

Introduction to Data Analysis of Gravitational Wave Signals.

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Overview

LIGO-VIRGO

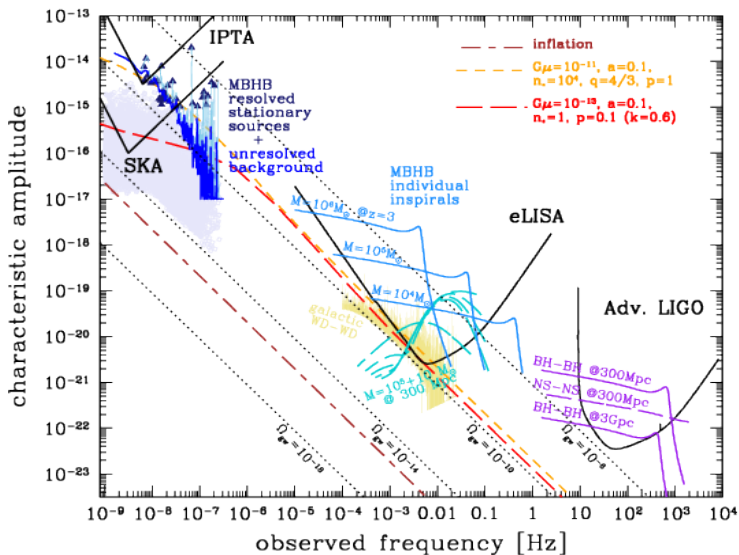
DA basics

LISA

GW sources in LISA band

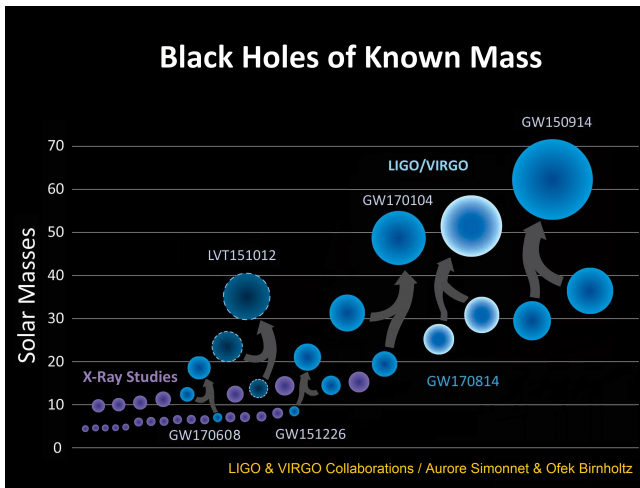
LISA data analysis

Gravitational waves landscape

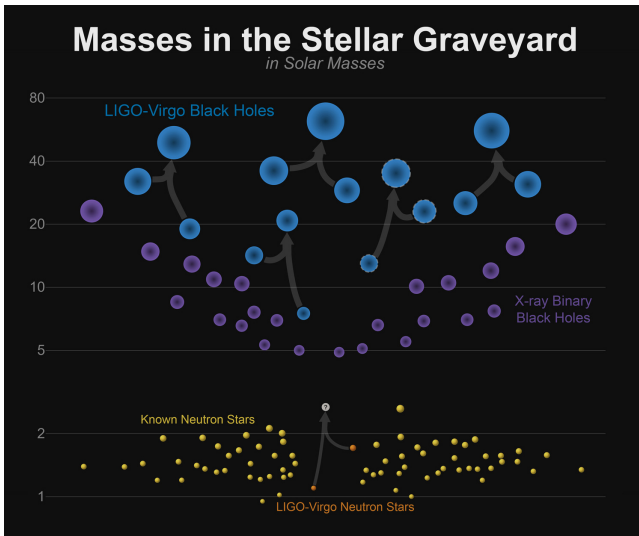


[credits: A. Sesana]

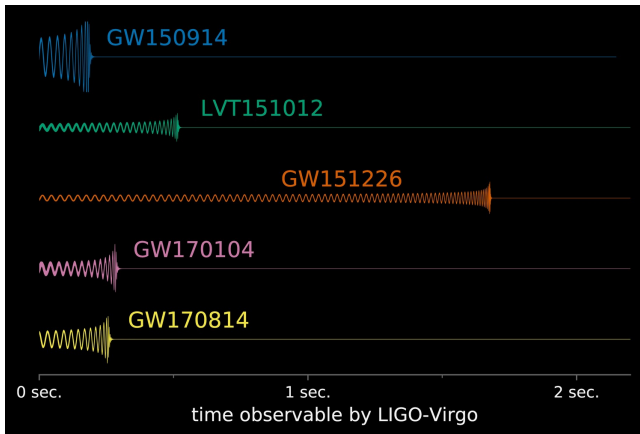
Detection of GW with LIGO



Detection of GW with LIGO



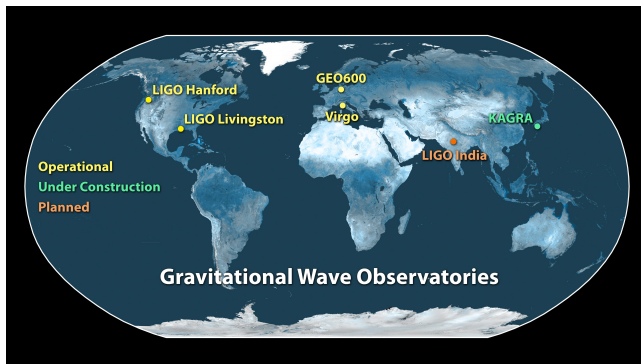
Detection of GW with LIGO



[Image credit: LIGO/B. Farr]

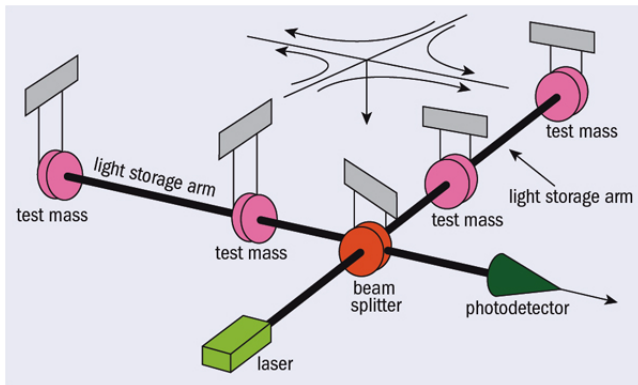
GW observatories on the ground

The network of the GW observatories on the ground.



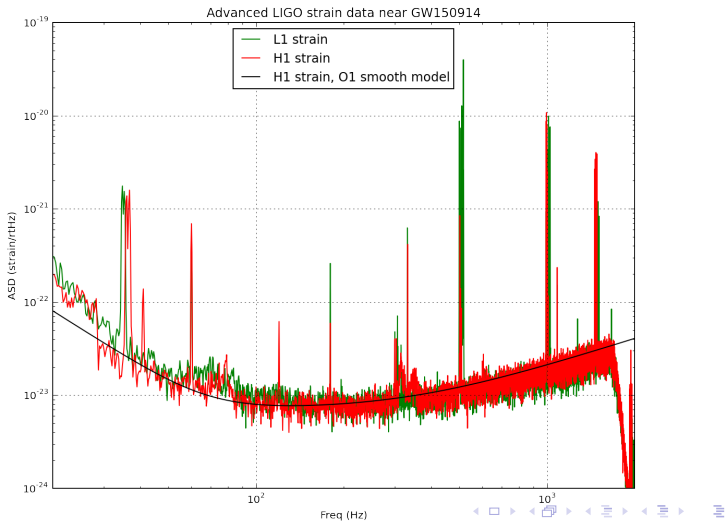
Laser interferometers as GW detector

Over-simplified scheme of LIGO interferometer.



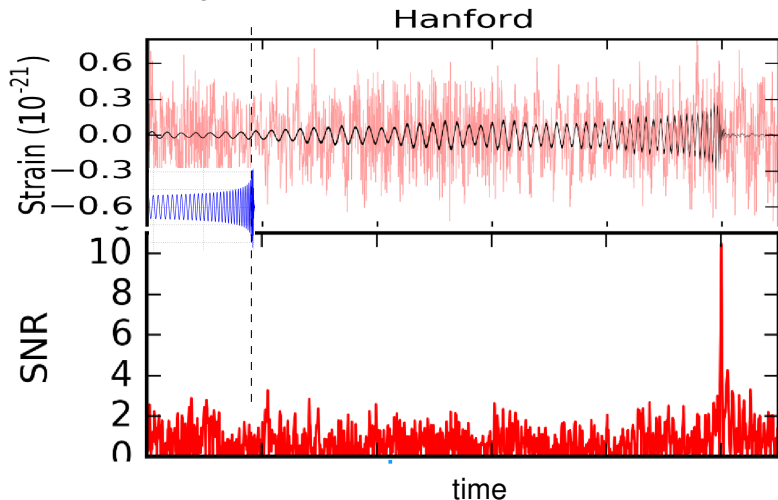
Laser interferometers as GW detector

Sensitivity during O1/O2



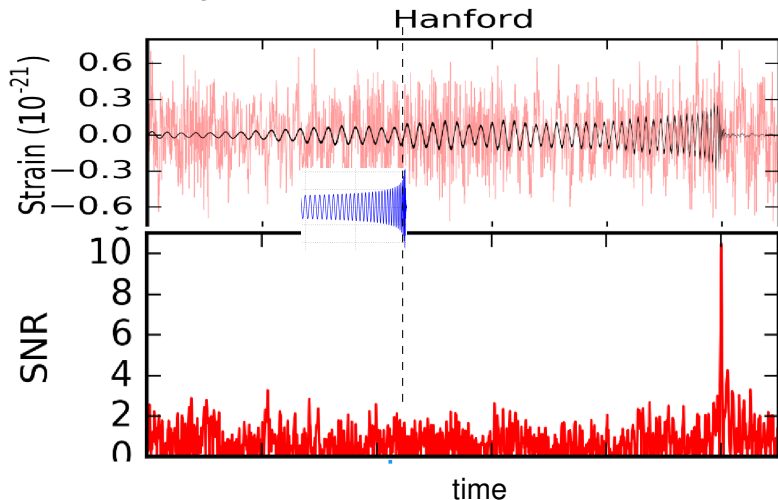
Matched filtering

We are searching for a signal of a specific shape buried in the noise: matched filtering.



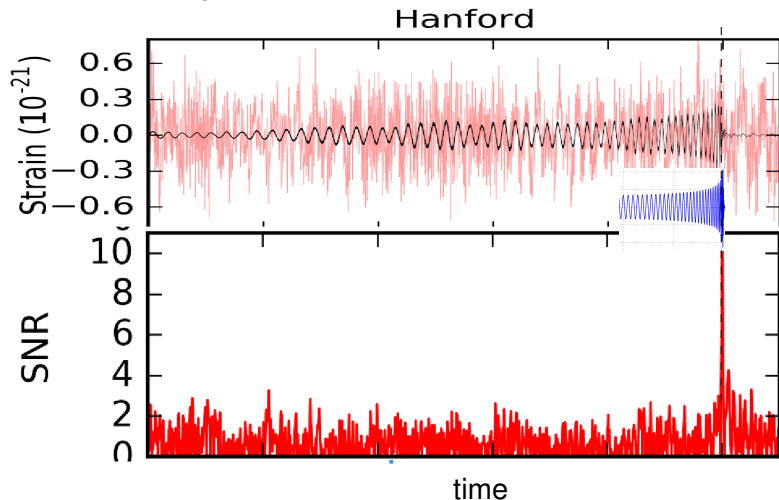
Matched filtering

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Matched filtering

We are searching for a signal of a specific shape buried in the noise: matched filtering.



Matched filtering

- Assume that the data $d(t)$ consists of the noise $n(t)$ and a signal $s(t)$: $d(t) = n(t) + s(t)$.
- The matched filtering is optimal for the signal of a known shape. It is done in frequency domain: $\tilde{d}(f) = FFT(d(t))$.
- The matched filtering takes into account that the sensitivity of the detector is not the same across the frequency band:

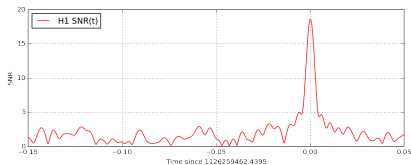
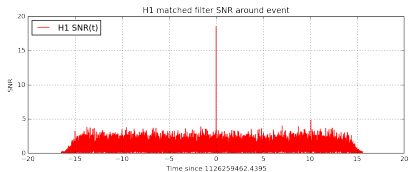
$$\rho = 4\Re \int_0^\infty df \frac{\tilde{d}(f)\tilde{h}^*(f)}{S_n(f)}$$

- Here we search for a signal of a specific shape $\tilde{h}(f)$ (or $h(t)$). Note that $h(t)$ and the signal $s(t)$ might not be identical, but need to be as close as possible.
- If we average over the noise realizations and assume that the template and the signal are identical:

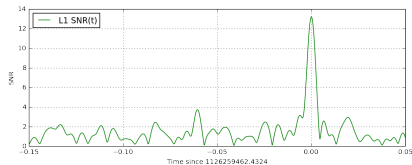
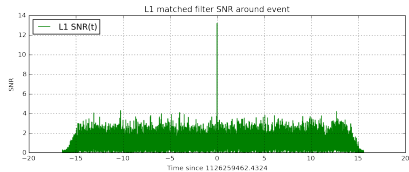
$$\bar{\rho} = 4\Re \int_0^\infty df \frac{\tilde{s}(f)\tilde{s}^*(f)}{S_n(f)} = SNR^2$$

Matched filtering: GW150914

H1



L1

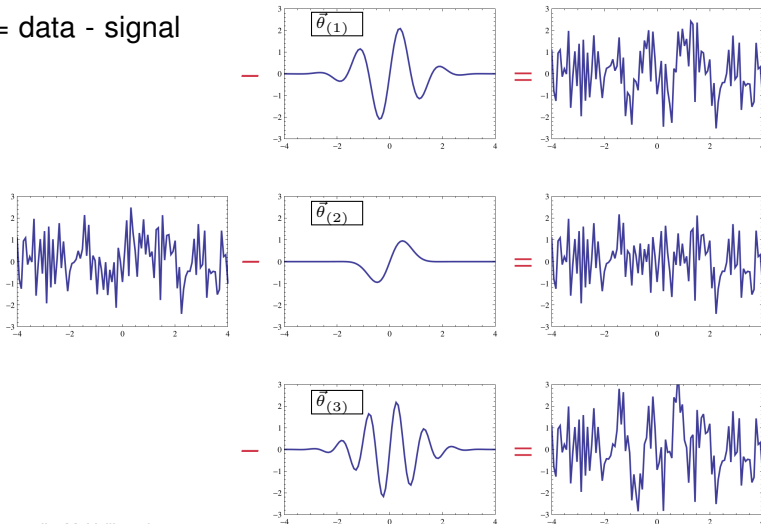


[LOSC: <https://losc.ligo.org/tutorials/>]



Matched filtering and parameter estimation

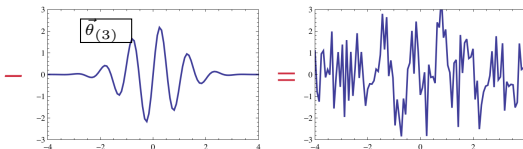
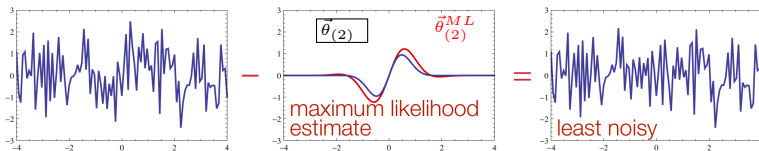
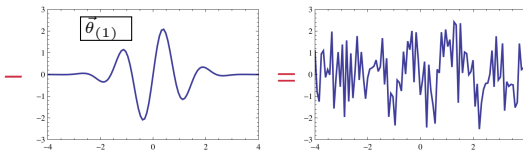
noise = data - signal



credits M. Vallisneri

Matched filtering and parameter estimation

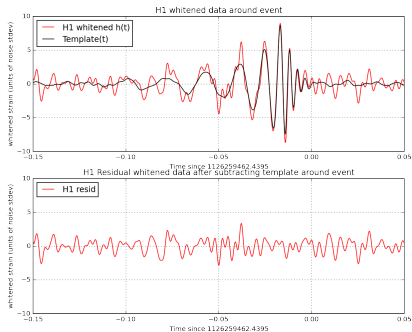
noise = data - signal
 $p(\text{signal parameters})$
 $= p(\text{noise residuals})$



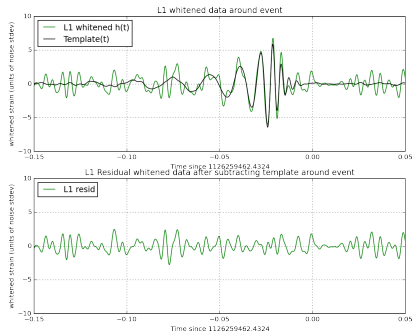
credits M. Vallisneri

Matched filtering and parameter estimation: GW150914

H1



L1



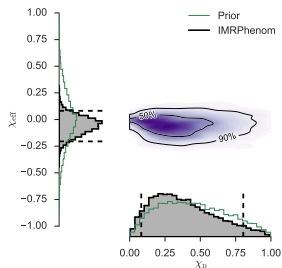
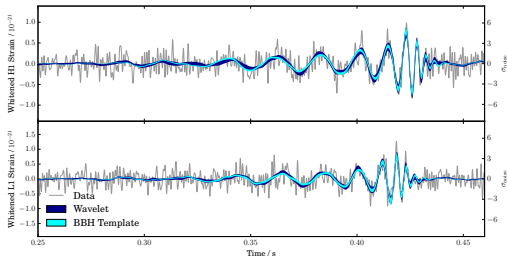
[LOSC: <https://losc.ligo.org/tutorials/>]



Parameter estimation

prior

$$p(\theta|d) = \frac{p(d|\theta)p(\theta)}{p(d)}$$



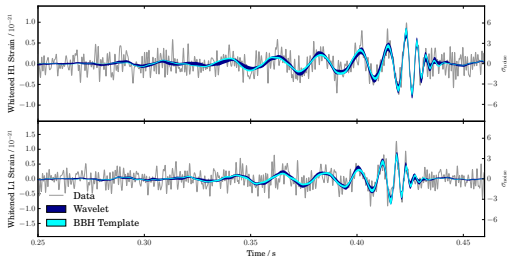
[GW150914, LSC+Virgo PRL (2016)]



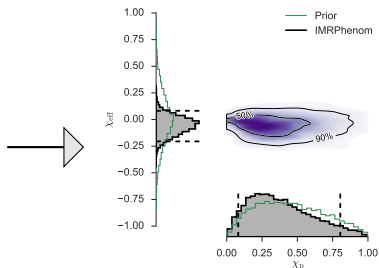
Parameter estimation

likelihood

$$p(\theta|d) = \frac{p(d|\theta)p(\theta)}{p(d)}$$



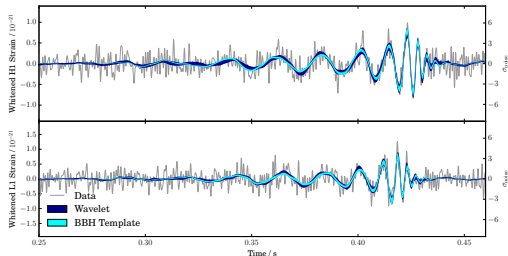
[GW150914, LSC+Virgo PRL (2016)]



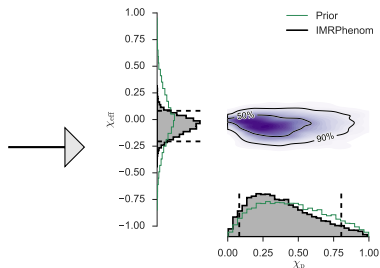
Parameter estimation

posterior

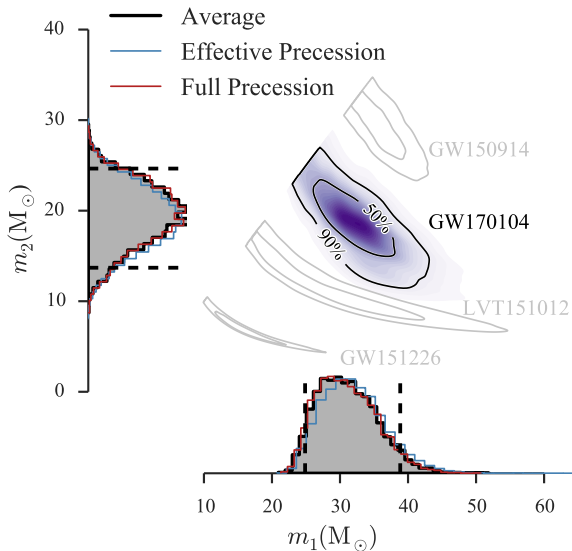
$$p(\theta|d) = \frac{p(d|\theta)p(\theta)}{p(d)}$$



[GW150914, LSC+Virgo PRL (2016)]

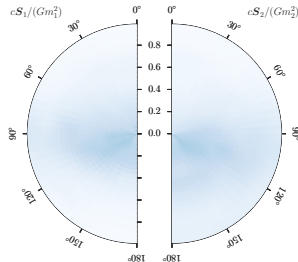


Inferred parameters of binary systems: masses

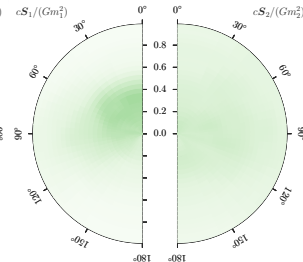


Inferred parameters of binary systems: spins

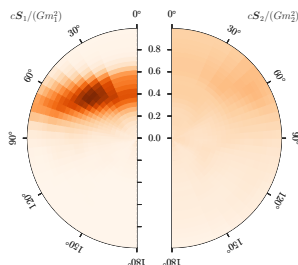
GW150914



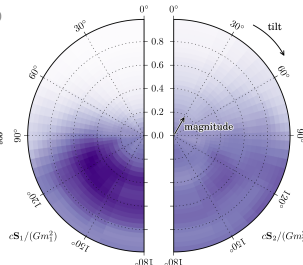
LVT151012



GW151226



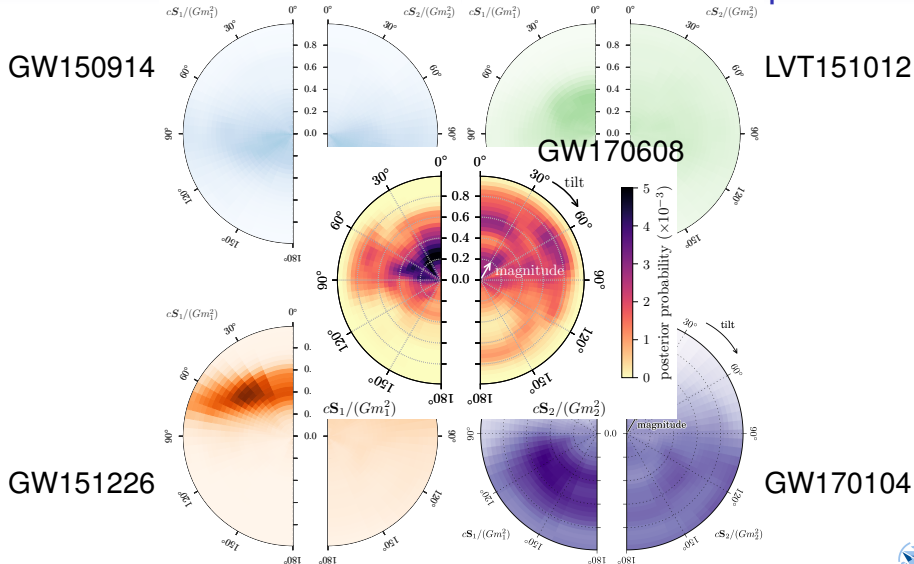
GW170104



[LSC+Virgo PRL (2016, 2017)]

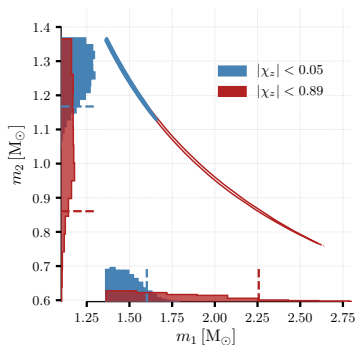
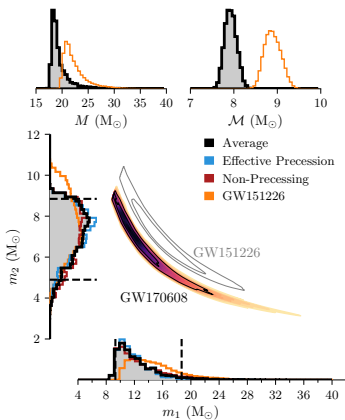


Inferred parameters of binary systems: spins



[LSC+Virgo PRL (2016, 2017)]

Correlations and degeneracies



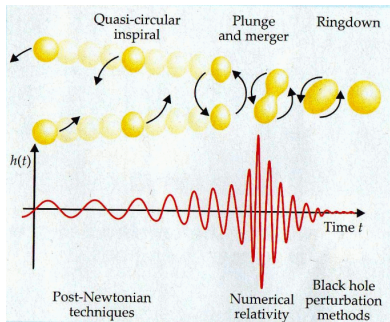
BNS: GW170817

[LVC PRL, 2017]

[LVC, ApJL, 2017]

Waveform modeling

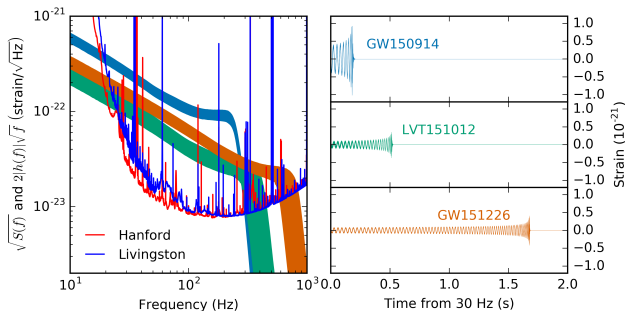
The GW signal from binary system can be conventionally split in “inspiral”, “merger” and “ringdown”



The most accurate description of GW signal is given by numerical solution of GR for two body problem. Requires large computational resources and long time: short waveforms (~ 20 cycles). Simulations are not very reliable for large spins and large mass ratio.

GW signals in frequency domain

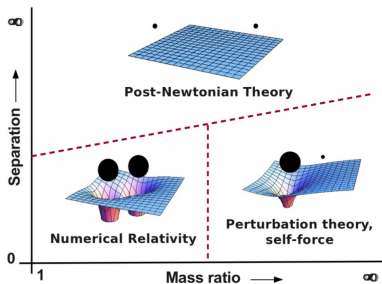
Low mass systems: we see mainly inspiral. High mass binaries we see mainly merger-ringdown.



Waveform modeling

- **Post-Newtonian (PN) approximation**: slow motion approximation ($v/c \ll 1$)
- **Perturbative theory**: small mass ratio ($m/M \ll 1$)
- **Effective-One-Body approach**: combination of PN, Perturbative approach and NR

Binary parameter space



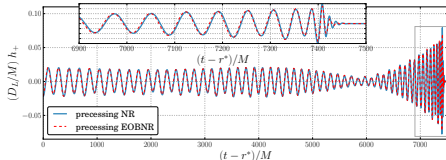
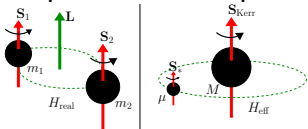
[Leor Barack]



Waveform modeling: Effective-One-Body and Phenomenological Model

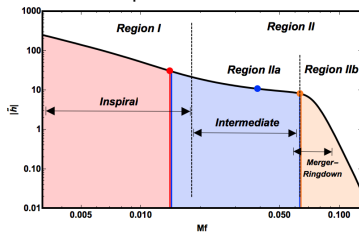
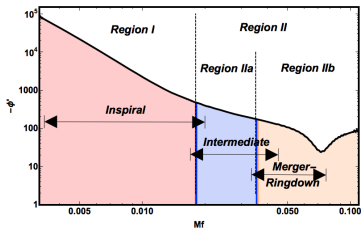
EOB: mapping two body problem to a test mass moving in effective perturbed Kerr spacetime

Real problem → **Effective problem**



[Babak+ PRD, 2016]

IMRPhenom: phenomenological approach uses PN for inspiral and fit for the late IMR

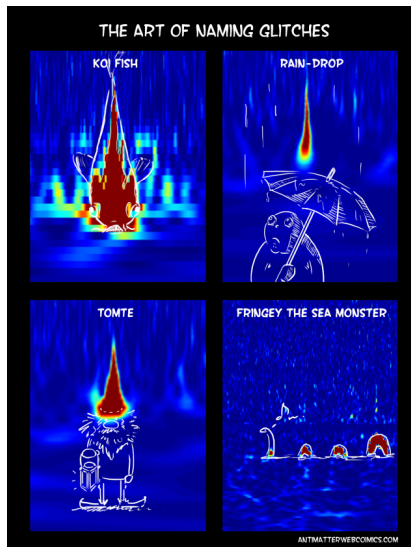


[Khan+ PRD, 2016]

Non-stationary noise

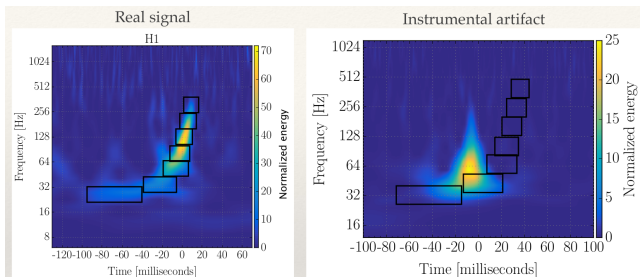
The noise is non-stationary.

- There are various artifacts (glitches): produce high SNR
- Cross-detector consistency eliminates most
- Some are understood, some not



Non-stationary noise

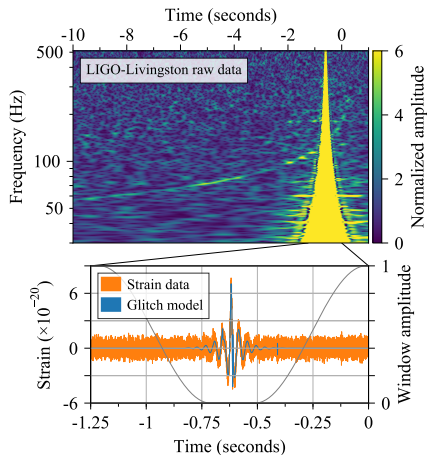
We check the consistency of the candidate event with what we expect from the GW signal in time-frequency.



[credits: I. Harry]

Non-stationary noise

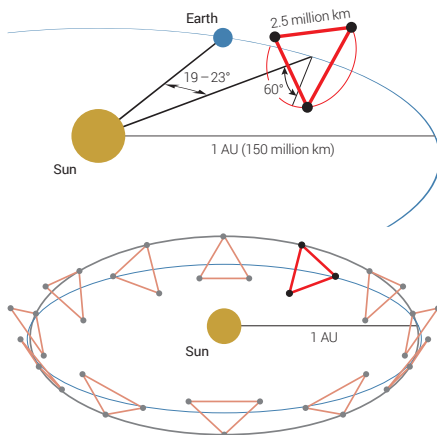
The most unfortunate case is the glitch on top of the GW signal: binary NS GW signals GW170817.



[LVC PRL, 2017]

LISA: laser interferometer in space.

- LISA is a future space based GW observatory, to be launched around 2034.
- LISA Pathfinder: very successful demonstration of LISA technologies.



LISA vs ground based ifo: what are the common problems

- Modelling GW signal from BH binaries: the signal is total mass invariant (t/M , fM)
- Continuous GW signals: single deformed NS in LIGO band, white dwarf binaries in LISA band
- Matched filtering: the main technique to search and estimate parameters
- Non-stationary noise: 🤔

LISA vs ground based ifo: what is different

- **Data size is small:** (does not require large data storage, but the data products could be significant)
- **Data is dominated by signals:** (requires global fit \rightarrow multi- (trans) dimensional fit)
- **TDI (time delay interferometry):** (the main source of noise is the laser frequency noise in measuring $\delta\nu/\nu$, requires time delayed combination of measurements to cancel this noise)
- **Non-trivial response function:** (many signal have $\lambda_{GW} \sim L$, it requires computation of response beyond the long wavelength approximation)
- **GW signals are long lived:** (signals from the same and different kind of sources are present simultaneously in the data, noise fluctuations are significant)
- **Some GW signals are strong:** (requirement on modelling and hierarchy of the search)



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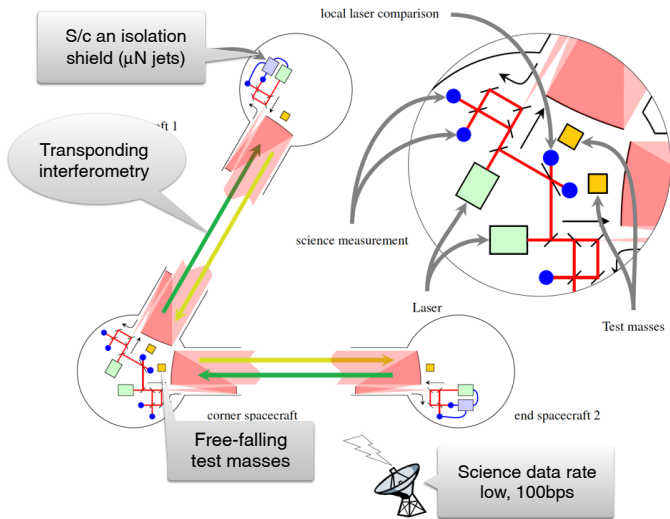
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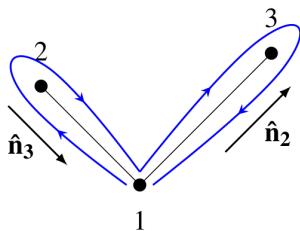
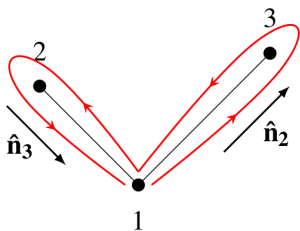
LISA' operating concept

LISA uses transponding laser interferometry.



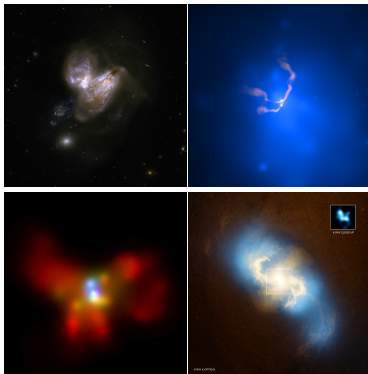
TDI: unequal arm problem.

The laser frequency noise emitted at s/c 1 travels to the s/c 2, 3. We need to take measurements after the light has traveled equal optical path in different directions and subtract them: noise cancels exactly, but not the GW signal.



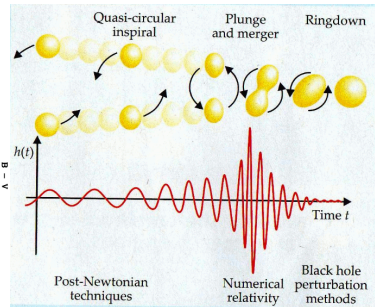
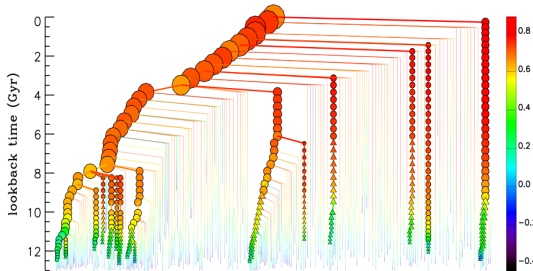
Sources: MBH binaries

- We expect that all galaxies host MBH in their nuclei. (Milky Way, S-stars: $4 \times 10^6 M_{\odot}$ BH)
- We know that galaxies merge: formation of MBH binaries
- We need to bring pair of MBHs close together (interaction with gas, stars)



Sources: MBH binaries

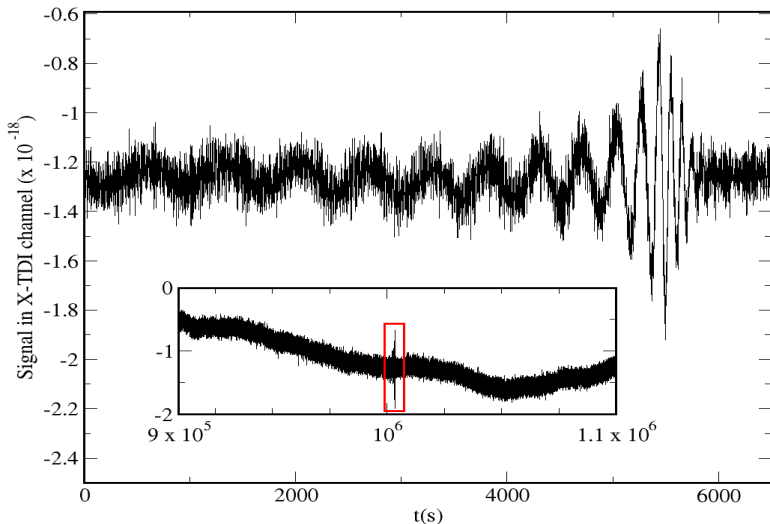
The MBHs are formed from initial seeds (large or small) by accreting gas and by mergers



[credits: Gabriella De Lucia, Baumgarte and Shapiro]

Main features of the signal from MBH binaries

Some signals are very strong and can be seen by eye:



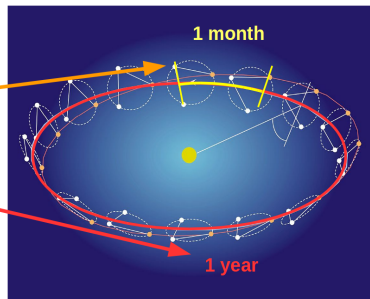
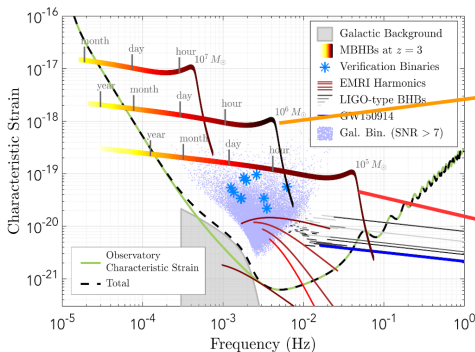
Main features of the signal from MBH binaries

- We use matched filtering: need to know the model for GW signal very well
- The signal depends (in general) on 15 parameter (might be eccentric)
- Duration of the signal: from few weeks to almost a year (source and detector dependent)
- We will be able to detect precession and subdominant modes.



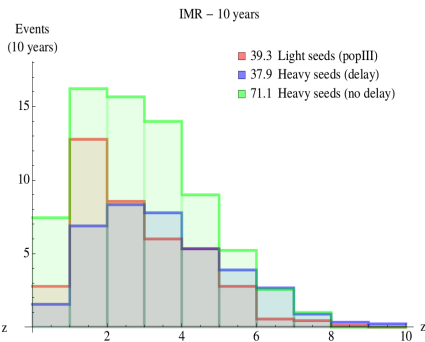
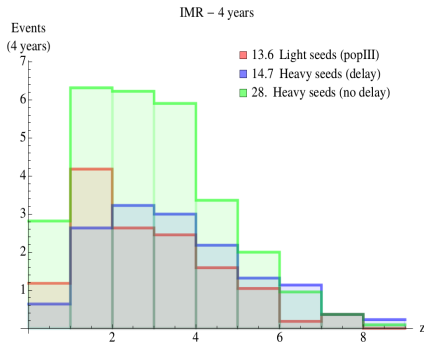
Estimating parameters of binary systems

- We are the most sensitive to the phase of GW signal: contains info about masses, spins of individual objects ($\delta M/M \sim 0.001 - 0.1, \delta\chi \sim 0.01 - 0.1, \delta\theta_{1,2} \sim 1 - 10^\circ$)
- Amplitude contains information about spin orientation, distance to the binary, polarization, sky location (but need to disentangle)
- Sky position: modulation of the amplitude (angular sensitivity as a function of time) and doppler modulation of the phase



MBHBs: Event rate

| | SNR > 8 | $\Delta\Omega <$ 10 deg^2 | LSST | SKA (radio only) | | | SKA + ELT | | | Total |
|-------------------------|------------|--|-------|------------------|------|-------|-----------|-------|-------|-------|
| | | | | Flare | Jet | Total | Spec | Photo | Total | |
| Insp+MR 4 years | 209 | 15.3 | 0.773 | 11.3 | 13.3 | 14.8 | 6.64 | 7.00 | 13.6 | 13.6 |
| | 30.6 | 15.9 | 2.45 | 15.6 | 11.5 | 15.9 | 6.91 | 7.82 | 14.7 | 14.7 |
| | 472 | 163 | 0.909 | 58.9 | 9.18 | 59.5 | 9.50 | 18.5 | 28.0 | 28.0 |
| Insp+MR 10 years | 563 | 45.8 | 1.67 | 34.2 | 40.2 | 44.2 | 17.9 | 21.4 | 39.3 | 39.3 |
| | 76.2 | 42.1 | 6.44 | 41.3 | 28.9 | 42.1 | 17.3 | 20.6 | 37.9 | 37.9 |
| | 1180 | 424 | 2.56 | 148 | 23.6 | 150 | 25.6 | 45.6 | 71.1 | 71.1 |



Extreme mass ratio inspirals (EMRIs)

- MBH in galactic nuclei is surrounded by a dense cusp of stars
- A compact object (NS, BH, WD) could be thrown toward MBH as a result of N-body interaction
- A compact object might be captured on a very eccentric orbit and slowly spirals toward central hole until the final plunge
- Extreme mass ratio inspiral (EMRI) - typical mass ratio $10^{-7} - 10^{-5}$
- Compact object spends $\sim 10^6$ orbits close the central MBH in the LISA band

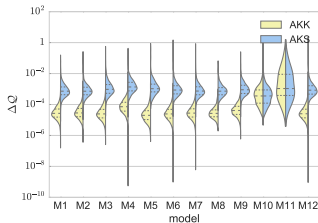
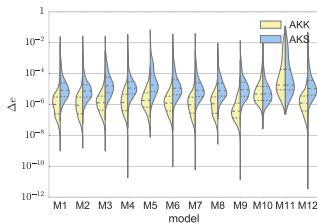
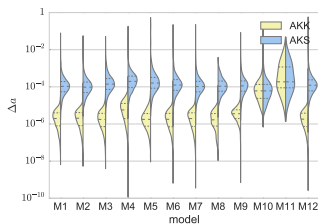
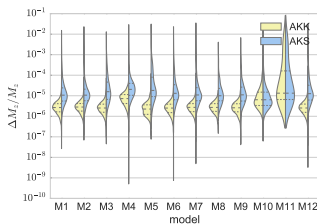


EMRIs waveform

- The GW signal from EMRIs is rich in structure: three-periodic motion with slowly evolving frequencies
- "Holidodesy" – mapping spacetime of the central object: good for testing GR.

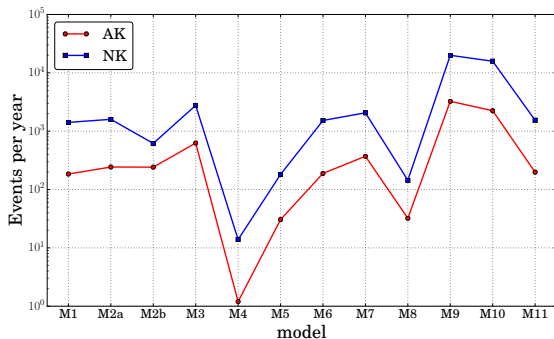


- We can detect EMRIs down to $\text{SNR} \sim 20$ (MLDC)
- Rich structure allows ultra-precise parameter estimation, including measuring the multipole moments of a central massive object (holiodesy)



EMRIs event rate

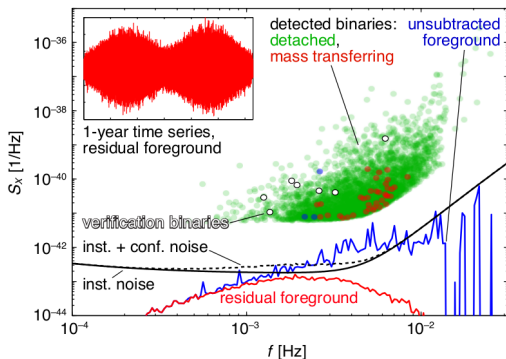
The expected event rate is very uncertain. We have used several models for MBHs distribution and for the compact objects (10 or 30 M_{\odot}). Green and red curves are bounding the event rate for each model.



[Babak+ (2017)]

Galactic white dwarf binaries

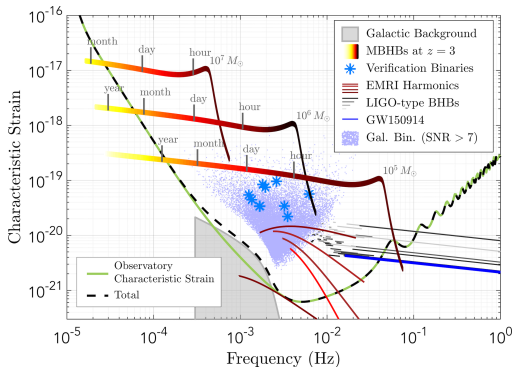
- LISA will observe $\sim 10^7$ Galactic WD binaries, only $\sim 10^4$ are resolvable others form GW stochastic signal
- GW signal is almost monochromatic with slow evolution of freq. (GW radiation and mass transfer)
- Verification binaries (GAIA, LSST)



LISA data analysis

- The data is signal dominated: many signal overlap in time and in frequency
- The problem is to detect and characterize as many signals as possible: LISA data challenge has restarted:

<https://lisa-ldc.lal.in2p3.fr/home>



[white paper arXiv:1305.5720]

Data Analysis: matched filtering

- **Matched filtering** has proven to be the way to accurately estimate parameters of the signals and to disentangle the GW signals. In LISA we will need to construct a multi-source/signal template allowing variable number of signals and glitches
- **Stochastic methods** are the most successful data analysis search methods. (as compared to the grid-based search adopted in LIGO-VIRGO data analysis)
- The stochastic methods: Parallel tempering Markov Chain Monte Carlo, Reverse Jump Markov Chain Monte Carlo, Nested sampling, Multimodal Genetic Algorithm



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Past Mock LISA Data Challenge

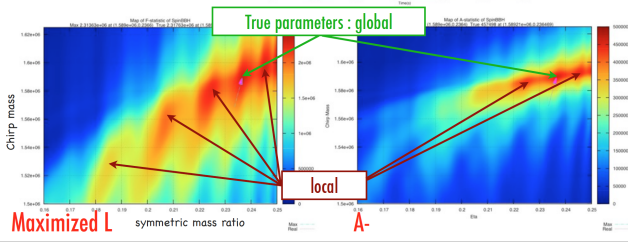
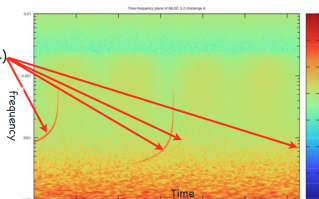
| | MLDC 1 | MLDC 2 | MLDC 1B | MLDC 3 | MLDC 4 |
|-----------------------|---|--|---|--|--|
| Galactic binaries | <ul style="list-style-type: none"> • Verification • Unknown isolated • Unknown interfering | <ul style="list-style-type: none"> • Galaxy 3×10^6 | <ul style="list-style-type: none"> • Verification • Unknown isolated • Unknown interfering | <ul style="list-style-type: none"> • Galaxy 6×10^7 chirping | <ul style="list-style-type: none"> • Galaxy 6×10^7 chirping |
| Massive BH binaries | <ul style="list-style-type: none"> • Isolated | <ul style="list-style-type: none"> • 4-6x, over "Galaxy" & EMRIs | <ul style="list-style-type: none"> • Isolated | <ul style="list-style-type: none"> • 4-6x spinning & precessing over "Galaxy" | <ul style="list-style-type: none"> • 4-6x spinning & precessing, extended to low-mass |
| EMRI | | <ul style="list-style-type: none"> • Isolated • 4-6x, over "Galaxy" & MBHs | <ul style="list-style-type: none"> • Isolated | <ul style="list-style-type: none"> • 5 together, weaker | <ul style="list-style-type: none"> • 3 x Poisson(2) |
| Bursts | | | | <ul style="list-style-type: none"> • Cosmic string cusp | <ul style="list-style-type: none"> • Poisson(20) cosmic string cusp |
| Stochastic background | | | | <ul style="list-style-type: none"> • Isotropic | <ul style="list-style-type: none"> • Isotropic |

Multimodality of the likelihood

The main problems of LISA is multidimensionality and multimodality.//

By plotting spectrogram
(time-frequency map)
we can clearly see 3(4)
signals

Petiteau, Babak, Shang Yu, CQG 2009,
PRD 2010.



Multimodality of the likelihood

The main problems of LISA is multidimensionality and multimodality.

MBHBs

| Source | mode | $\frac{\delta M}{M}$ $\times 10^{-3}$ | $\frac{\delta \eta}{\eta}$ $\times 10^{-3}$ | Δt_c (sec) | ΔSky (deg) | $\Delta \alpha_1$ $\times 10^{-3}$ | $\Delta \alpha_2$ $\times 10^{-3}$ | $\Delta D/D$ $\times 10^{-3}$ | $\Delta \tilde{\mathcal{S}}_1$ (deg) | $\Delta \tilde{\mathcal{S}}_2$ (deg) | $\Delta \tilde{\mathcal{L}}$ (deg) | $\Delta \phi_c$ $\times 10^{-2}$ | θ | \mathcal{F}_{full} |
|--------|---------|--|--|-----------------------|------------------------------|---------------------------------------|---------------------------------------|----------------------------------|---|---|---------------------------------------|-------------------------------------|----------|----------------------|
| srcMC1 | True | 1.3 | 4.4 | 6.1 | 1.18 | 2.7 | 6.2 | 6.2 | 8.30 | 6.06 | 1.07 | 7.5 | 1.0 | 1387732 |
| | A | 10.1 | 9.2 | 25.7 | 1.92 | 0.7 | 44.3 | 13.1 | 64.39 | 79.59 | 84.02 | 259.6 | 0.999870 | 1387772 |
| | B, C | 13.5 | 8.6 | 24.6 | 8.04 | 6.7 | 28.9 | 22.4 | 9.39 | 15.70 | 7.70 | 14.7 | 0.999944 | 1387946 |
| MBH-1 | True | 10.8 | 6.3 | 24.2 | 7.04 | 7.1 | 21.8 | 20.8 | 48.18 | 19.07 | 6.20 | 40.6 | 0.999952 | 1387914 |
| | A | 4.3 | 7.2 | 9.1 | 0.82 | 2.9 | 5.3 | 7.2 | 1.52 | 3.29 | 0.95 | 2.9 | 1.0 | 355588 |
| | B, C | 13.6 | 4.8 | 29.0 | 1.33 | 2.8 | 28.3 | 19.8 | 104.65 | 55.92 | 138.55 | 4.1 | 0.999939 | 355755 |
| srcMC2 | True | 11.0 | 92.5 | 154.6 | 176.51 | 24.1 | 3.5 | 24.7 | 55.37 | 54.86 | 81.31 | 135.7 | 0.999827 | 355769 |
| | A | 15.6 | 44.6 | 158.9 | 169.3 | 52.4 | 15.0 | 66.1 | 16.49 | 64.82 | 14.68 | 25.0 | 0.997845 | 354301 |
| | B, C | 10.1 | 55.3 | 26.7 | 0.47 | 29.2 | 151.4 | 138.7 | 22.90 | 65.30 | 16.17 | 102.2 | 1.0 | 12814.2 |
| srcMC3 | True | 9.2 | 139.0 | 40.5 | 0.34 | 55.9 | 390.1 | 181.3 | 159.55 | 74.93 | 63.38 | 7.3 | 0.999311 | 12834.4 |
| | A, B, C | 17.7 | 8.5 | 234.0 | 179.48 | 96.5 | 506.7 | 319.6 | 60.55 | 87.22 | 42.61 | 413.2 | 0.998723 | 12818.8 |
| | MBH-4 | | | | | | | | | | | | | |

EMRI(s)

| Source | Group | SNR | $\frac{\delta M}{M}$ $\times 10^{-3}$ | $\frac{\delta \mu}{\mu}$ $\times 10^{-3}$ | $\frac{\delta \nu_0}{\nu_0}$ $\times 10^{-5}$ | δe_0 $\times 10^{-3}$ | $\delta S $ $\times 10^{-3}$ | $\frac{\delta \lambda_{SL}}{\lambda_{SL}}$ $\times 10^{-3}$ | δspin (deg) | δsky (deg) | $\frac{\delta D}{D}$ |
|---------------------------|-----------|--------|--|--|--|----------------------------------|----------------------------------|--|-------------------------------|------------------------------|----------------------|
| EMRI-3 (19.507) | MTAPCIOA | 19.598 | 1.62 | 0.38 | -0.10 | -0.35 | -0.94 | -3.0 | 5.0 | 3.0 | -0.04 |
| | BabakGair | 21.392 | 1.77 | 1.01 | 1.95 | -1.2 | -0.68 | -2.3 | 116 | 4.5 | 0.13 |
| | BabakGair | 21.364 | 2.26 | 1.88 | 2.71 | -2.0 | -0.69 | -2.5 | 65 | 6.1 | 0.14 |
| | BabakGair | 21.362 | 1.51 | 1.01 | 2.09 | -1.3 | -0.50 | -1.7 | 7.6 | 6.2 | 0.14 |
| | EtFAG | — | 54.0 | 4.88 | -7375 | 26 | 17 | — | — | 32 | 0.83 |

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|--------|---------|--|--|-----------------------|------------------------------|---------------------------------------|---------------------------------------|----------------------------------|---|---|---------------------------------------|-------------------------------------|----------|-----------------------------|
| srcMC1 | True | (1.3) | 4.4 | 6.1 | 1.18 | 2.7 | 6.2 | 6.2 | 8.30 | 6.06 | 1.07 | 7.5 | 1.0 | 1387732 |
| | A | 10.1 | 9.2 | 25.7 | 1.92 | 0.7 | 44.3 | 13.1 | 64.39 | 79.59 | 84.02 | 259.6 | 0.999870 | 1387772 |
| | B, C | 13.5 | 8.6 | 24.6 | 8.04 | 6.7 | 28.9 | 22.4 | 9.39 | 15.70 | 7.70 | 14.7 | 0.999944 | 1387946 |
| MBH-1 | True | 10.8 | 6.3 | 24.2 | 7.04 | 7.1 | 21.8 | 20.8 | 48.18 | 19.07 | 6.20 | 40.6 | 0.999952 | 1387914 |
| | A | (4.3) | 7.2 | 9.1 | 0.82 | 2.9 | 5.3 | 7.2 | 1.52 | 3.29 | 0.95 | 2.9 | 1.0 | 355588 |
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| srcMC3 | True | 9.2 | 139.0 | 40.5 | 0.34 | 55.9 | 390.1 | 181.3 | 159.55 | 74.93 | 63.38 | 7.3 | 0.999311 | 12834.4 |
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| | BabakGair | 21.364 | 2.26 | 1.88 | 2.71 | -2.0 | -0.69 | -2.5 | 65 | 6.1 | 0.14 |
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| | EtFAG | — | 54.0 | 4.88 | -7375 | 26 | 17 | — | — | 32 | 0.83 |

Constrains on the GW signal modelling

- Stochastic error: noise pushes the maximum likelihood away from the true parameters. Scales with SNR ($\sim 1/SNR$).
- Systematic errors: error in modelling GW signal. For loud signals could become a dominant source of error.
- MBHB signal: requires complete (IMR) model with eccentricity, arbitrary spins, subdominant modes
- EMRI signal: requires going beyond the first order in perturbation theory to model long signal (\sim year)



Summary

GW astronomy on the ground

- LIGO and VIRGO have detected GWs from handful of BBHs and one BNS
- GW detectors undergo upgrade: start operating with improved sensitivity in O3
- Expect more signals (could be 1/week), looking forward to detecting more BNSs and BH-NS
- LIGO-India and KAGRA: extending the network in the near future (better sky coverage, localization, ...)
- Study the population of GW sources. Combine events to have stringent constraints on deviations from GR



Summary

GW astronomy in space

- LISA is very strong now! LPF and GWs with LIGO: full speed ahead with space-based project.
- Data will be signal dominated, requires new/improved methods for analysing the data.
- Stringent requirements on the models of GW signals from binary systems
- LISA: amazing astrophysical and fundamental physics laboratory
- LISA data challenge:
<https://lisa-ldc.lal.in2p3.fr/home>
- LISA symposium:
<http://ciera.northwestern.edu/LISA12.php>