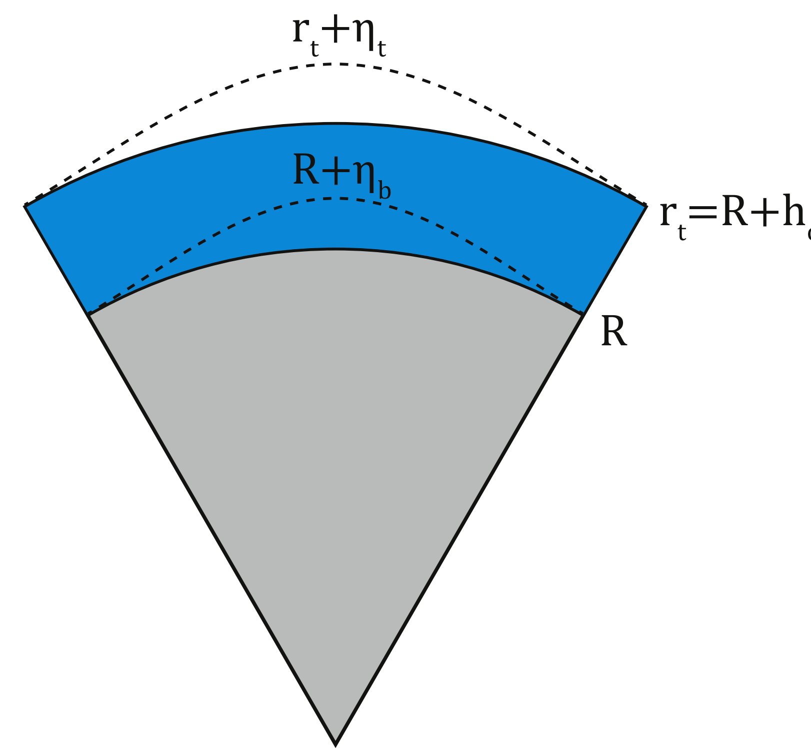
$$U = (1 + k^T)U^T + U^L$$
$$\delta R = h^T U^T / g$$

+deformation of the satellite in response to U^L

$$U = (1 + k^T)U^T + (1 + k^L)U^L$$
$$\delta R = h^T U^T / g + h^L U^L / g$$

Surface Oceans

- Laplace tidal equations describing ocean tides in a thin, incompressible surface ocean



  - Mass conservation:

$$\partial_t h_o + \nabla \cdot (h_o \mathbf{u}) = 0$$

  - Momentum conservation:

$$\partial_t \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} + \alpha \mathbf{u} + \frac{c_D}{h_o} |\mathbf{u}| \mathbf{u} = -g(1 - \xi \gamma^L) \nabla \eta + \gamma^T \nabla U^T$$

Additional potential due to ocean tides (U^L): $U^L = g\xi\eta, \quad \xi \equiv \frac{3}{2n+1} \frac{\rho_o}{\bar{\rho}}$

Deformation of the satellite in response to U^L: $\gamma^L \equiv 1 + k^L - h^L$

Deformation of the satellite in response to U^T: $\gamma^T \equiv 1 + k^T - h^T$

- Energy dissipation per unit time and surface area: $F_{diss} = \alpha \rho_o h_o \mathbf{u} \cdot \mathbf{u} + c_D \rho_o (\mathbf{u} \cdot \mathbf{u})^{3/2}$

Surface Oceans: equilibrium tide

- Laplace tidal equations describing ocean tides in a thin, incompressible surface ocean
  - Mass conservation:

$$\partial_t h_o + \nabla \cdot (h_o \mathbf{u}) = 0$$

  - Momentum conservation:

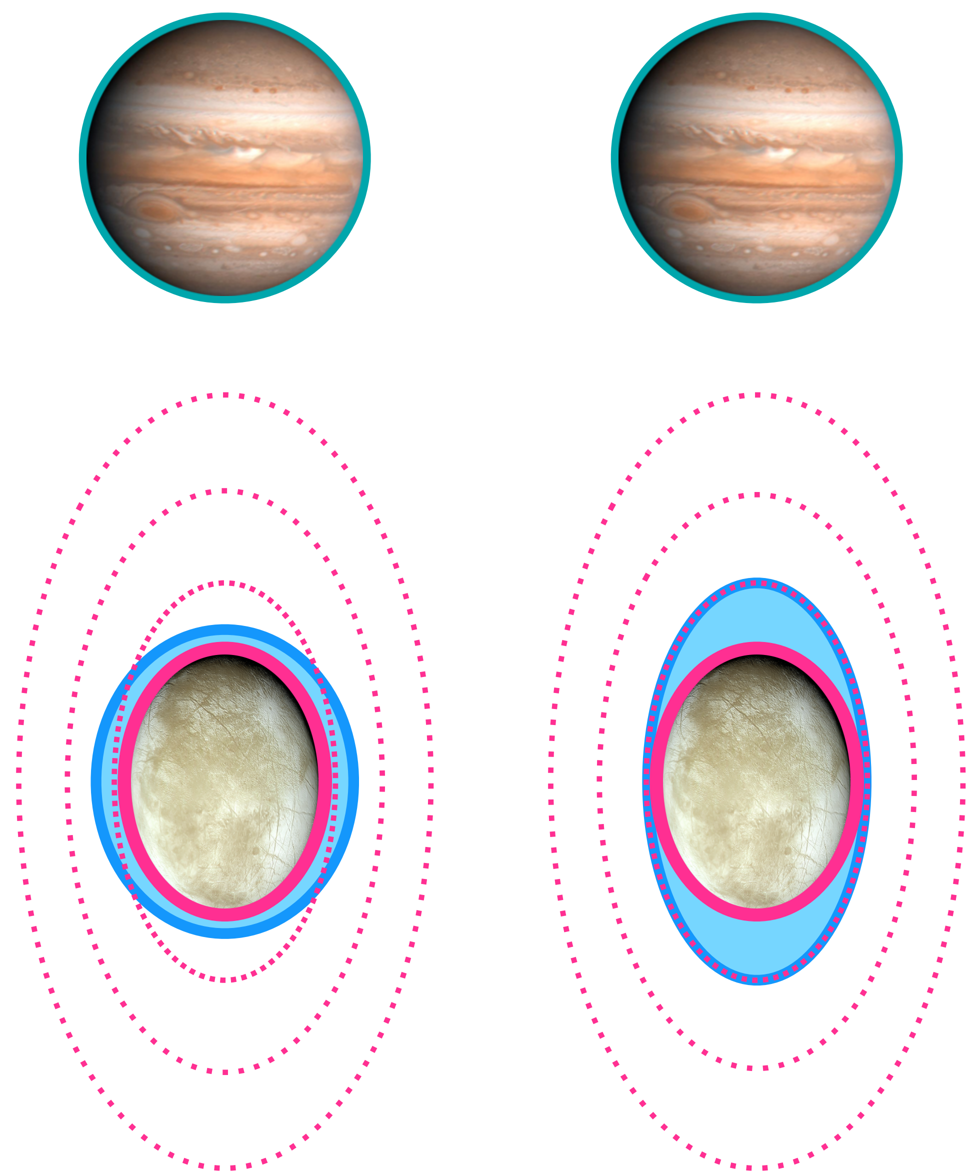
$$\cancel{\partial_t \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} + \alpha \mathbf{u} + \frac{c_D}{h_o} |\mathbf{u}| \mathbf{u}} = -g(1 - \xi\gamma^L)\nabla\eta + \gamma^T \nabla U^T$$

$$\mathbf{u} = 0$$

    - Equilibrium tides (u=0):

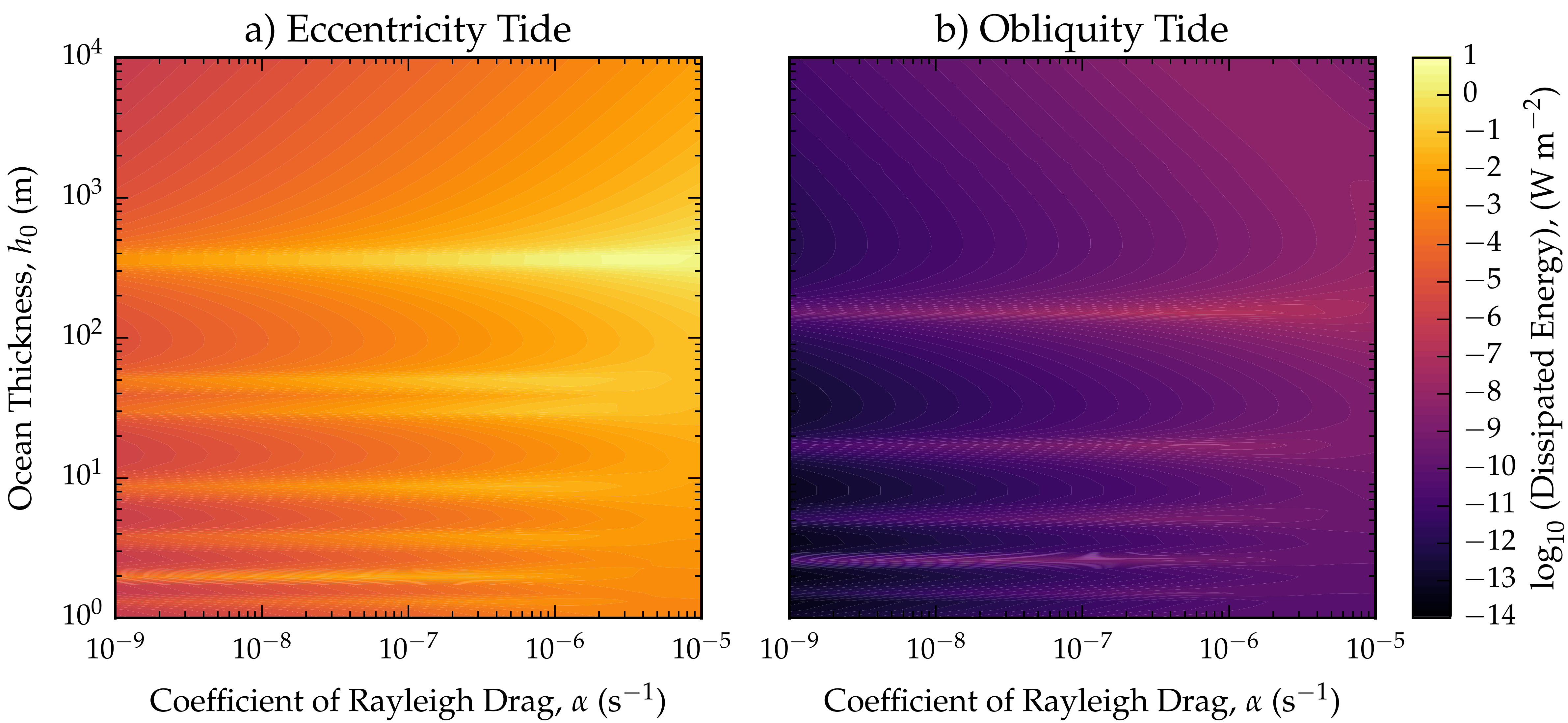
$$\eta = \eta_{eq} \equiv \frac{\gamma^T}{(1 - \xi\gamma^L)} \frac{U^T}{g}$$

    - Dynamic tides (u≠0)



$$\eta = \eta_{eq}$$

Enceladus: Surface Ocean Dissipation

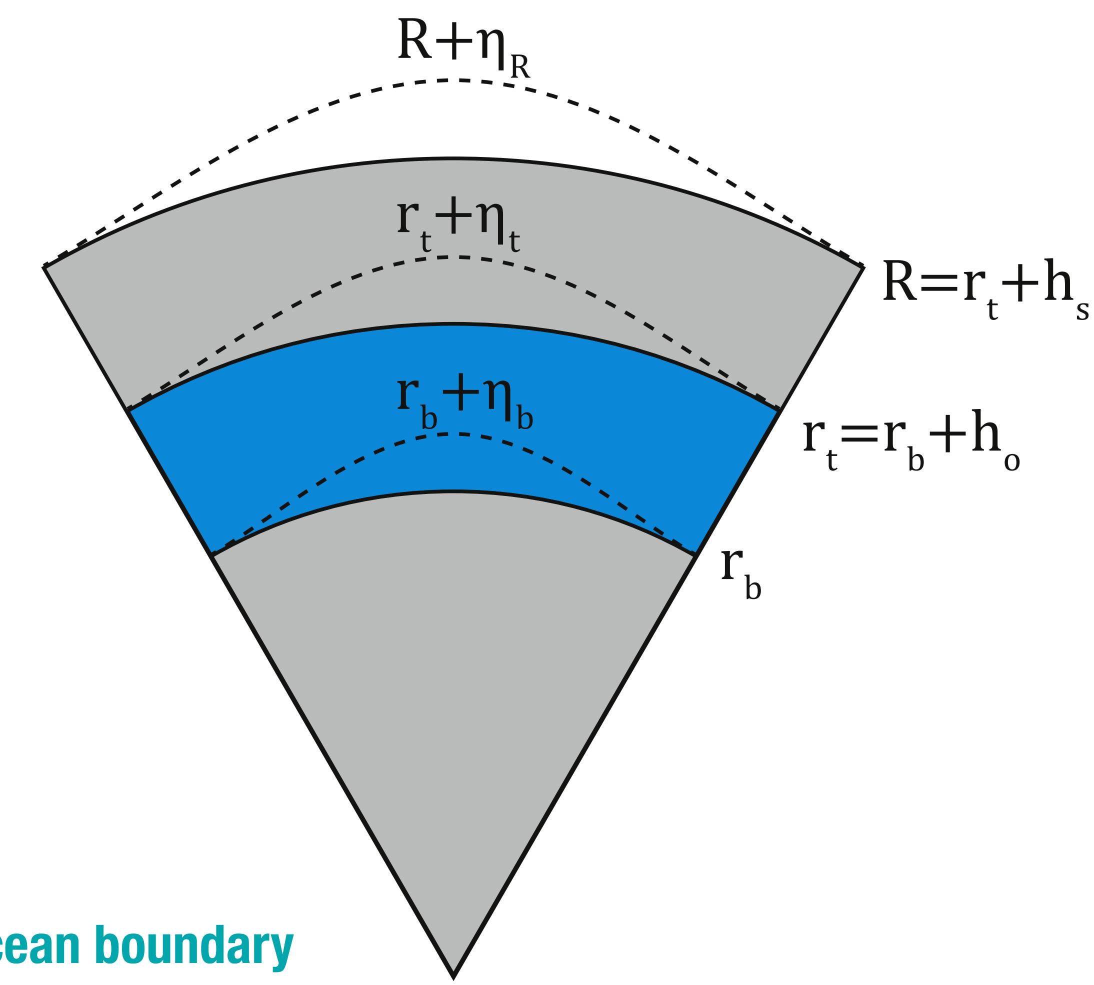


Hay & Matsuyama et al. 2016

- Ocean dissipation can be resonantly enhanced (Tyler 2008, 2009, 2011; Matsuyama, 2014; Hay & Matsuyama, 2016)
  - But resonances correspond to very thin oceans
- Obliquity tidal heating << eccentricity tidal heating due Enceladus' small obliquity (Chen & Nimmo, 2011; Chen et al. 2014)
  - e=0.0047, obliquity=0.00045 deg assuming a Cassini state (Chen & Nimmo, 2011; Baland et al. 2016)

Subsurface Oceans

- Modified Laplace tidal equations (LTE) describing ocean tides in a thin, incompressible subsurface ocean



  - Mass conservation:

$$\partial_t h_o + \nabla \cdot (h_o \mathbf{u}) = 0$$

  - Momentum conservation:

$$\partial_t \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} + \alpha \mathbf{u} + \frac{c_D}{h_o} |\mathbf{u}| \mathbf{u} = -\frac{1}{\rho_o} \nabla P + \nabla U$$

  - Pressure: $P = \rho_o g(r_t + \eta_t - r) + \boxed{\sigma_{rr}^T + \sigma_{rr}^P}$

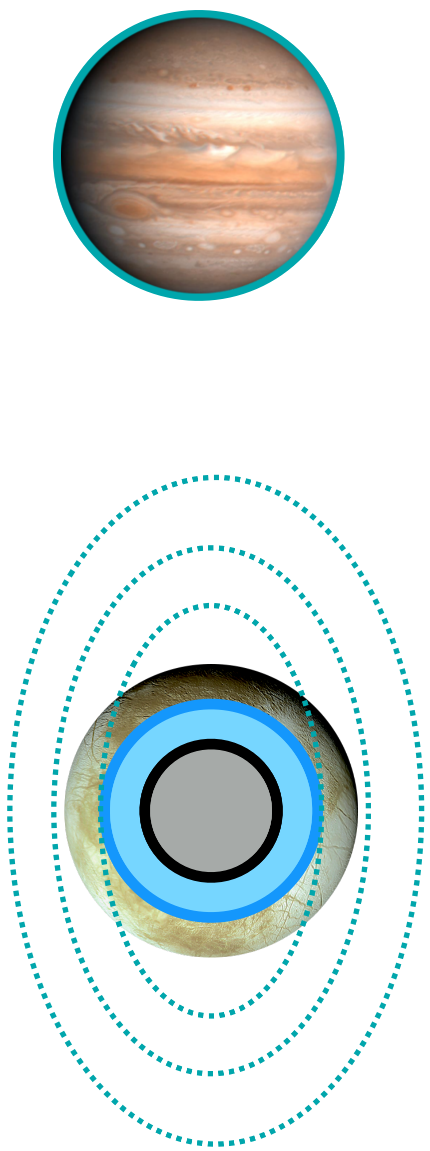
radial stress at the shell-ocean boundary due to tidal (T) and dynamic pressure (P) forcing

  - Forcing potential: U

  - Use tidal and pressure Love numbers to describe the static and dynamic parts of the deformation in response to tidal forcing.

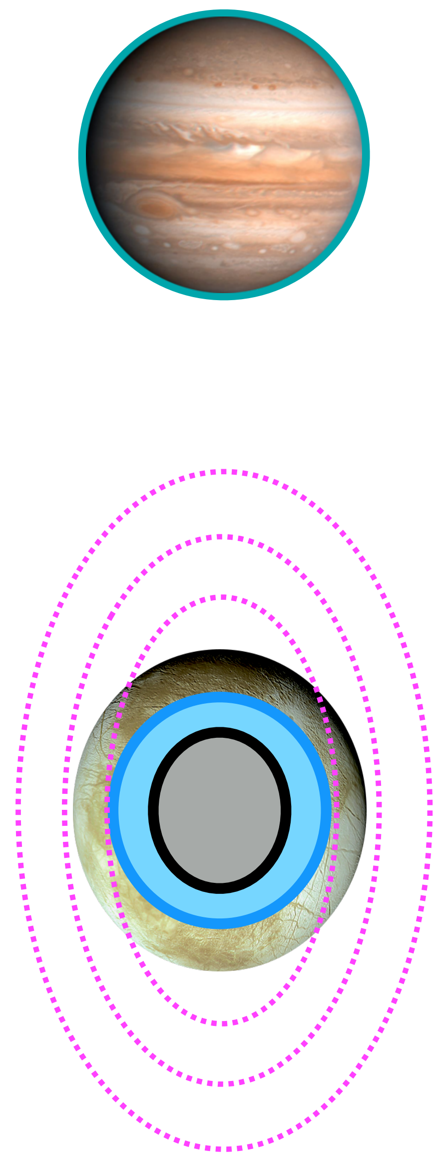
\

Subsurface oceans: Forcing Potential



Tide raising primary

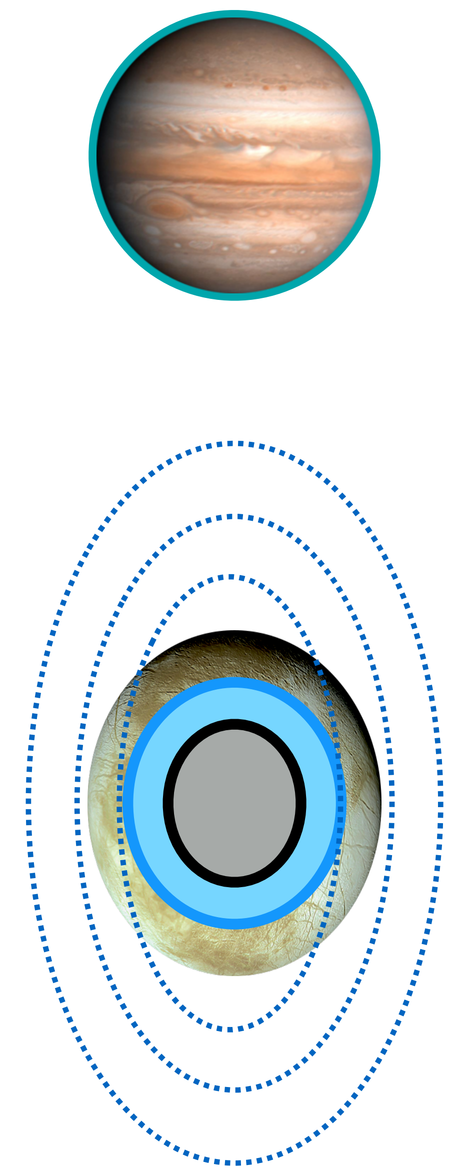
$$U = U^T$$



+deformation of the satellite in response to U^T

$$U = (1 + k^T)U^T$$

$$\delta R = h^T U^T / g$$



+deformation in response to dynamic pressure forcing (U^P)

$$U = (1 + k^T)U^T + k^P U^P$$

$$\delta R = h^T U^T / g + h^P U^P / g$$

Subsurface Oceans

- Laplace tidal equations describing ocean tides in a thin, incompressible surface ocean

  - Mass conservation:

$$\partial_t h_o + \nabla \cdot (h_o \mathbf{u}) = 0$$

  - Momentum conservation:

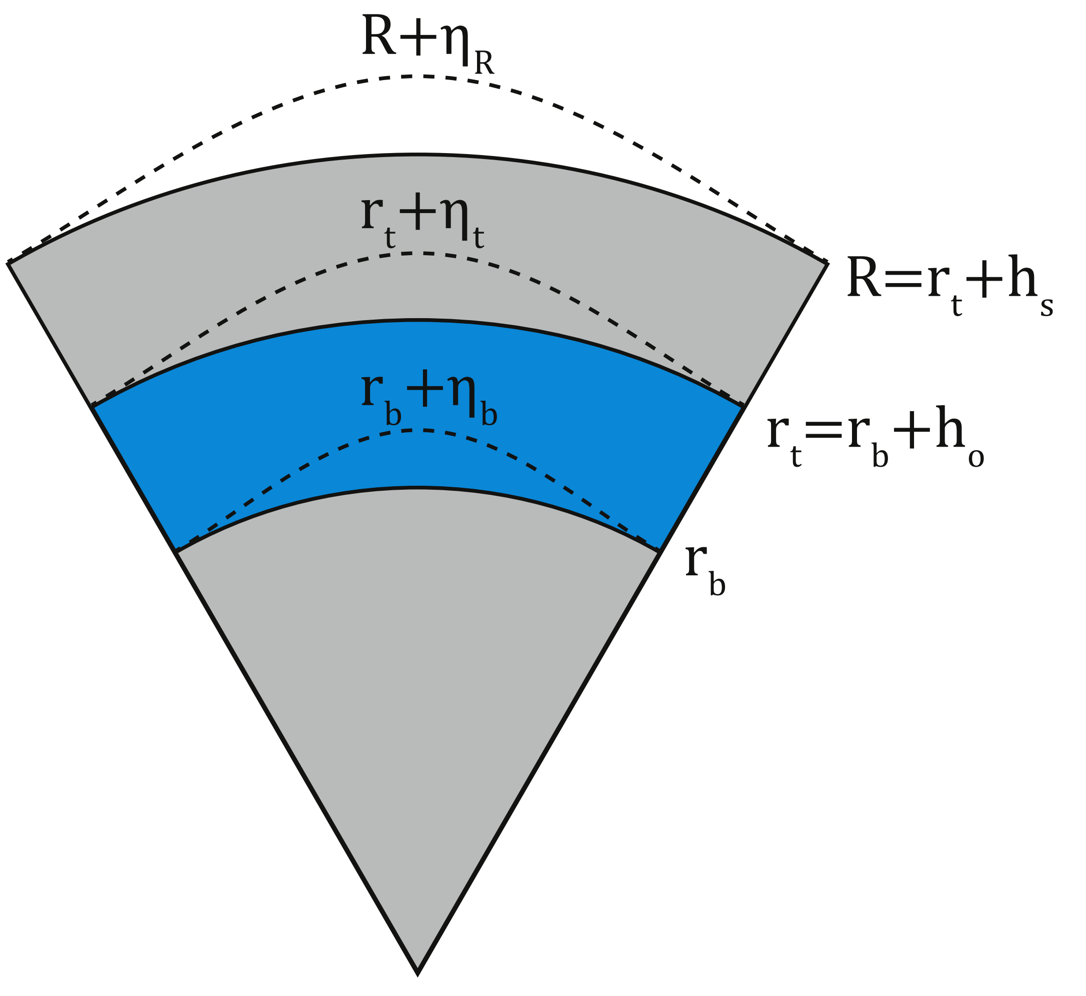
$$\partial_t \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} + \alpha \mathbf{u} + \frac{c_D}{h_o} |\mathbf{u}| \mathbf{u} = -g\beta \nabla \eta + \upsilon \nabla U^T$$

Deformation of the satellite in response to U^T:

$$\upsilon = \frac{h^T(r_t) - h^T(r_b)}{h^P(r_t) - h^P(r_b)}$$

Deformation in response to dynamic pressure forcing (U^P):

$$\beta = \frac{1}{h^P(r_t) - h^P(r_b)}$$



- Linear drag (Beuthe 2016; Matsuyama et al. 2018) and non-linear bottom drag (Hay & Matsuyama, 2019)

- Energy dissipation per unit time and surface area: $F_{diss} = \alpha \rho_o h_o \mathbf{u} \cdot \mathbf{u} + c_D \rho_o (\mathbf{u} \cdot \mathbf{u})^{3/2}$

Subsurface Oceans: equilibrium tide

- Laplace tidal equations describing ocean tides in a thin, incompressible surface ocean
  - Mass conservation:

$$\partial_t h_o + \nabla \cdot (h_o \mathbf{u}) = 0$$

  - Momentum conservation

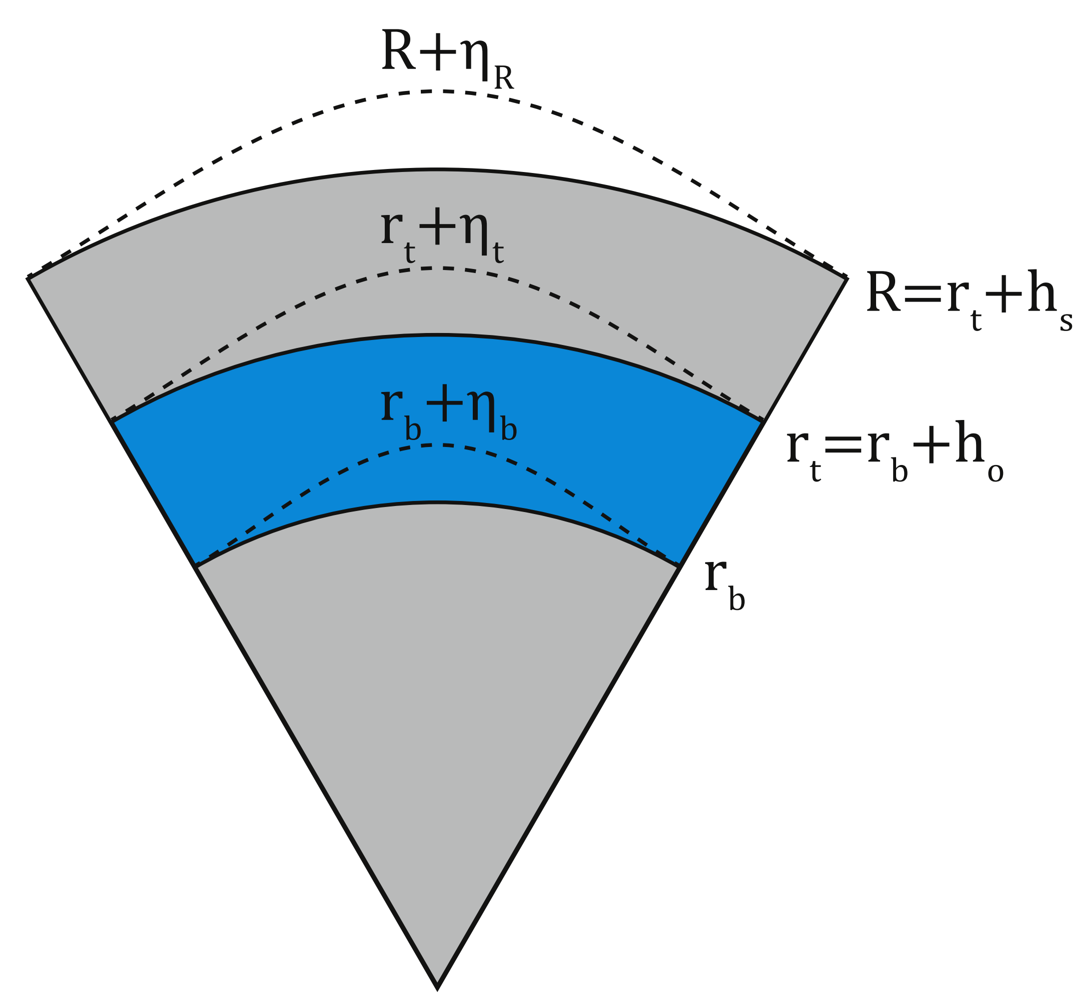
$$\cancel{\partial_t \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} + \alpha \mathbf{u} + \frac{c_D}{h_o} |\mathbf{u}| \mathbf{u}} = -g\beta \nabla \eta + \upsilon \nabla U^T$$

$$\mathbf{u} = 0$$

    - Equilibrium tides (u=0):

$$\eta = \eta_{eq} \equiv \frac{\upsilon}{\beta} \frac{U^T}{g} = h^T(r_t)\frac{U^T}{g} - h^T(r_b)\frac{U^T}{g}$$

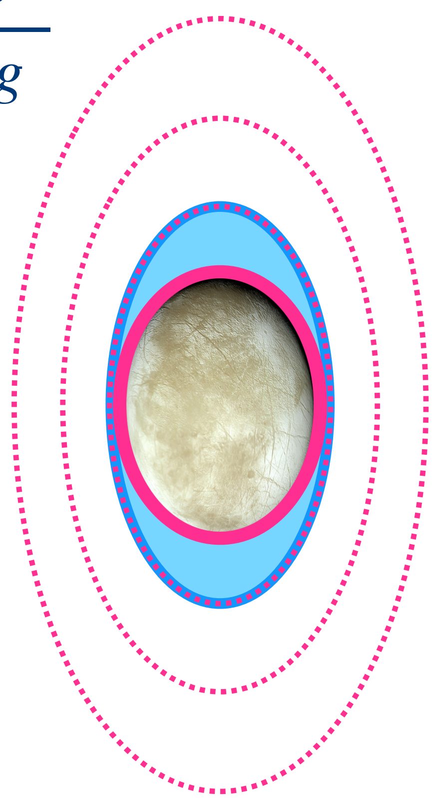
    - Dynamic tides (u≠0)



Equilibrium tides

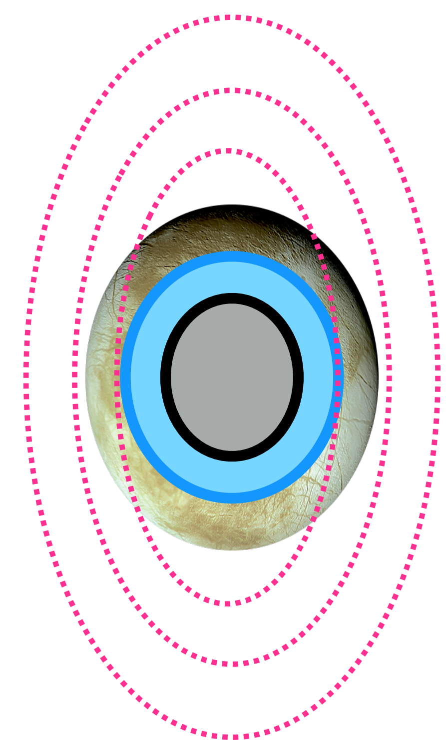
Surface ocean

$$\eta_{eq} = \frac{\gamma^T}{(1 - \xi\gamma^L)}\frac{U^T}{g}$$



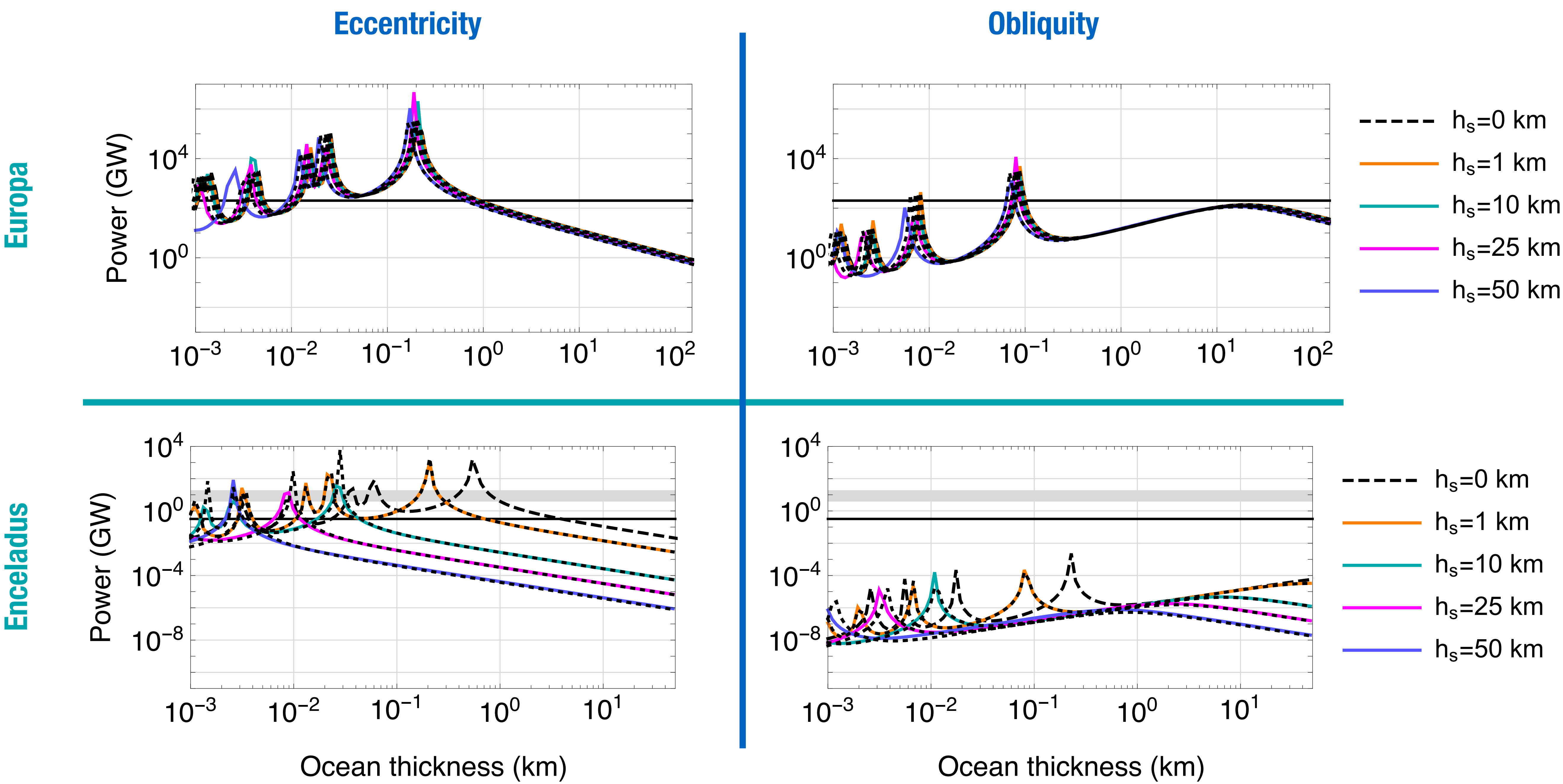
Subsurface ocean

$$\eta_{eq} \equiv \frac{\upsilon}{\beta}\frac{U^T}{g}$$



- Stresses in the shell prevent the ocean from following an equipotential surface

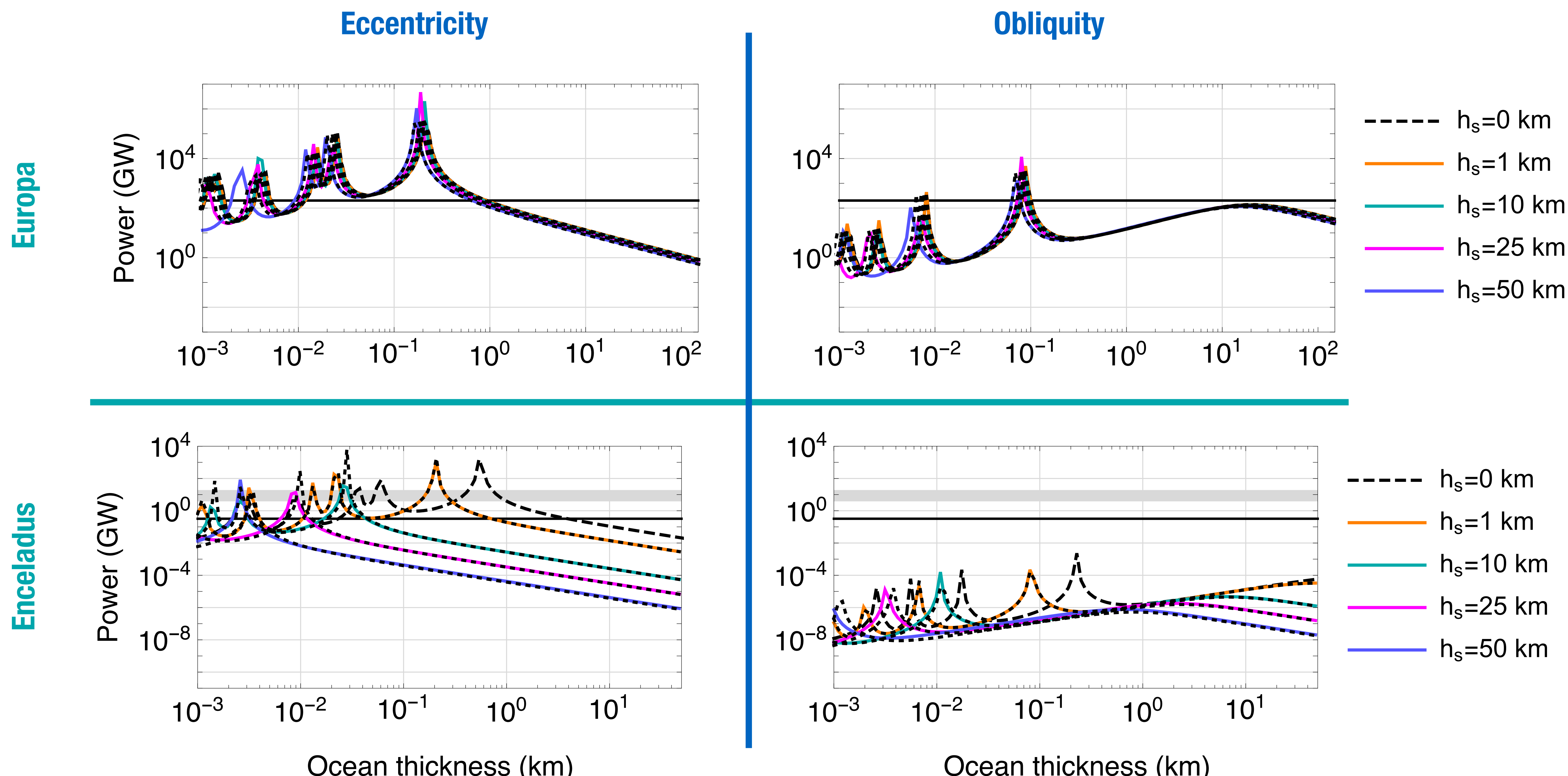
Linear friction Results (α~10^{-7} s^{-1})



Matsuyama et al. 2018

- Core density is adjusted to satisfy the mean density constraint
- The effect of an overlying shell is smaller for Europa (due to its small effective rigidity)
- The shell's resistance to ocean tides increases with shell thickness, reducing tidal heating
- Resonant ocean thicknesses decrease with shell thickness

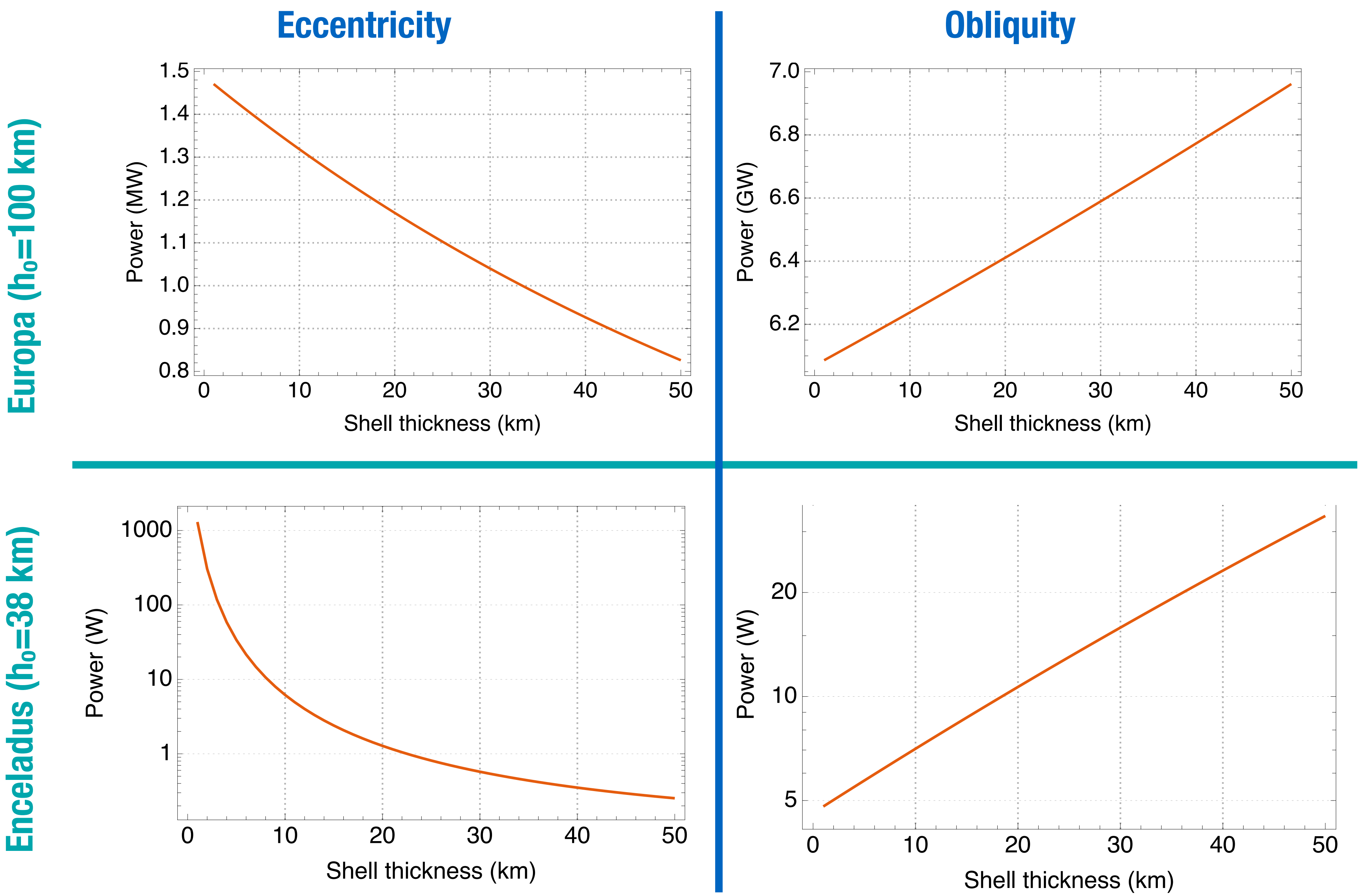
Linear friction Results (α~10^{-7} s^{-1})



Matsuyama et al. 2018

- Not possible to explain Enceladus' endogenic power radiated from the south polar terrain (Spencer et al. 2016) assuming the shell and ocean thicknesses inferred from gravity and topography data (shell thickness ~20 km, ocean thickness ~40 km, Beuthe et al 2016, Hemingway et al. 2017)
- Ocean tidal heating is generally weaker than radiogenic heating
    - Europa: obliquity tidal heating is comparable to radiogenic heating if ocean thickness ~20 km

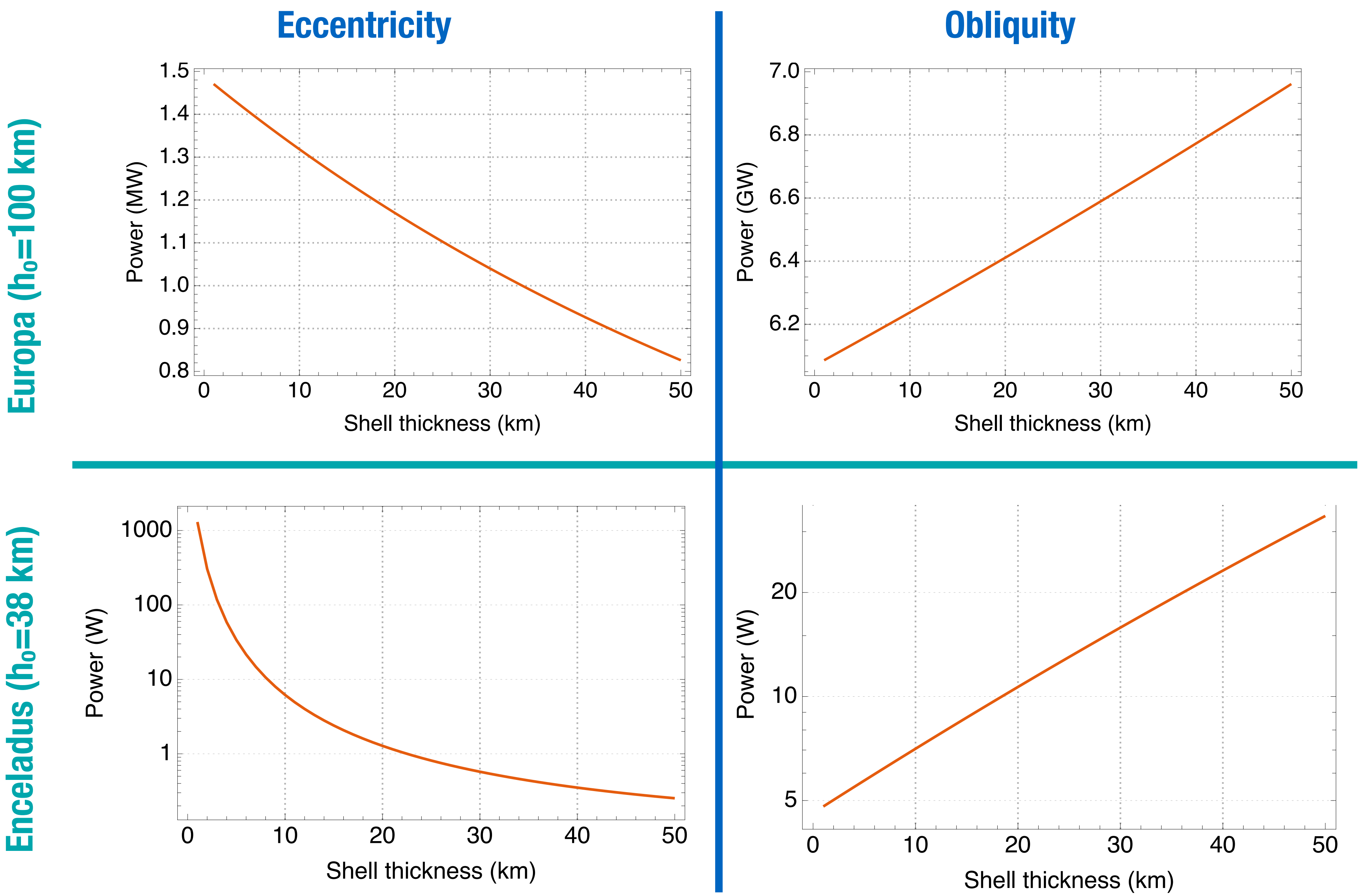
Bottom friction Results ($c_D=4\times10^{-3}$)

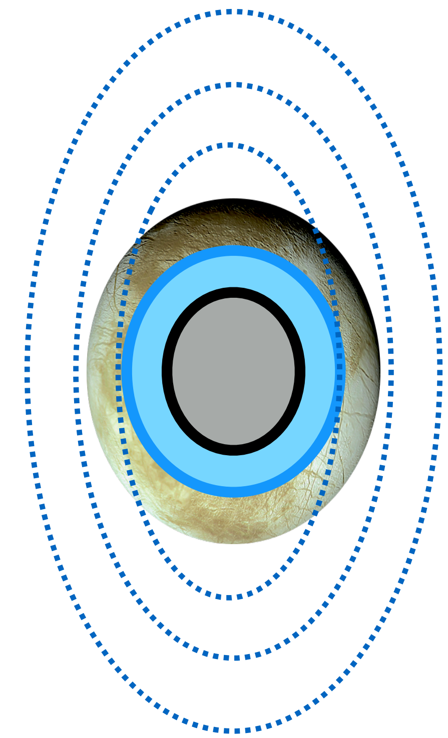


Scaling law solutions from Hay & Matsuyama (2019), based on Chen & Nimmo (2014)

- Core density is adjusted to satisfy the mean density constraint
- The effect of an overlying shell is smaller for Europa (due to its small effective rigidity)
- Ocean tidal heating is weaker than radiogenic heating (~0.3 GW for Enceladus, ~200 GW for Europa)

Bottom friction Results ($c_D=4\times10^{-3}$)

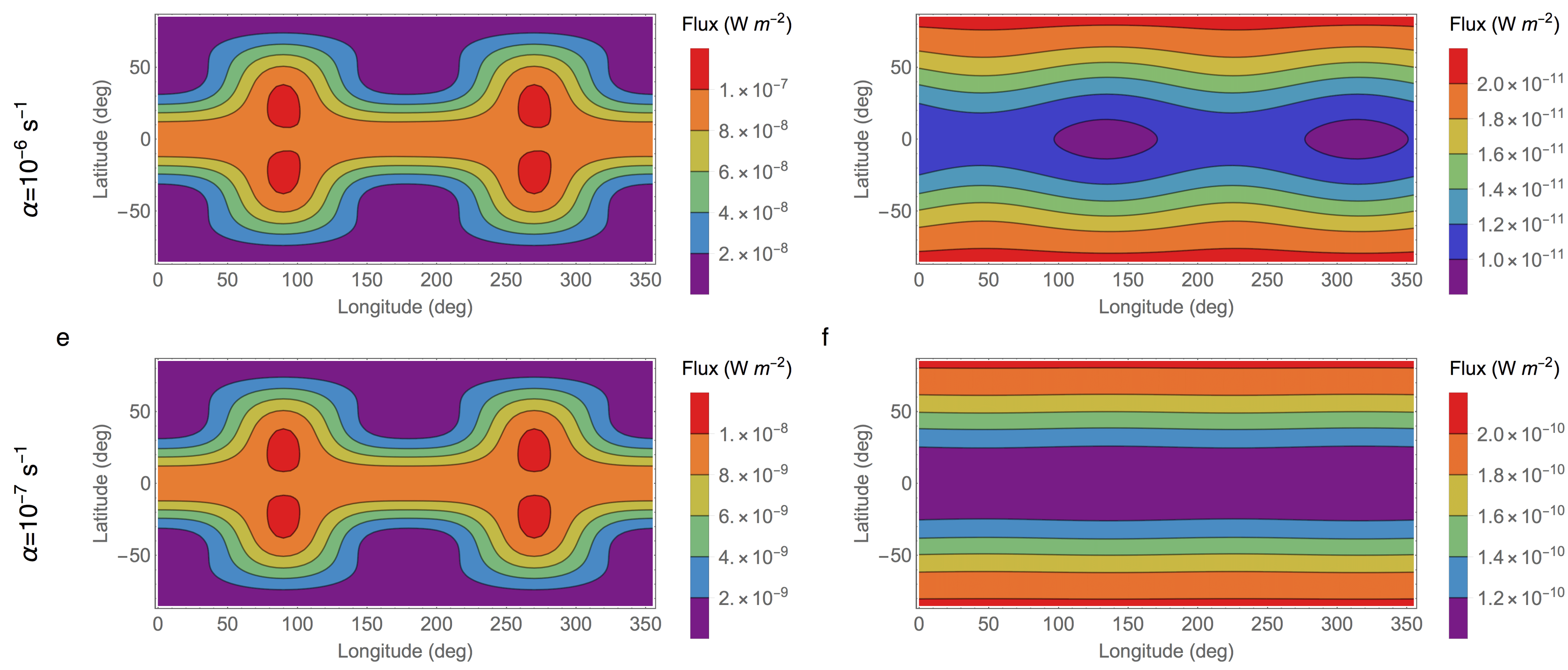




- Energy dissipation can increase or decrease with shell thickness
  - Stresses in the shell damp radial displacements
  - A thicker, deformed shell can generate a larger amplification of the forcing potential, increasing energy dissipation
  - Eccentricity forcing generates gravity waves with radial displacements
  - Obliquity forcing generates gravity waves and Rossby waves with no radial displacement

Enceladus: time-averaged heat flux (linear drag)

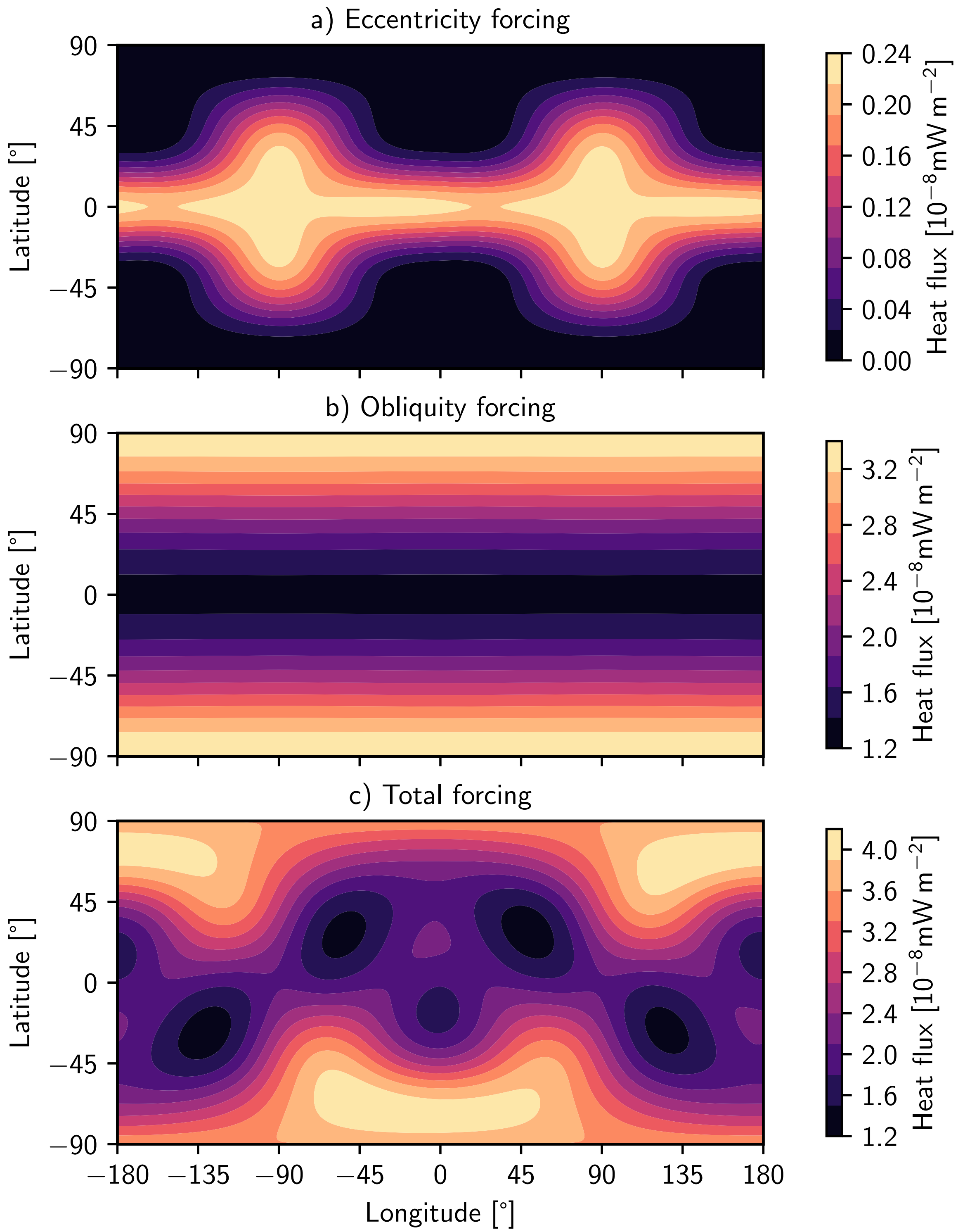
Eccentricity **Obliquity**

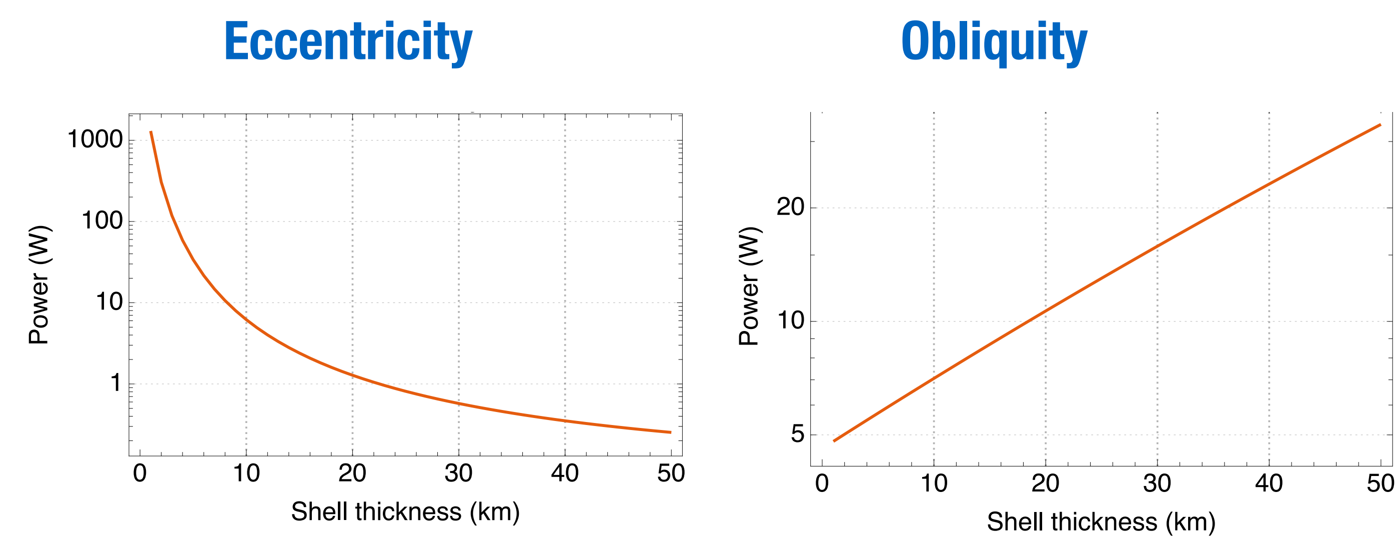


Matsuyama et al. 2018

- Assuming the likely shell and ocean thicknesses inferred from gravity and topography constraints (shell thickness = 23 km, ocean thickness = 38 km, Beuthe et al. 2016)
- The time-averaged surface distribution of ocean tidal heating is different from that due to dissipation in the solid shell
- This can lead to unique shell thickness variations if the shell is conductive

Enceladus: time-averaged heat flux (cD=4×10^{-3})



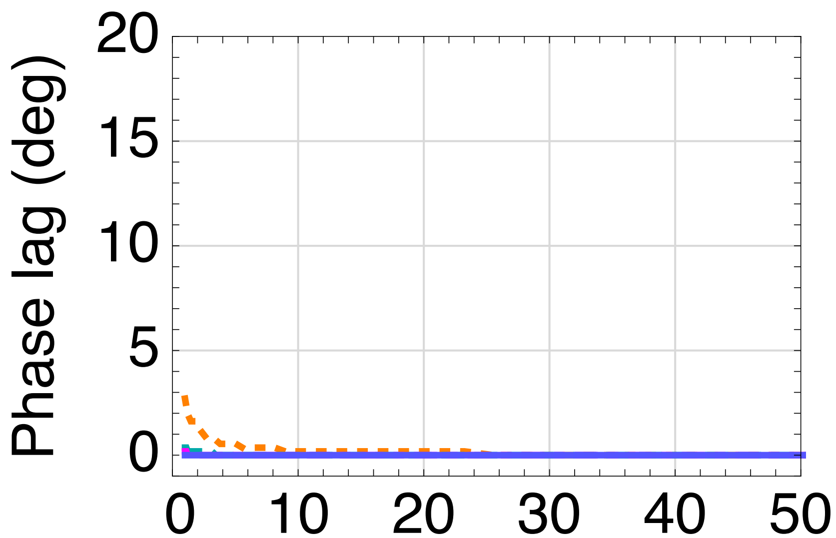


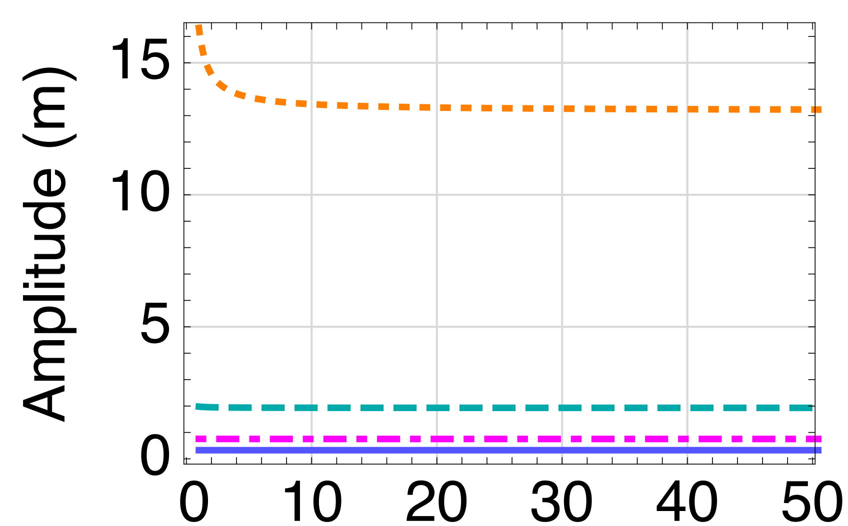
- Eccentricity and obliquity forcing contributions become comparable despite Enceladus' small obliquity
  - e=0.0047, obliquity=0.00045 deg assuming a Cassini state (Chen & Nimmo, 2011; Baland et al. 2016)
- Unique ocean tidal heating pattern
- But the dissipation flux is very small

Hay & Matsuyama (2019)

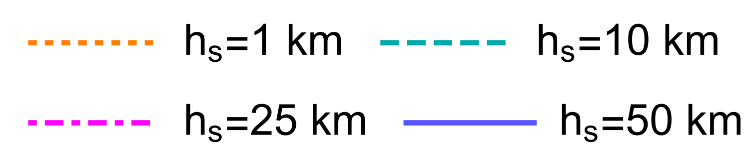
Enceladus: dynamic surface displacement phase lag due to the delayed ocean response

Eccentricity tide, α=10^{-5} s^{-1}

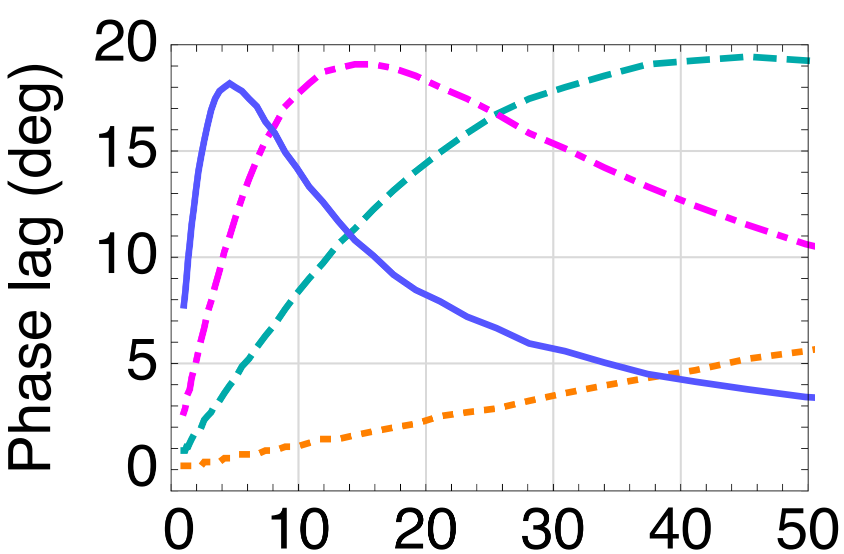


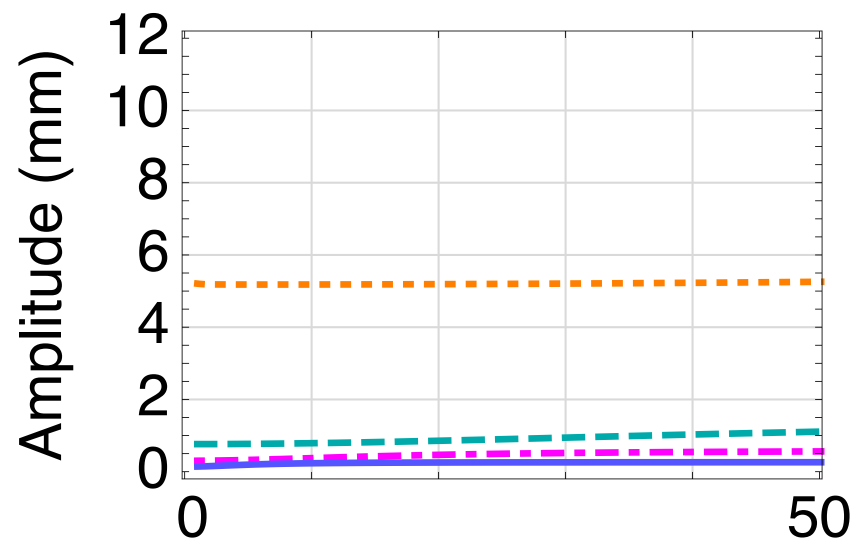


- **Phase lag < 2.5 deg**
- **Amplitude:**
  - **~13 m (1 km thick shell)**
  - **~1 m (10 km thick shell)**



Obliquity tide, α=10^{-8} s^{-1}





Ocean thickness (km)

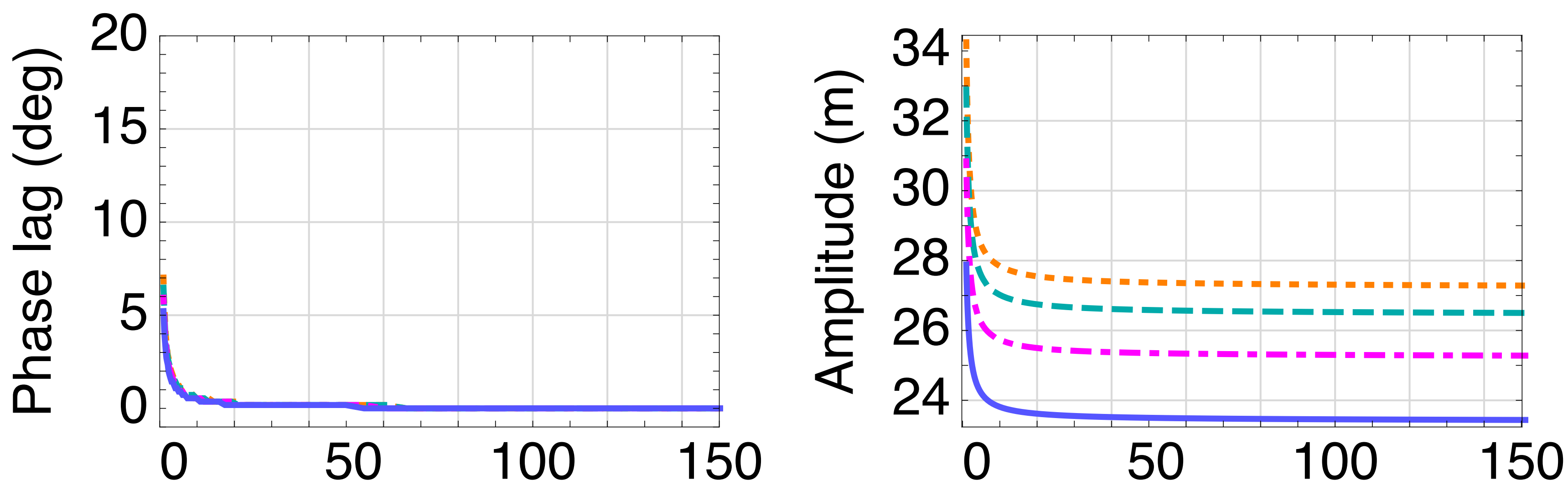
Ocean thickness (km)

- **Phase lag can be as large as ~20 deg, sensitive to ocean thickness**
- **Amplitude:**
  - **~6 mm (1 km thick shell)**
  - **~1 mm (10 km thick shell)**

Matsuyama et al. 2018

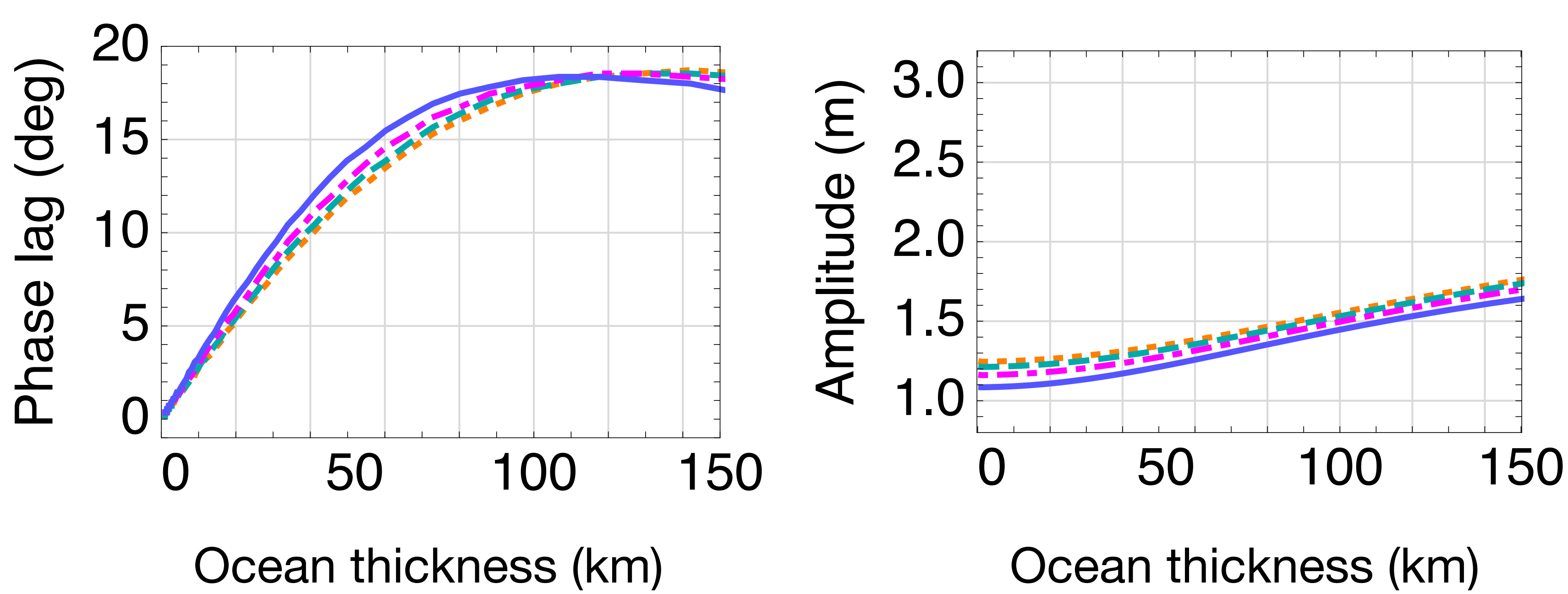
Europa: dynamic surface displacement phase lag due to the delayed ocean response

Eccentricity tide, $\alpha=10^{-5}\ s^{-1}$



- **Phase lag < 7 deg**
- **Amplitude: ~23 - 27 m**

Obliquity tide, $\alpha=10^{-8}\ s^{-1}$



- **Phase lag can be as large as ~20 deg, sensitive to ocean thickness**
- **Amplitude: ~1-2 m**

Matsuyama et al. 2018

Summary

- Ocean tidal heating can be resonantly enhanced but this requires very thin oceans
- The shell's resistance to radial tides increases with shell thickness, reducing tidal heating
- The deformed shell's gravitational potential can enhance tidal heating
- Obliquity tidal heating becomes comparable or larger than eccentricity tidal heating when the effect of an overlying shell is taken into account
- The time-averaged surface distribution of ocean tidal heating is distinct from that due to dissipation in the solid shell
- The dynamic surface displacement can have phase lags relative to the forcing tidal potential due to the delayed ocean response
  - Measurement of the obliquity phase lag (e.g. by Europa Clipper) would provide a probe on ocean thickness
- Characterizing the expected horizontal shell thickness variations requires:
  - Solving the coupled thermal-orbital evolution problem
  - Coupling ocean dissipation with shell dissipation (enhanced tidal deformation due to dynamic ocean tides would also enhance dissipation in the shell)