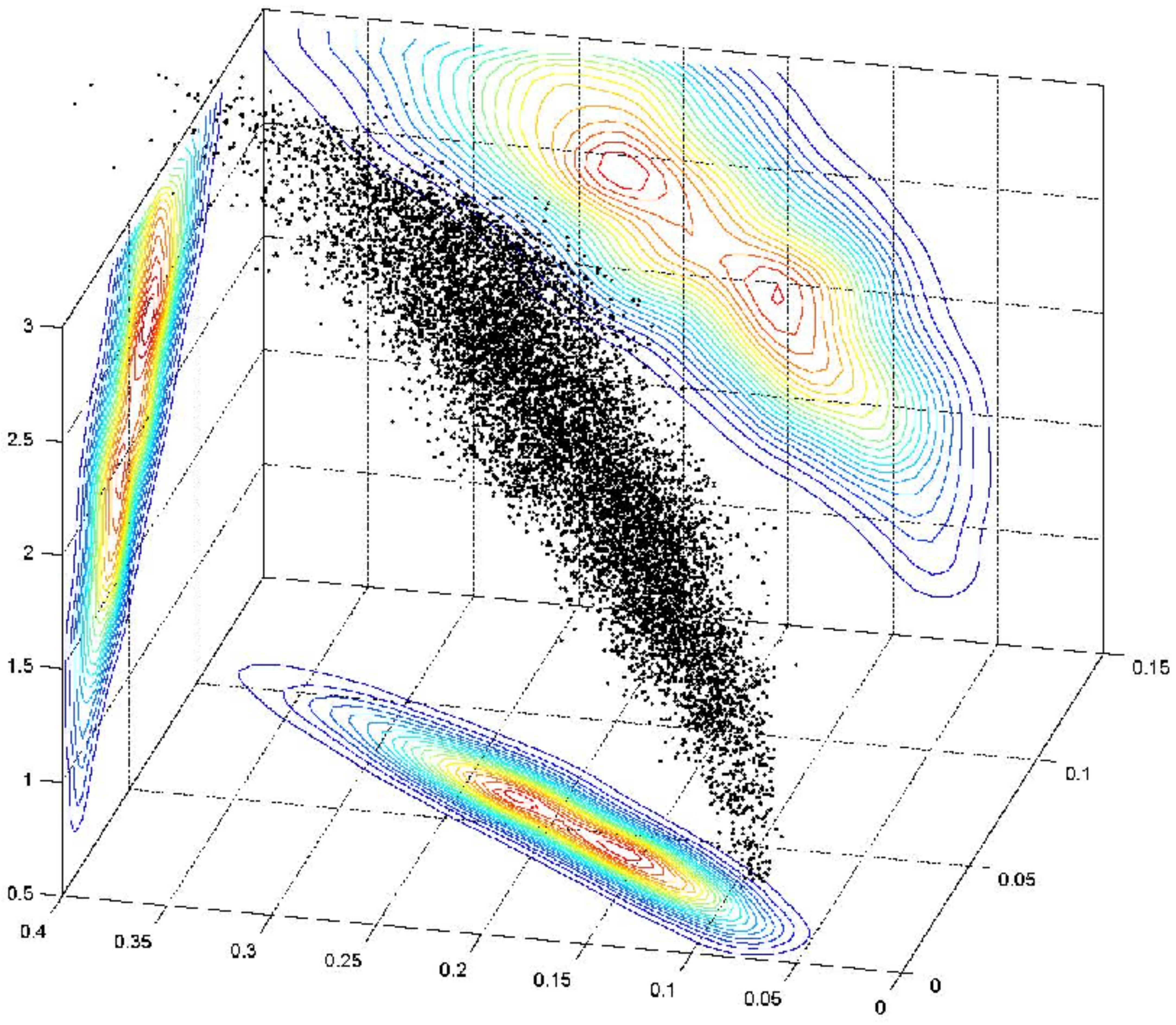
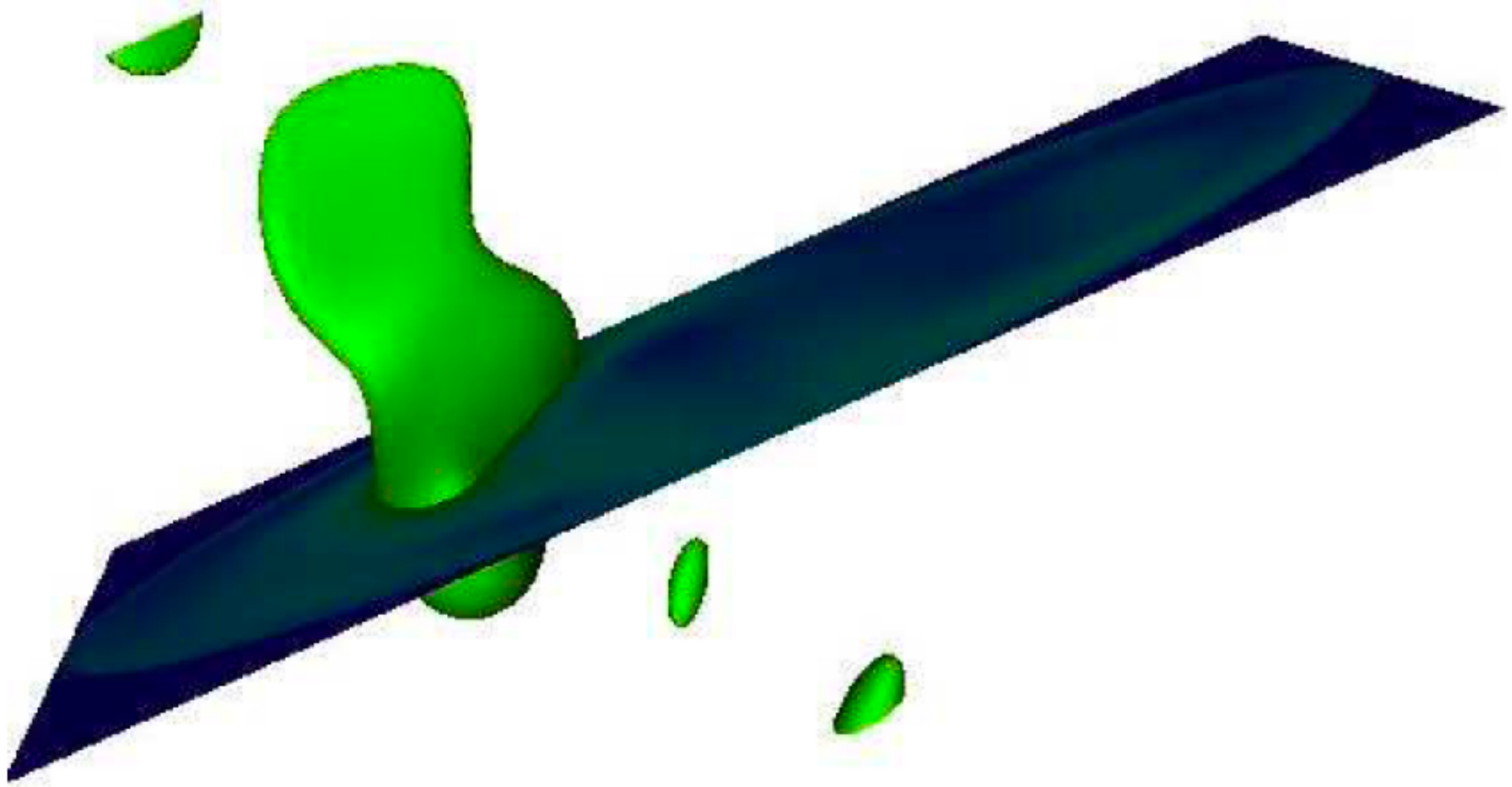
# Stochastic Methods in LISA Searches




MONTANA STATE UNIVERSITY

Neil J. Cornish

extreme Gravity Institute

# 11th Pacific Coast Gravity Meeting
# 24-25 Mar 1995. Pasadena, California

Saturday, 25 March 1995

8:30 Coffee and donuts

**** Experimental Gravity and Gravitational Radiation ****

8:45 Koya Suehiro — "Operation of 20m Fabry-Perot prototype with modecleaner"
sue@gravity.mtk.nao.ac.jp
National Astronomical Observatory, Japan

9:00 Joe Weber — "Gravitational Antenna Observations"
???
UC Irvine

9:15 Albert Lazzarini — "Overview and Status of Ligo Project"
lazz@ligo.caltech.edu
Caltech

9:30 Aaron Gillespie — "Installation of New Test Masses in the 40-meter Interferometer"
aaron@ligo.caltech.edu
Caltech

9:45 Torrey Lyons — "Recombination of the 40-m Interferometer"
torrey@ligo.caltech.edu
Caltech

10:00 Alan Wiseman — "Gravitational Wave Signals from Coalescing Binaries"
agw@tapir.caltech.edu
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10:15 Bill Folkner — "LISA - Laser Interferometer Space Antenna for Gravitational Wave Measurements"
wmf@logos.jpl.nasa.gov
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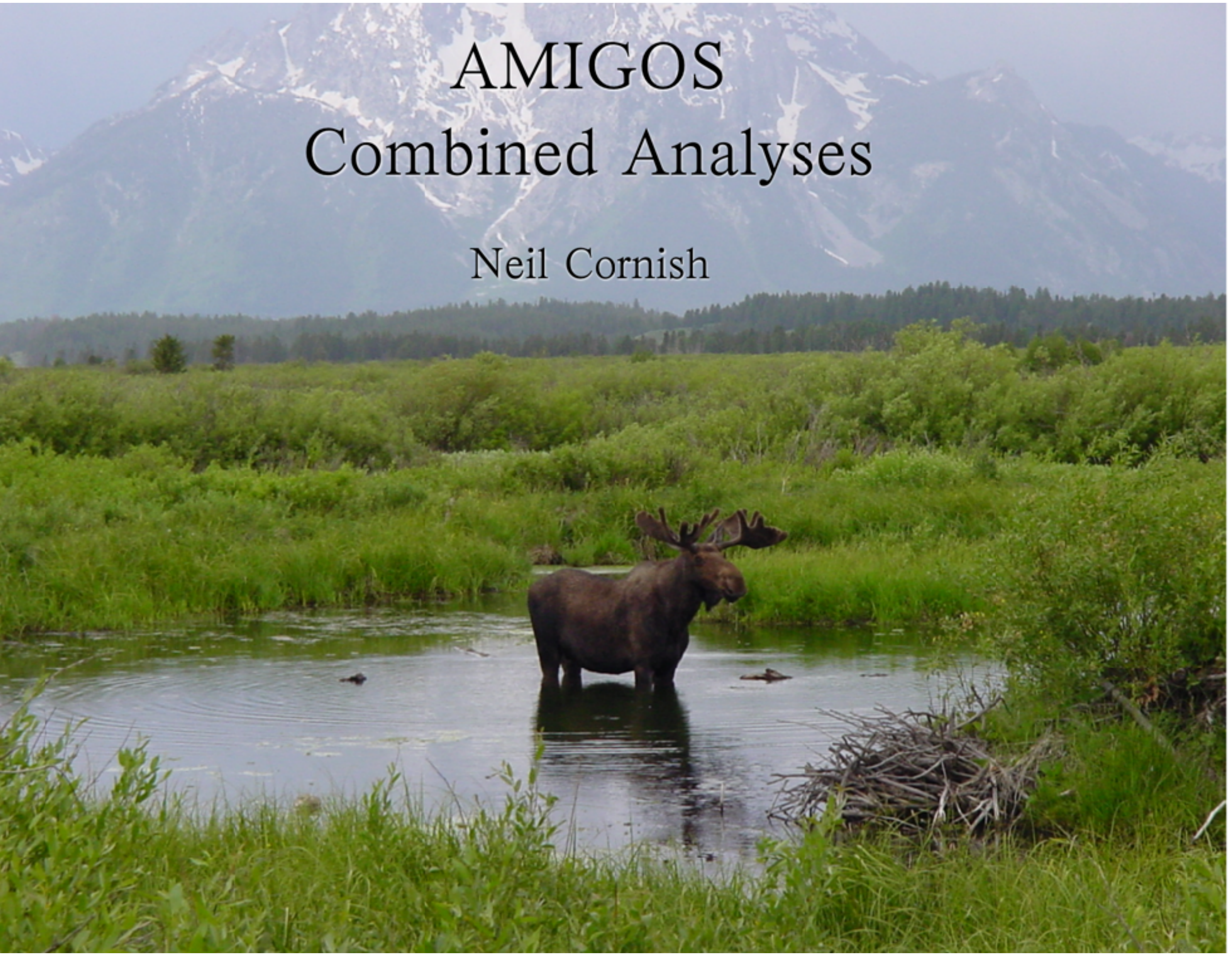
Déjà vu: Analysis Methods for Interferometric Gravitational-wave
Observations from Space (AMIGOS)
Caltech, Oct 13-15, 2005




## AMIGOS Section 7.3.3

### Risks

- Not thinking about combined analyses early enough
- Data Gaps/Disturbances - may significantly worsen confusion problem
- Magnitude of problem not understood (what effort needs to be devoted, not enough, early enough)

http://www.vallis.org/publications/amigos-2005.pdf

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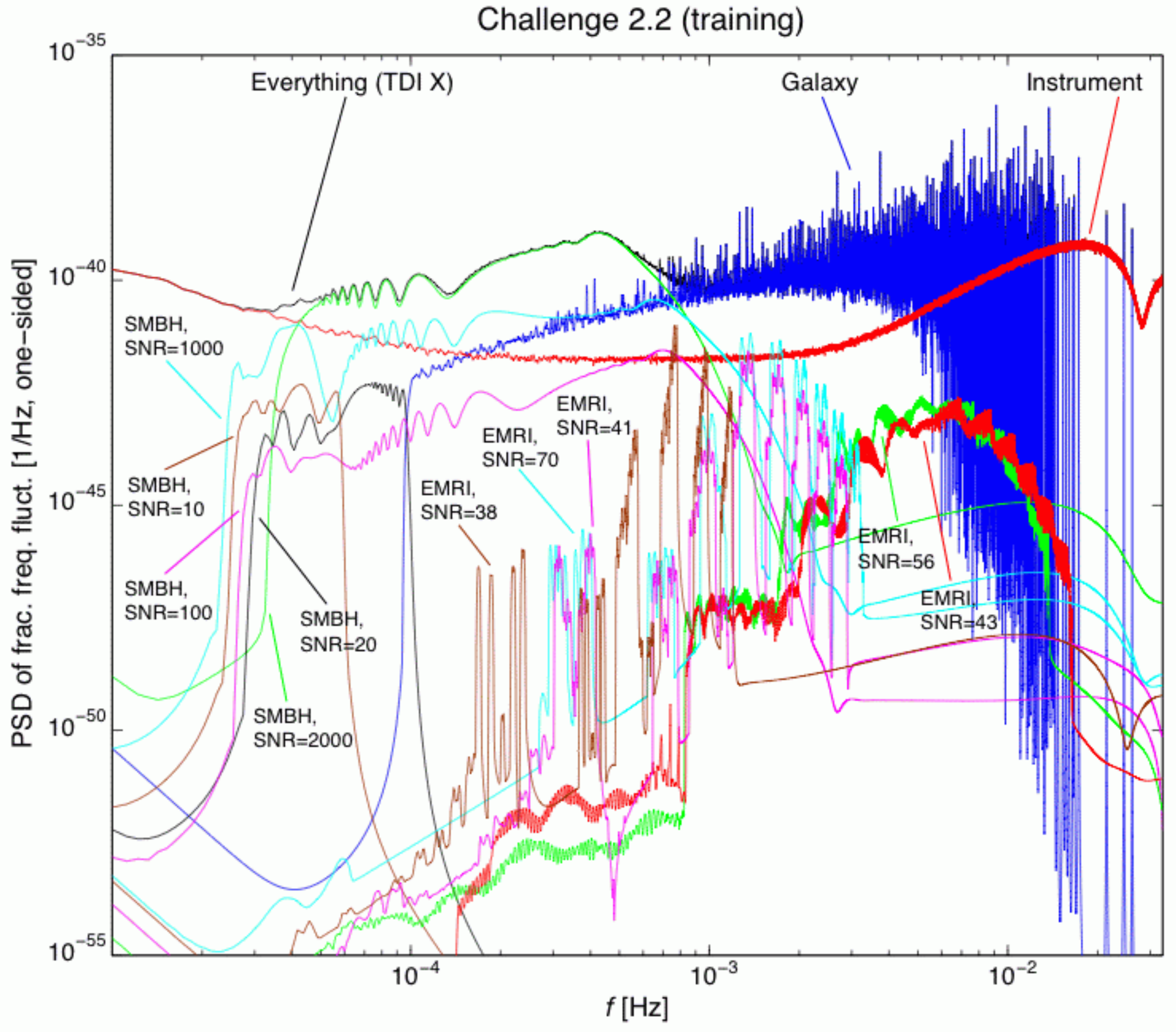
# Outline

- Signal and Noise model
- Likelihood function for non-stationary noise
- Trans-dimensional Bayesian modeling
- Galactic binaries
- EMRIs

# Signal & Noise models




$$p(\mathbf{d}|\mathbf{h}) = \frac{1}{\sqrt{\det(2\pi\mathbf{C})}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})^{\dagger}\mathbf{C}^{-1}(\mathbf{d}-\mathbf{h})}$$

Noise will be non-stationary and possibly non-Gaussian

$$\mathbf{h} = \sum_{j=1}^{N} \mathbf{h}_j(\vec{\lambda}_j)$$

Signal model is the sum over all *N* resolvable signals (convolved with the instrument response)

The number *N* of resolvable signals is *a priori* unknown. Parameters of the signals also unknown

Some of the signals are extremely complex (e.g. EMRIs)

# Likelihood for Non-stationary Gaussian Noise

$$p(\mathbf{d}|\mathbf{h}) = \frac{1}{\sqrt{\det(2\pi\mathbf{C})}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})^\dagger \mathbf{C}^{-1} (\mathbf{d}-\mathbf{h})}$$

Cost of computing the likelihood is far less in a representation where the noise correlation matrix **C** is diagonal

e.g. Colored stationary noise has a diagonal noise correlation matrix in the Fourier domain

Pulsar Timing has to deal with colored, non-stationary data and un-even sampling - analysis performed directly in the time domain. Clever tricks have been developed to speed up the costly matrix inversions and sums

# Likelihood for Non-stationary Gaussian Noise

$$p(\mathbf{d}|\mathbf{h}) = \frac{1}{\sqrt{\det(2\pi\mathbf{C})}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})^\dagger \mathbf{C}^{-1}(\mathbf{d}-\mathbf{h})}$$

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For a large class of discrete wavelet transformations and locally stationary noise [1]

$$C_{(i,j)(k,l)} = \delta_{ij}\delta_{kl}\, C_{ik}$$ [2]

Time ($i$) Frequency ($k$)

This is the likelihood used by the LIGO coherent WaveBurst algorithm

[**1.** "Fitting time series models to nonstationary processes". Dahlhaus, Ann. Statist., 25, 1 (1997)]

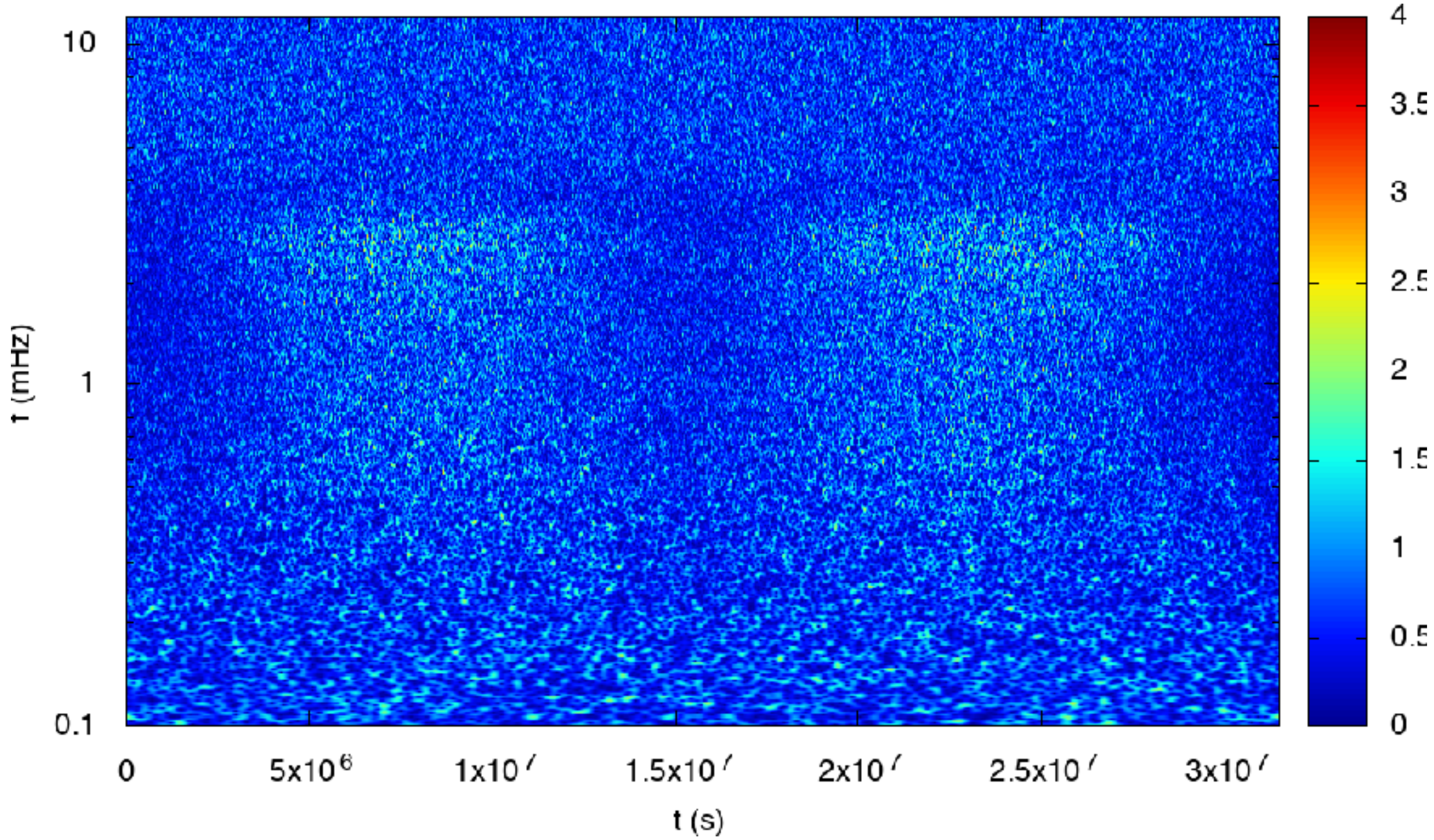
[**2.** "Wavelet processes and adaptive estimation of the evolutionary wavelet spectrum", Nason, von Sachs, & Kroisandt, J. R. Statist. Soc. Series B**62**, 271 (2000)]

# Likelihood for Non-stationary Gaussian Noise

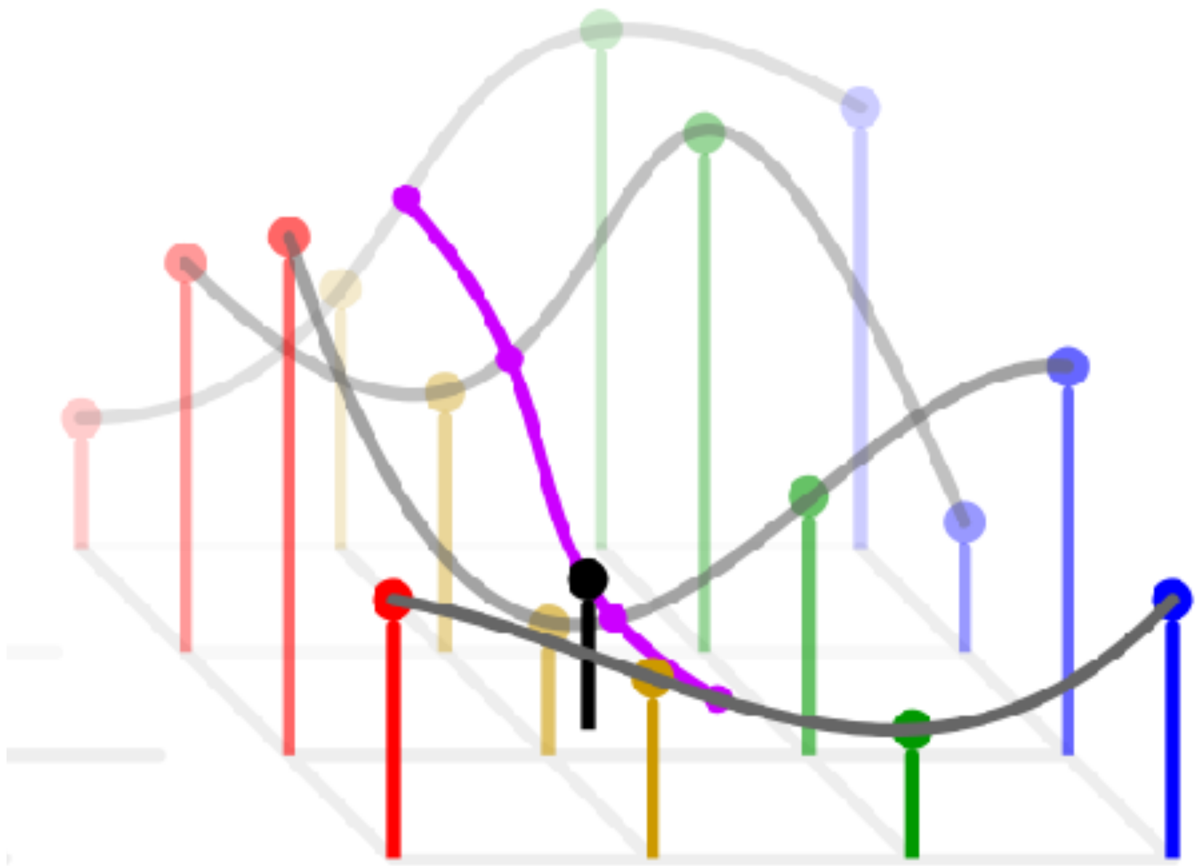
$$p(\mathbf{d}|\mathbf{h}) = \frac{1}{\sqrt{\det(2\pi\mathbf{C})}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})^\dagger \mathbf{C}^{-1}(\mathbf{d}-\mathbf{h})}$$

Propose that we use a discrete wavelet based likelihood for LISA data analysis

$$C_{(i,j)(k,l)} = \delta_{ij}\delta_{kl}\, C_{ik}$$




Model the wavelet spectrum $C_{ik}$ as a smooth function in frequency and time. E.g. Trans-dimensional Bicubic spline



Bicubic

# Likelihood for Non-stationary Gaussian Noise

$$p(\mathbf{d}|\mathbf{h}) = \frac{1}{\sqrt{\det(2\pi\mathbf{C})}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})^\dagger \mathbf{C}^{-1}(\mathbf{d}-\mathbf{h})}$$

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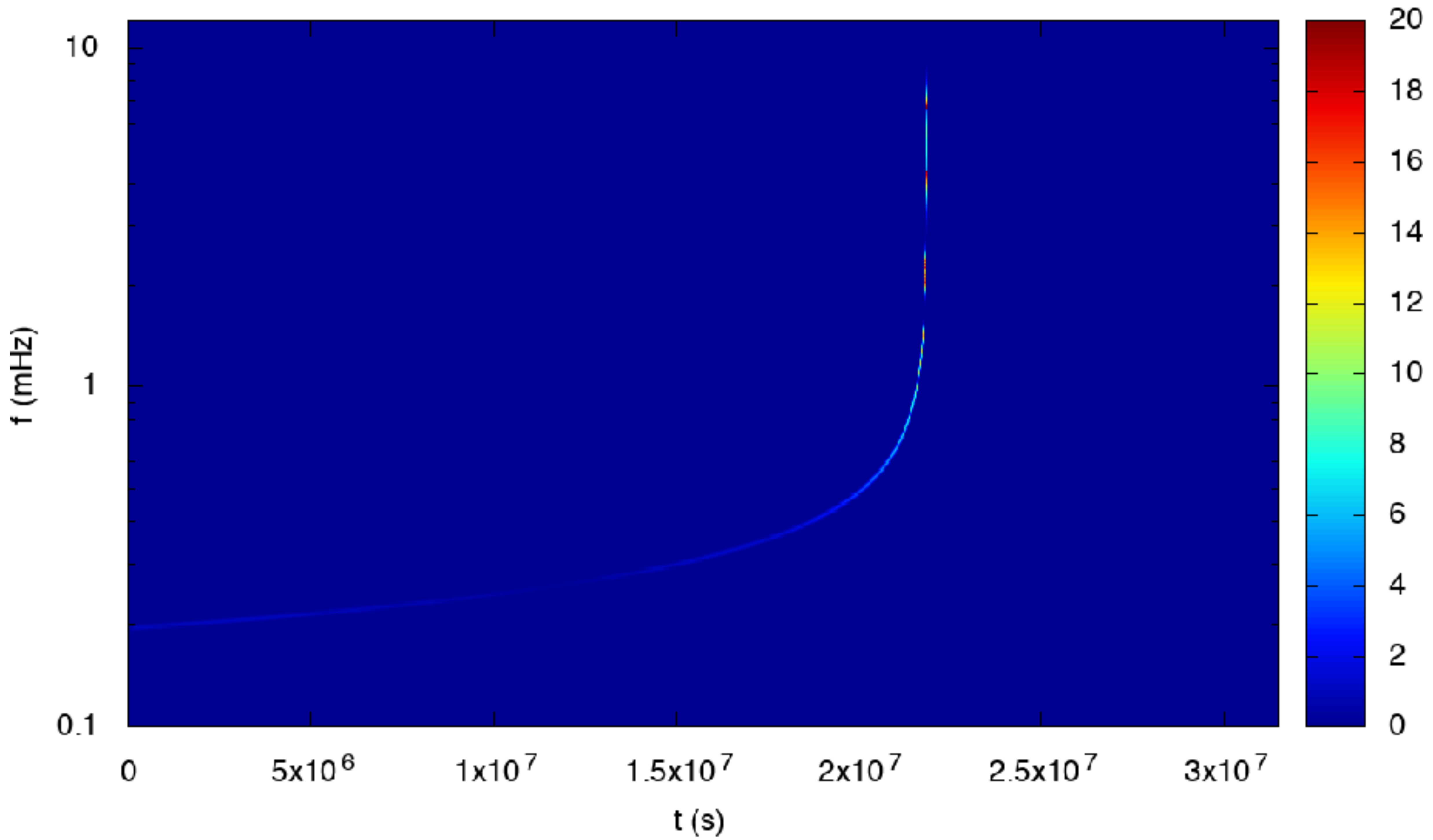


Need fast wavelet transforms of the signals for computational efficiency

Use SPA to derive analytic wavelet domain waveforms?

# Likelihood for Non-stationary Gaussian Noise

$$p(\mathbf{d}|\mathbf{h}) = \frac{1}{\sqrt{\det(2\pi\mathbf{C})}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})^{\dagger}\mathbf{C}^{-1}(\mathbf{d}-\mathbf{h})}$$

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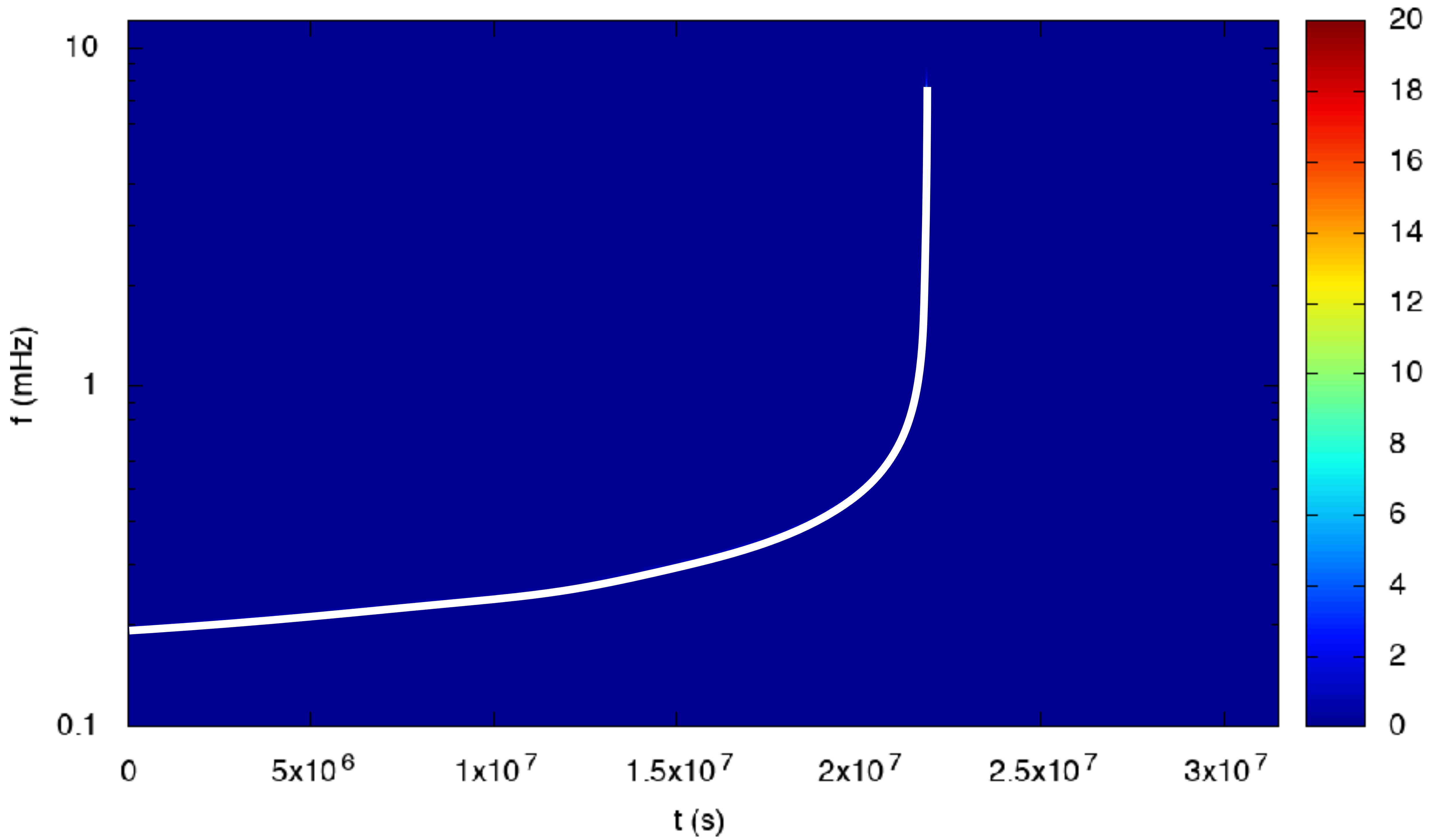



Need fast wavelet transforms of the signals for computational efficiency

Use SPA to derive analytic wavelet domain waveforms?

Only compute wavelets along predicted t-f track?

# The Global Solution

$$\chi^2 = (\mathbf{d} - \mathbf{h}|\mathbf{d} - \mathbf{h}) = (\mathbf{d}|\mathbf{d}) - 2\log\Lambda$$

Relative likelihood $\log\Lambda = (\mathbf{d}|\mathbf{h}) - \frac{1}{2}(\mathbf{h}|\mathbf{h})$

Single source RL $\log\Lambda_i = (\mathbf{d}|\mathbf{h}_i) - \frac{1}{2}(\mathbf{h}_i|\mathbf{h}_i)$

$$\log\Lambda = \sum_i \log\Lambda_i - \sum_{i>j}(\mathbf{h}_i|\mathbf{h}_j)$$

Source confusion

The typical overlaps between any two signals are very small (exceptions are WD binaries with ~same frequency and similar sky location). But, there are many signals, so the confusion adds up.

Any unresolved signals act as an effective noise term (though non-Gaussian and non-stationary)

[Cutler & Harms, Phys.Rev. D**73** 042001 (2006)]

[Robson & Cornish, Class.Quant.Grav. **34** 244002 (2017) ]

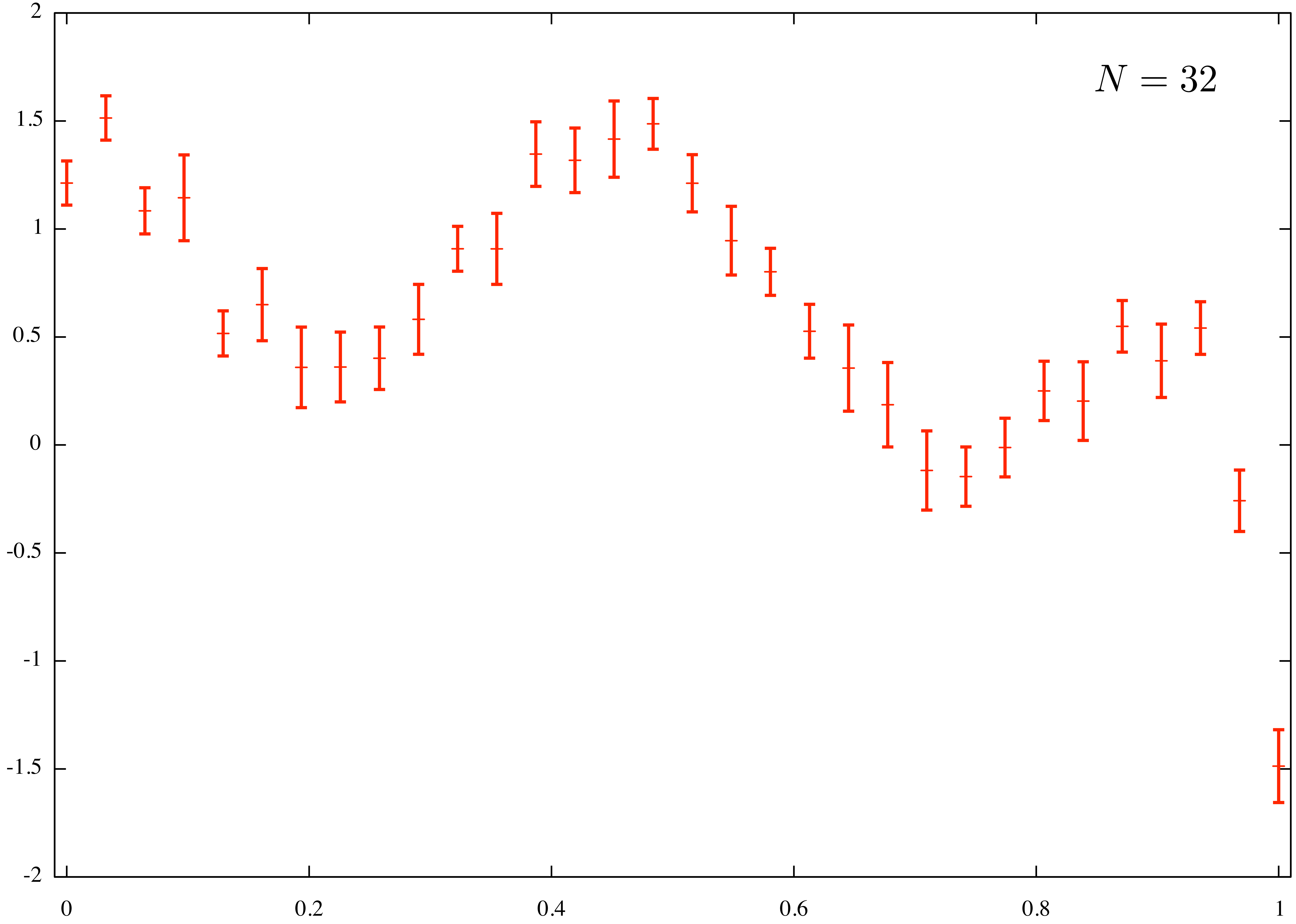
*Model everything and let the data sort it out*

Unknown number of signals and source parameters. Noise model of unknown complexity

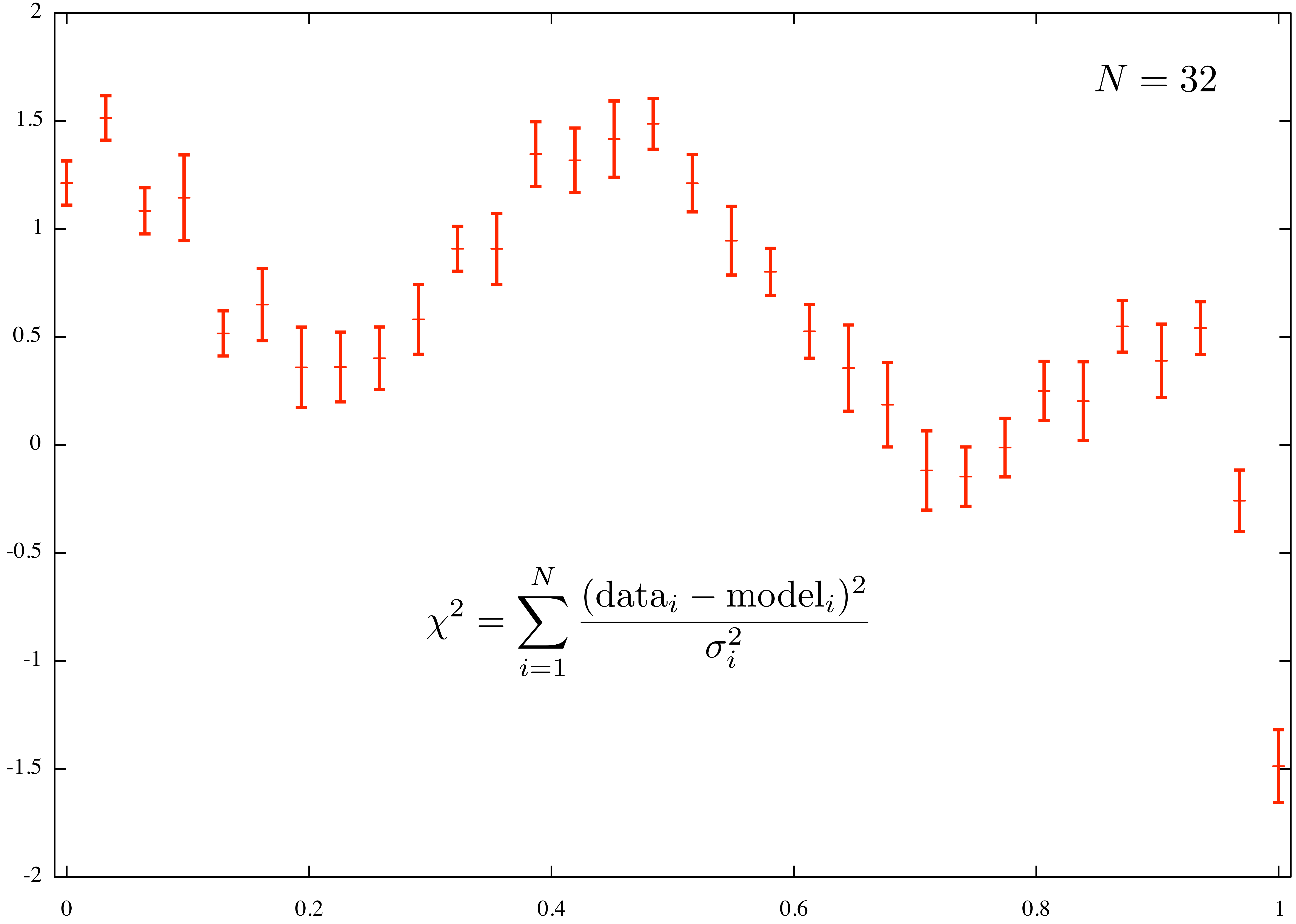
$\Rightarrow$ **Let the data decide the model dimension**

$\Rightarrow$ **Make the model dimension a parameter**

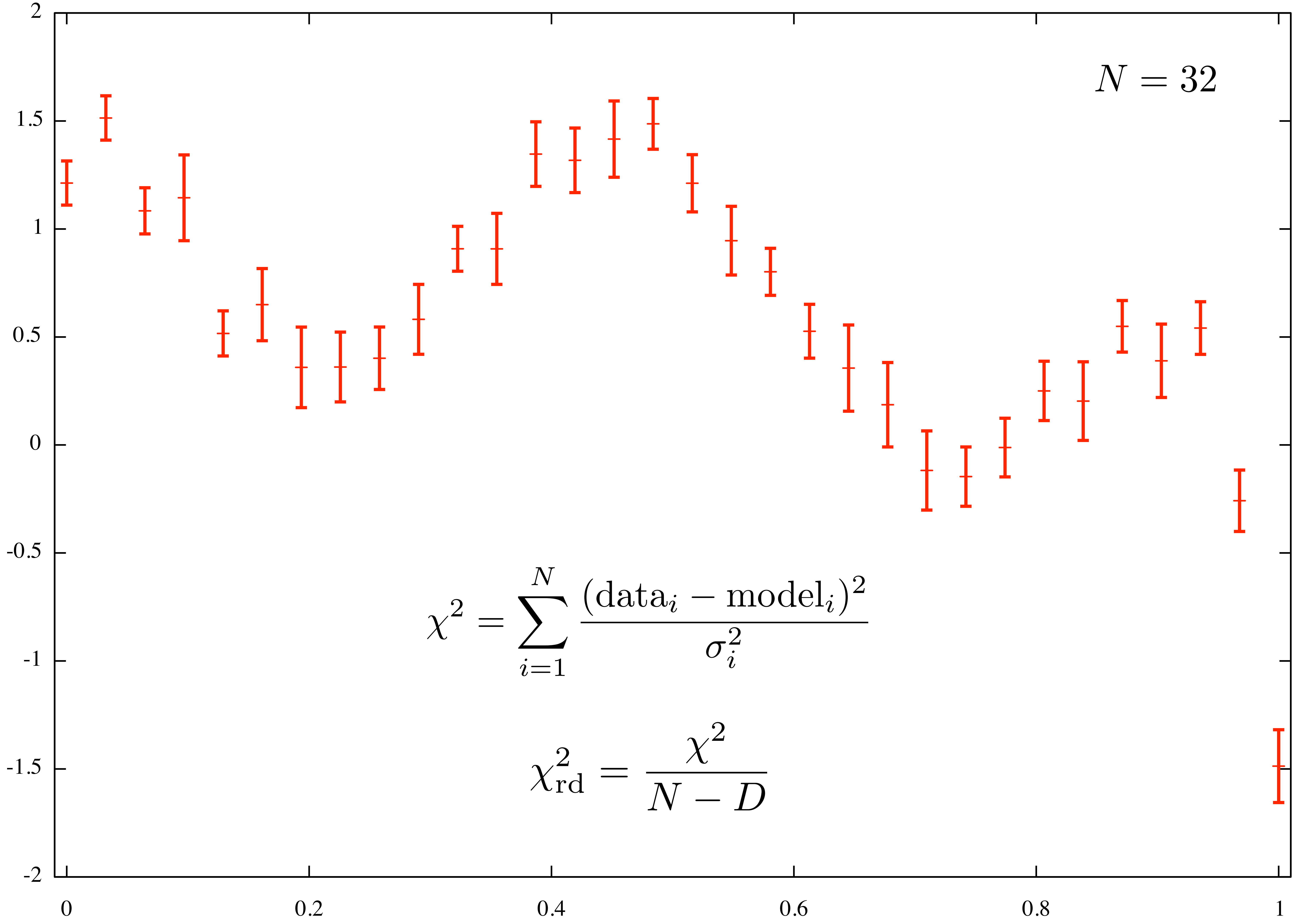
Trans-dimensional Markov Chain Monte Carlo

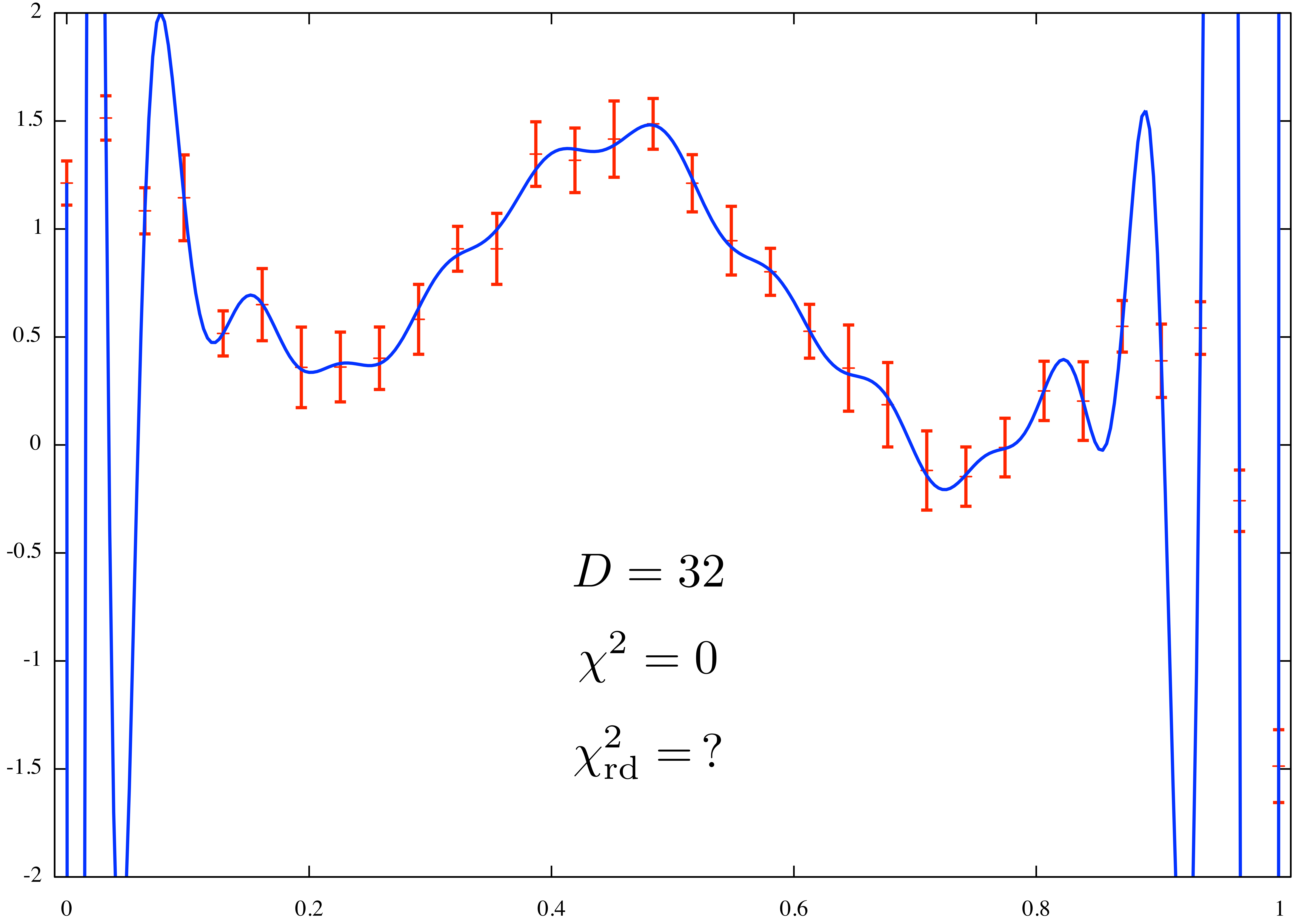


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.2
0.4
0.6
0.8
1
$N = 32$

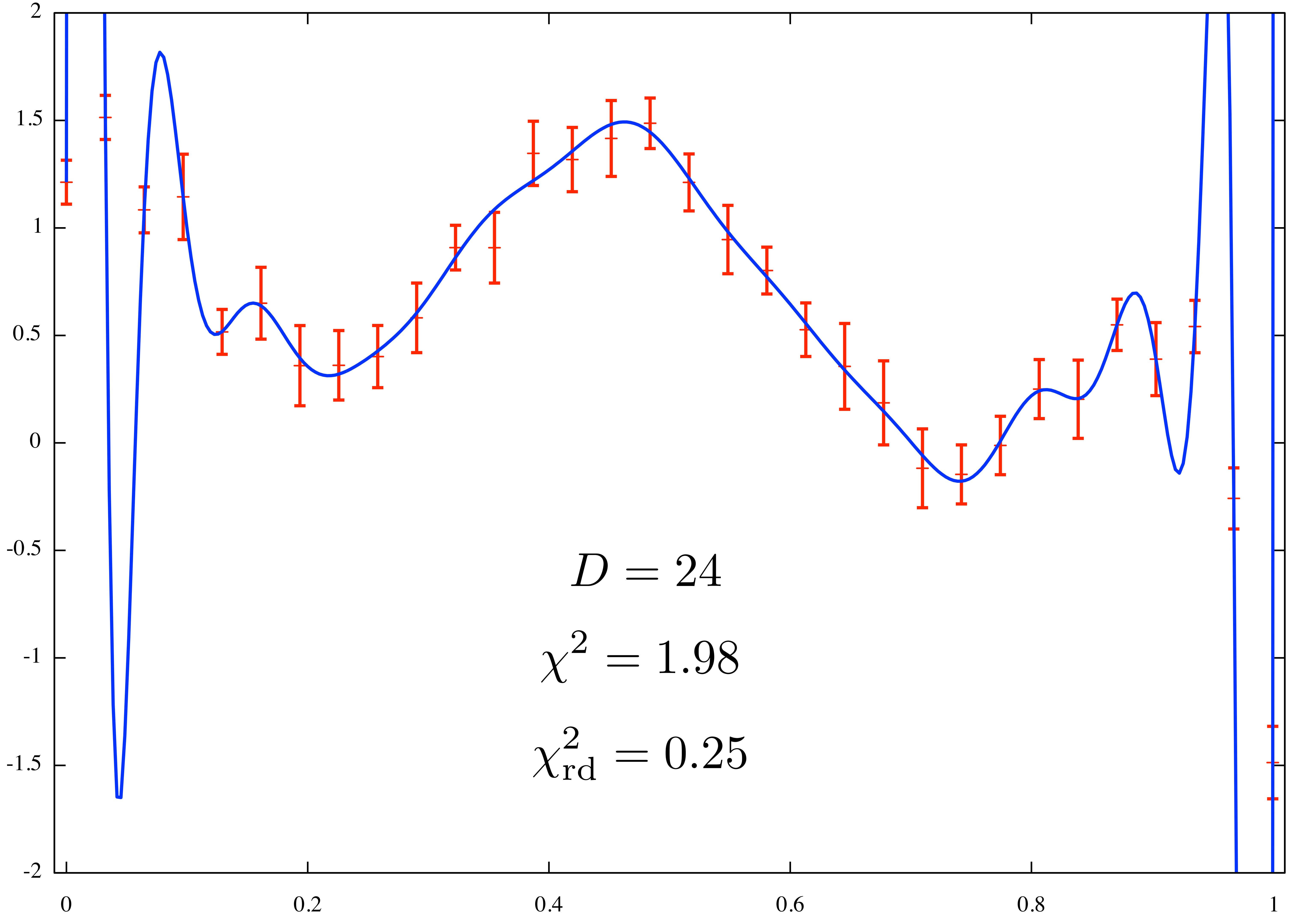


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.2
0.4
0.6
0.8
1
$N = 32$
$\chi^2 = \sum_{i=1}^{N} \frac{(\text{data}_i - \text{model}_i)^2}{\sigma_i^2}$

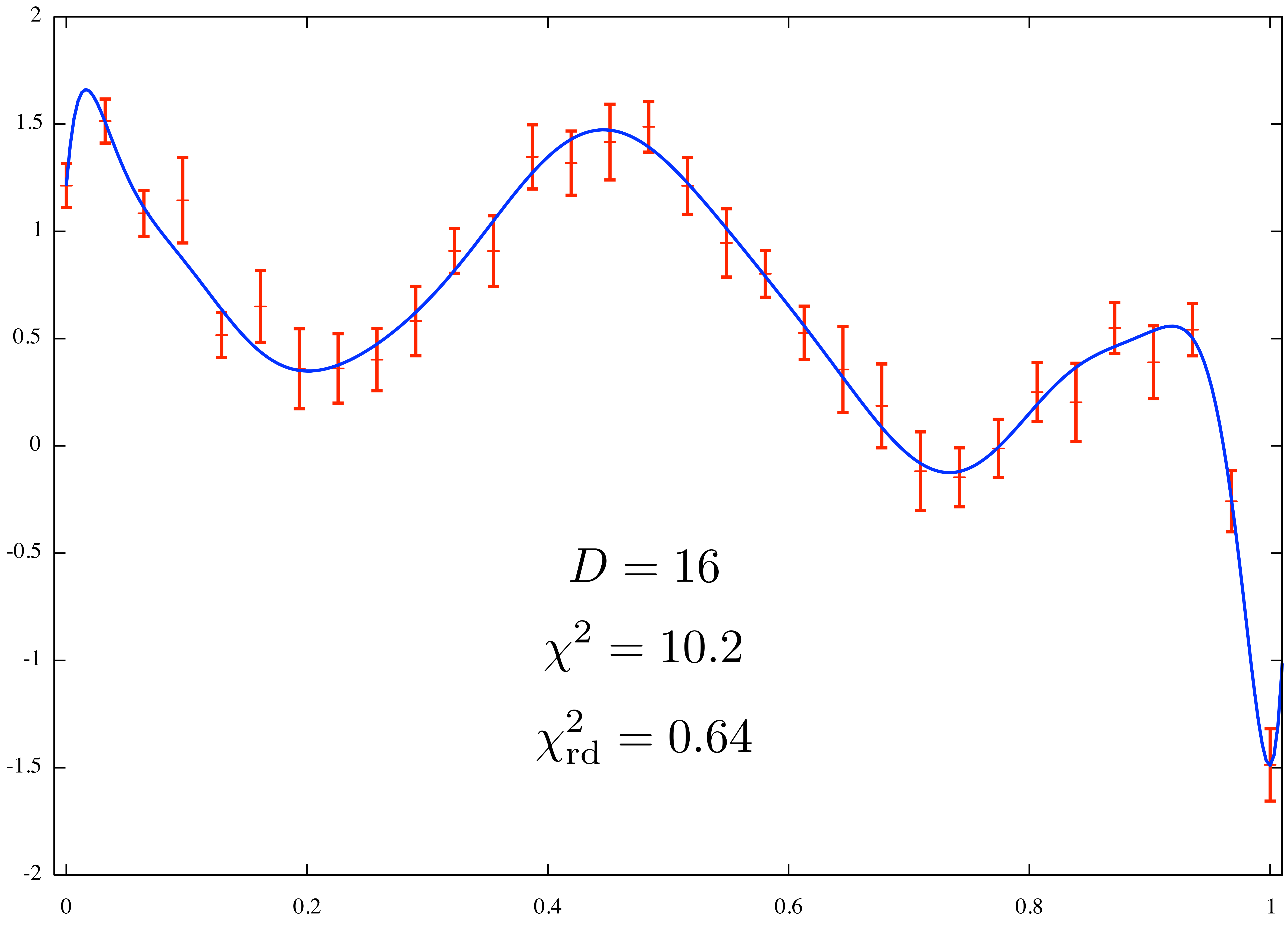

$N = 32$
$$\chi^2 = \sum_{i=1}^{N} \frac{(\mathrm{data}_i - \mathrm{model}_i)^2}{\sigma_i^2}$$
$$\chi^2_{\mathrm{rd}} = \frac{\chi^2}{N - D}$$
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.2
0.4
0.6
0.8
1



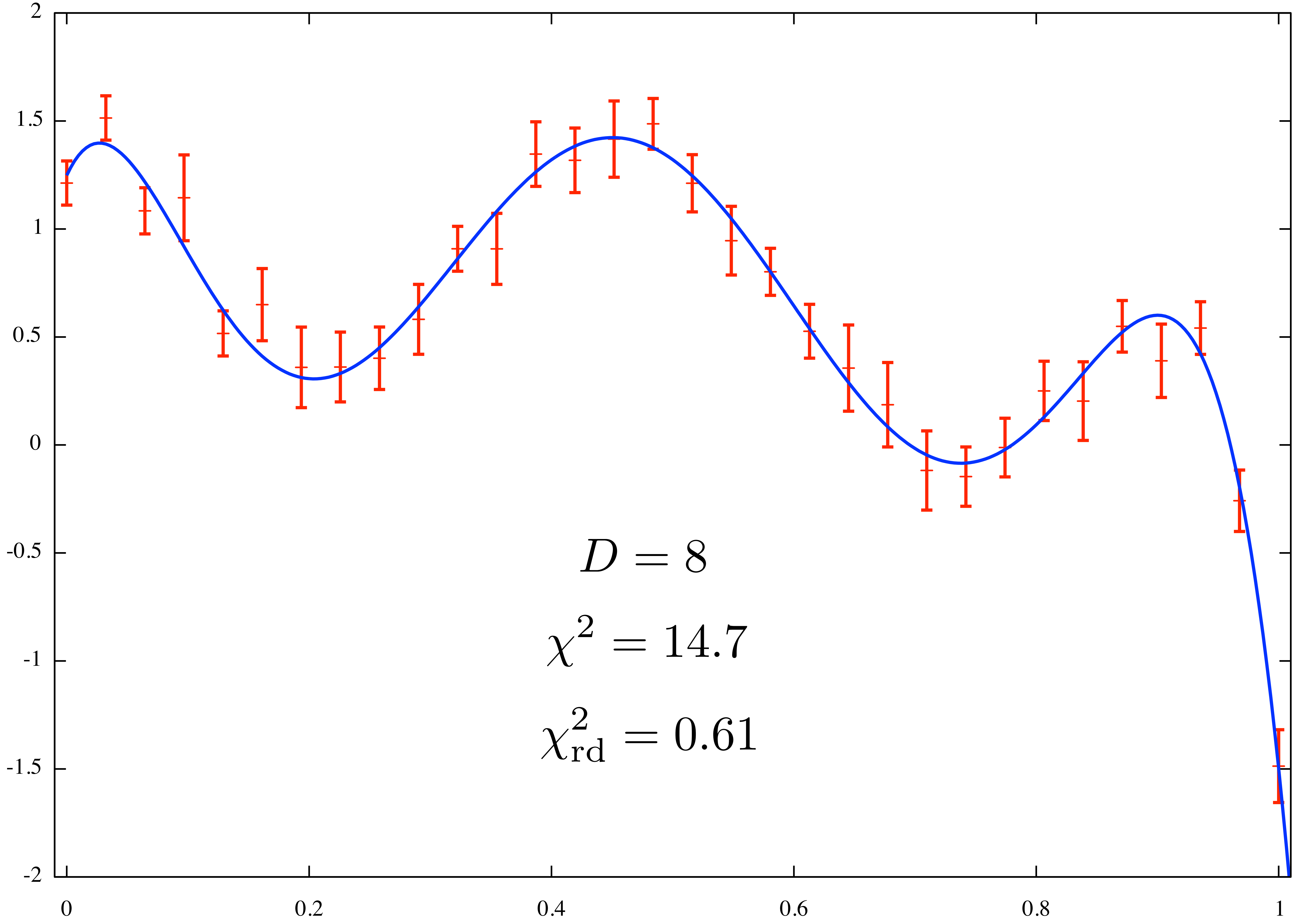
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.2
0.4
0.6
0.8
1
$D = 32$
$\chi^2 = 0$
$\chi^2_{\rm rd} = ?$



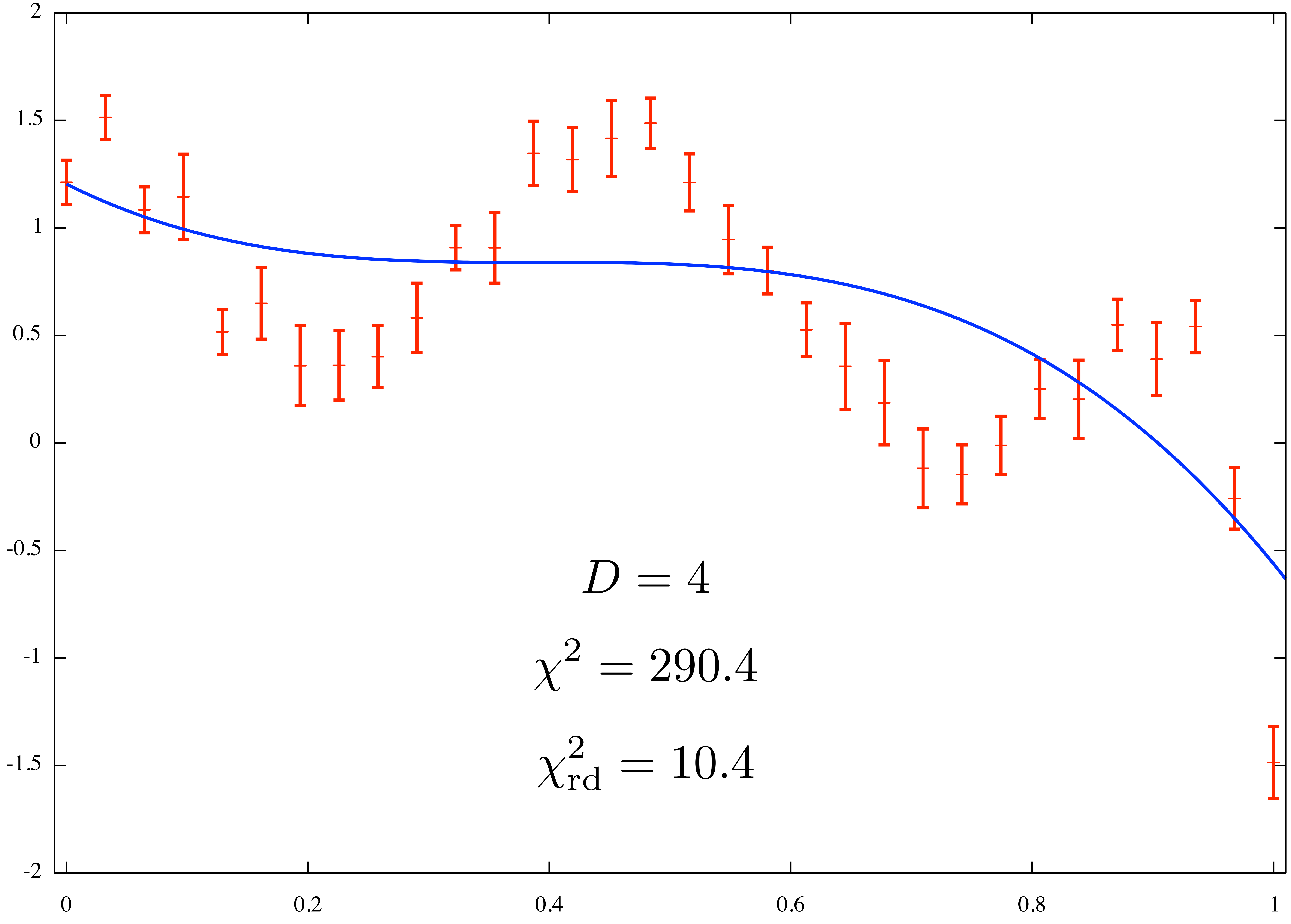
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.2
0.4
0.6
0.8
1
$D = 24$
$\chi^2 = 1.98$
$\chi^2_{\rm rd} = 0.25$



2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.2
0.4
0.6
0.8
1
$D = 16$
$\chi^2 = 10.2$
$\chi^2_{\rm rd} = 0.64$

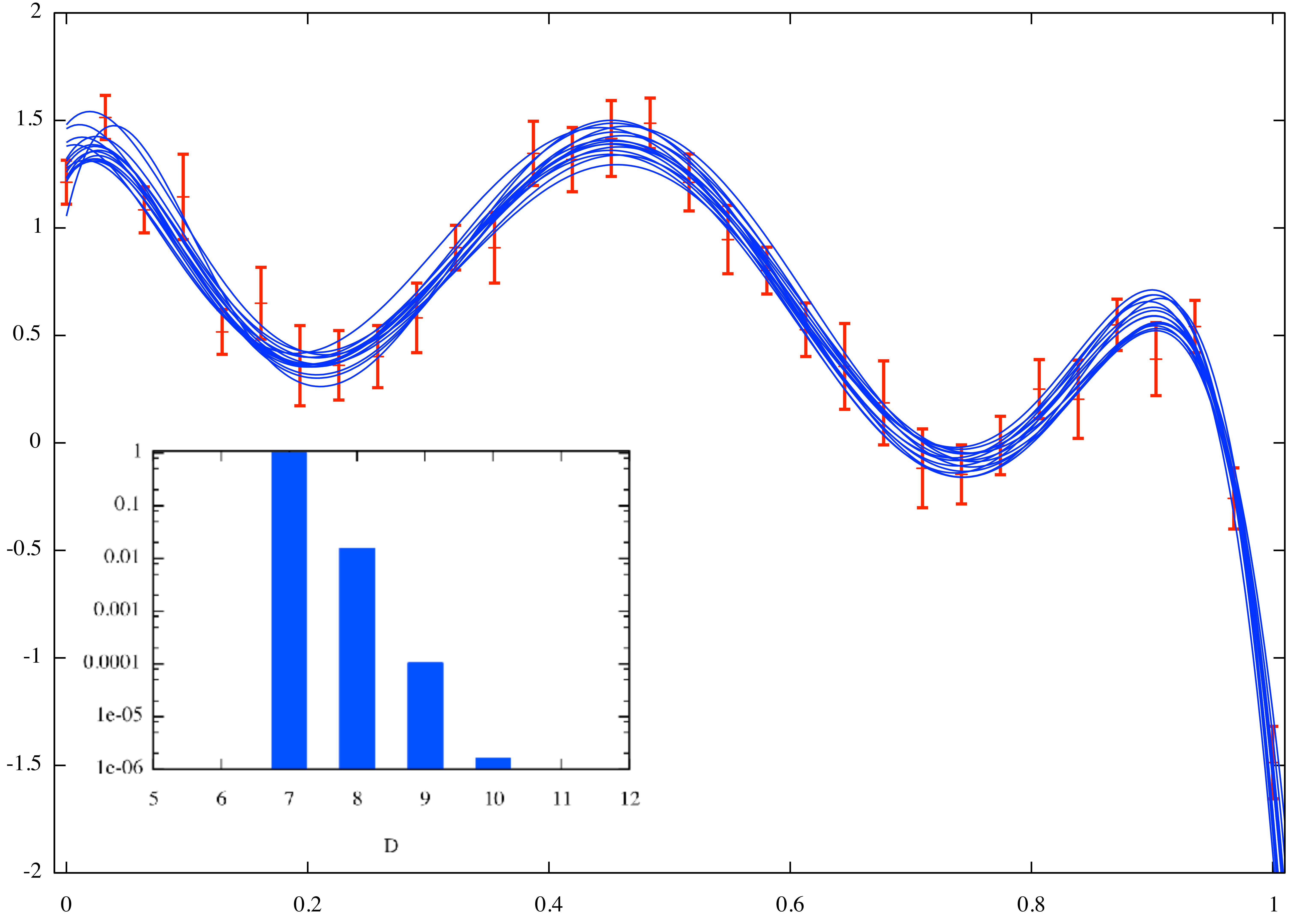


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.2
0.4
0.6
0.8
1
$D = 8$
$\chi^2 = 14.7$
$\chi^2_{\rm rd} = 0.61$



2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.2
0.4
0.6
0.8
1
$D = 4$
$\chi^2 = 290.4$
$\chi^2_{\mathrm{rd}} = 10.4$

# Trans-dimensional Markov Chain Monte Carlo

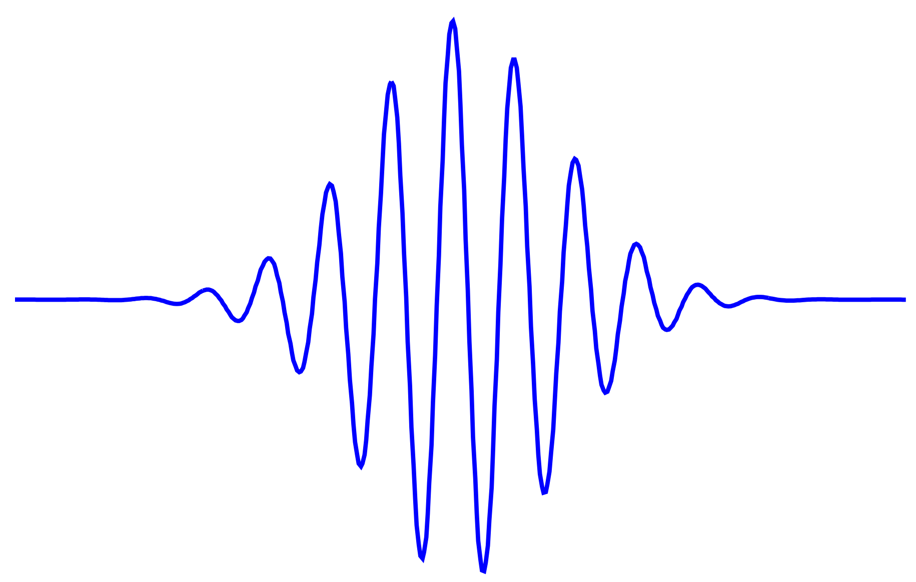


# BayesWave

Cornish & Littenberg 2015

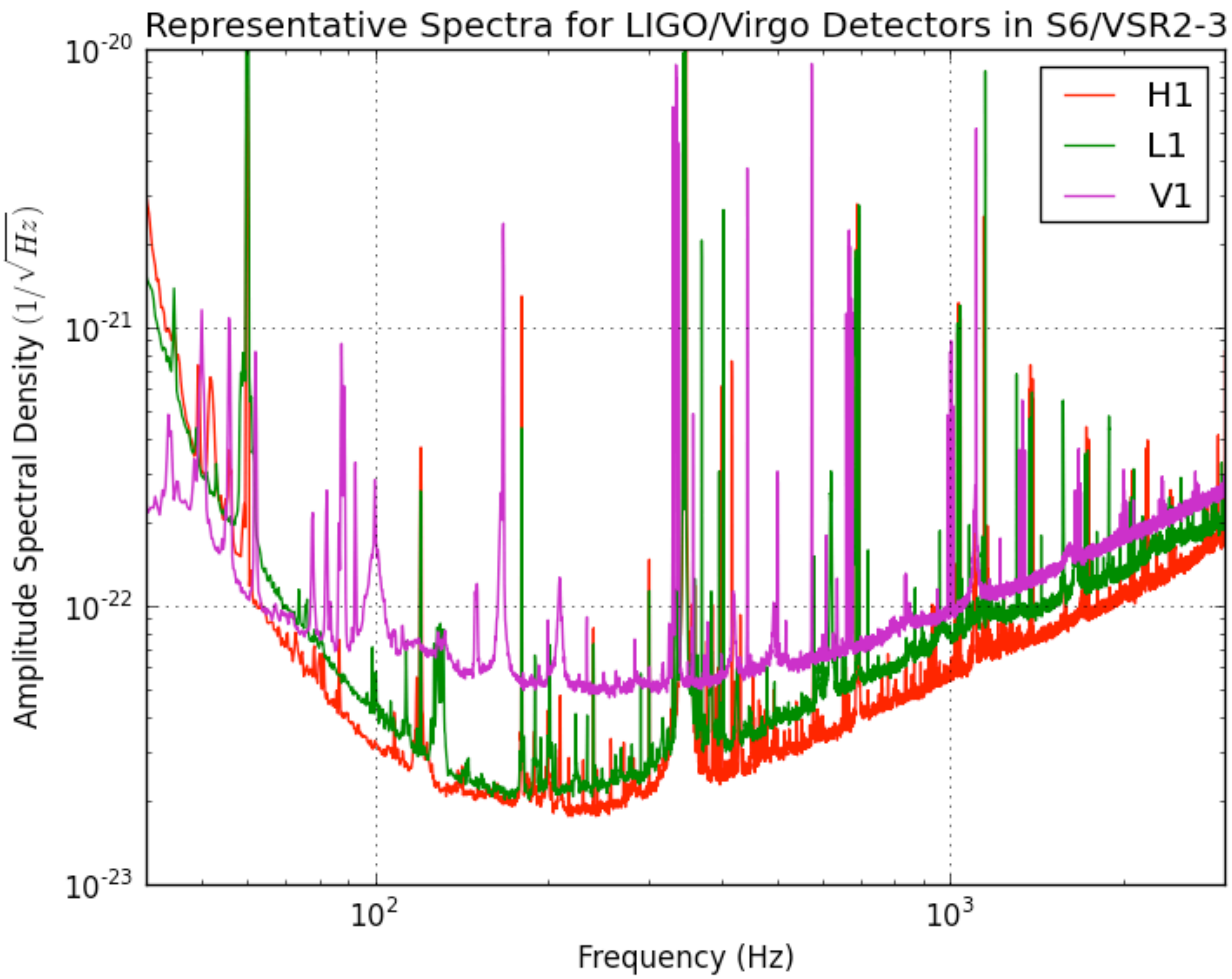
Ellis & Cornish 2016

- Bayesian model selection
  - Three part model (signal, glitches, gaussian noise)
  - Trans-dimensional Markov Chain Monte Carlo

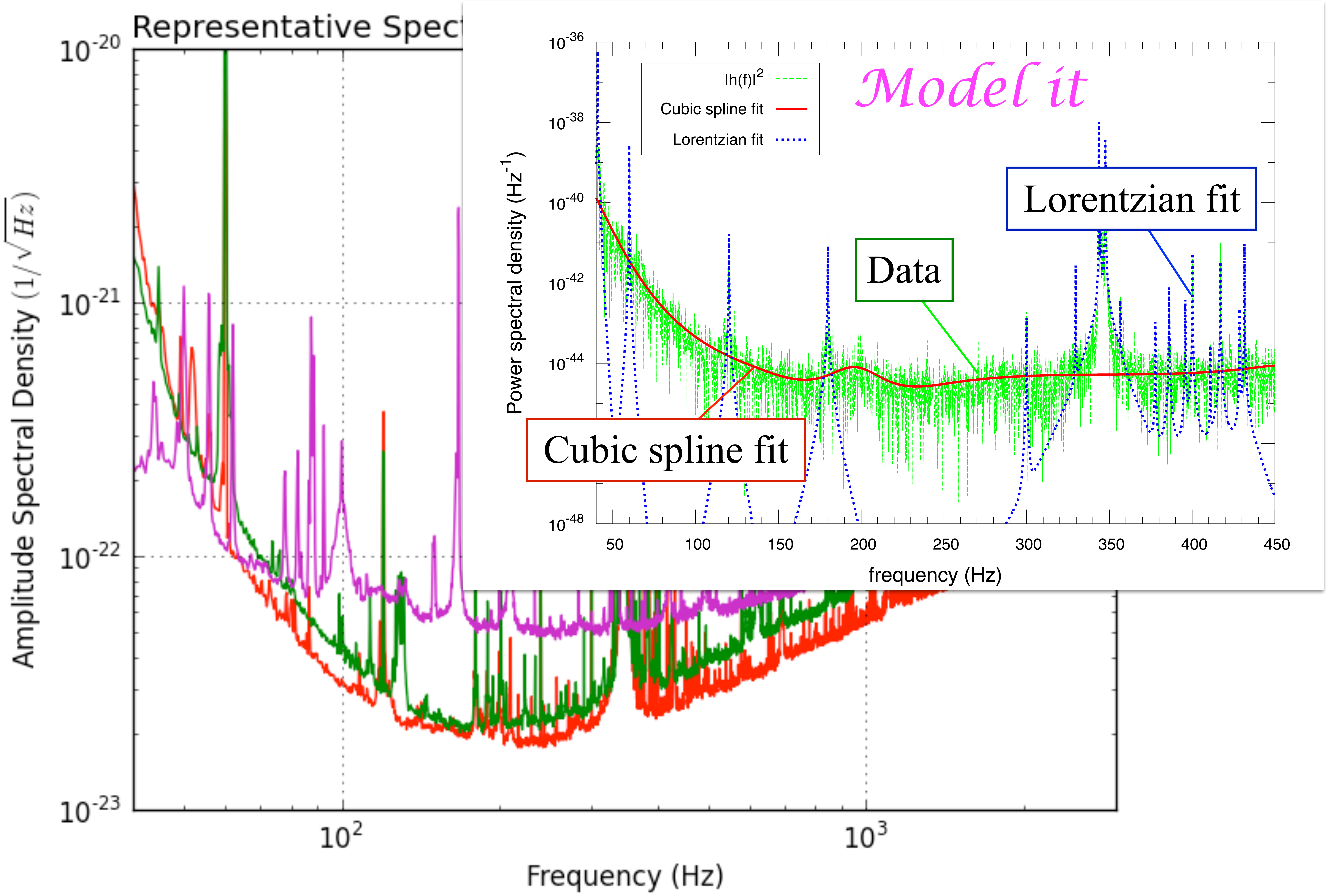


- Wavelet decomposition
  - Glitch & GW modeled by wavelets
  - Number, amplitude, quality and TF location of wavelets varies

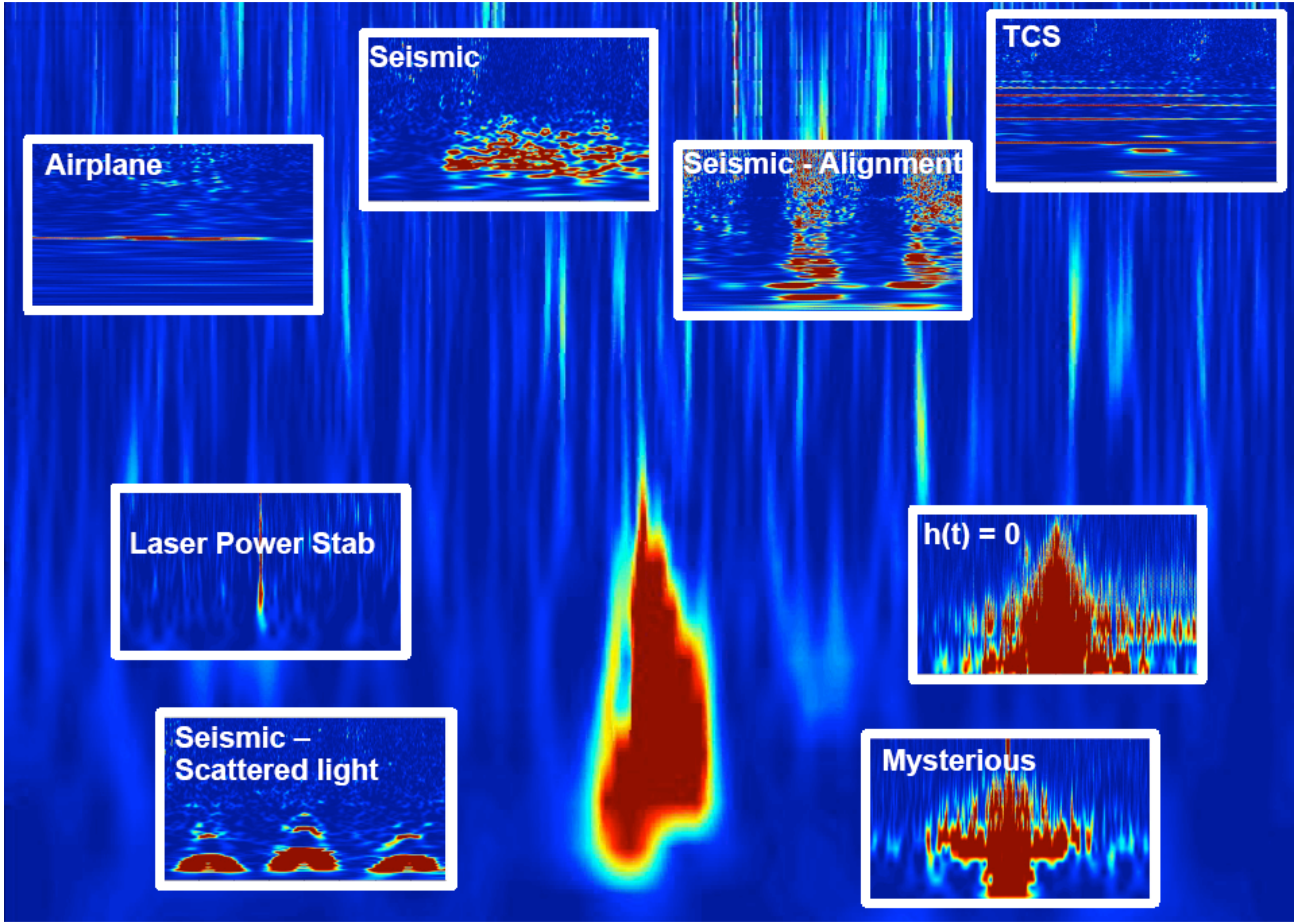
# Lines and a drifting noise floor



# Lines and a drifting noise floor

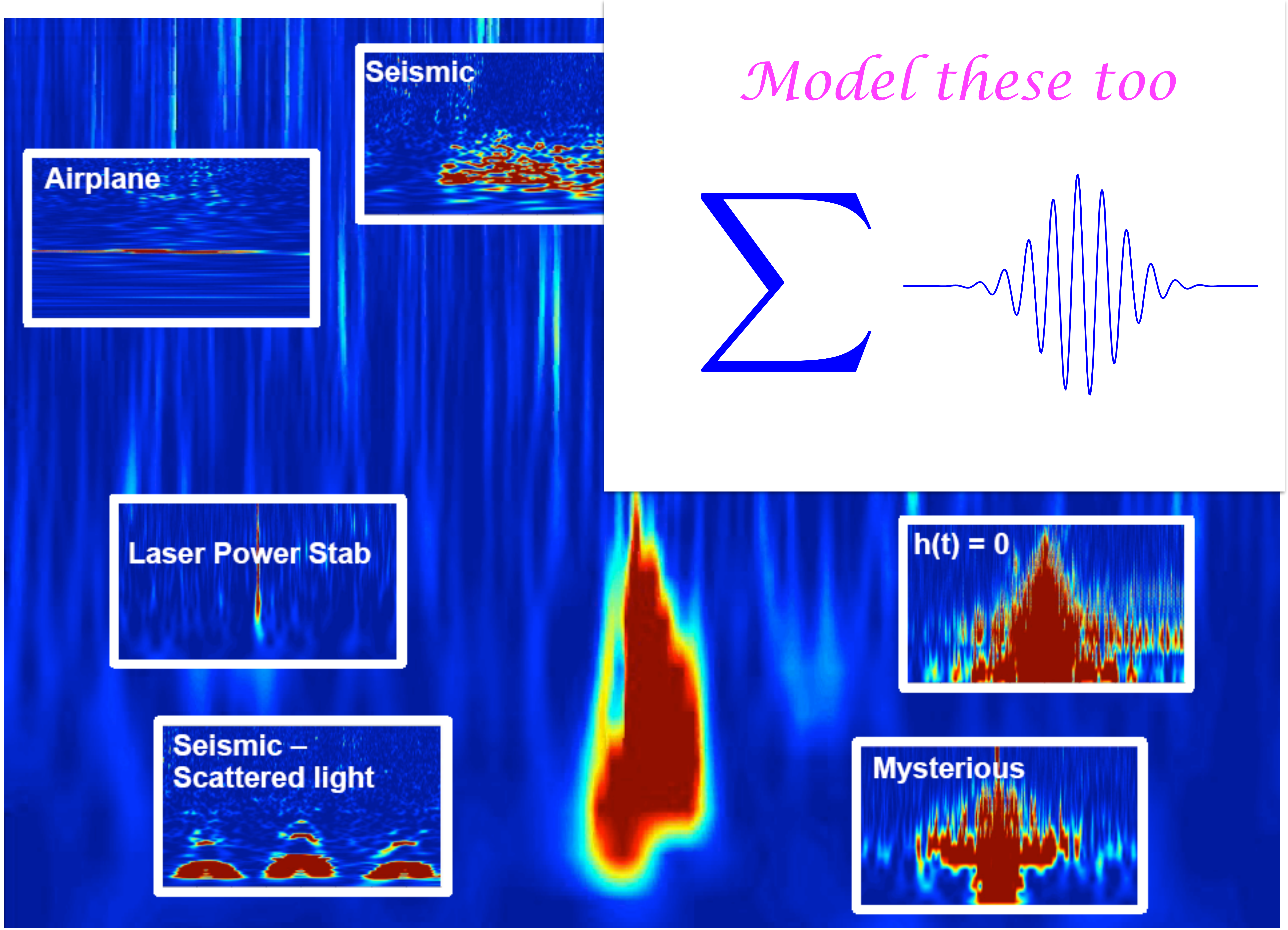


# Glitches

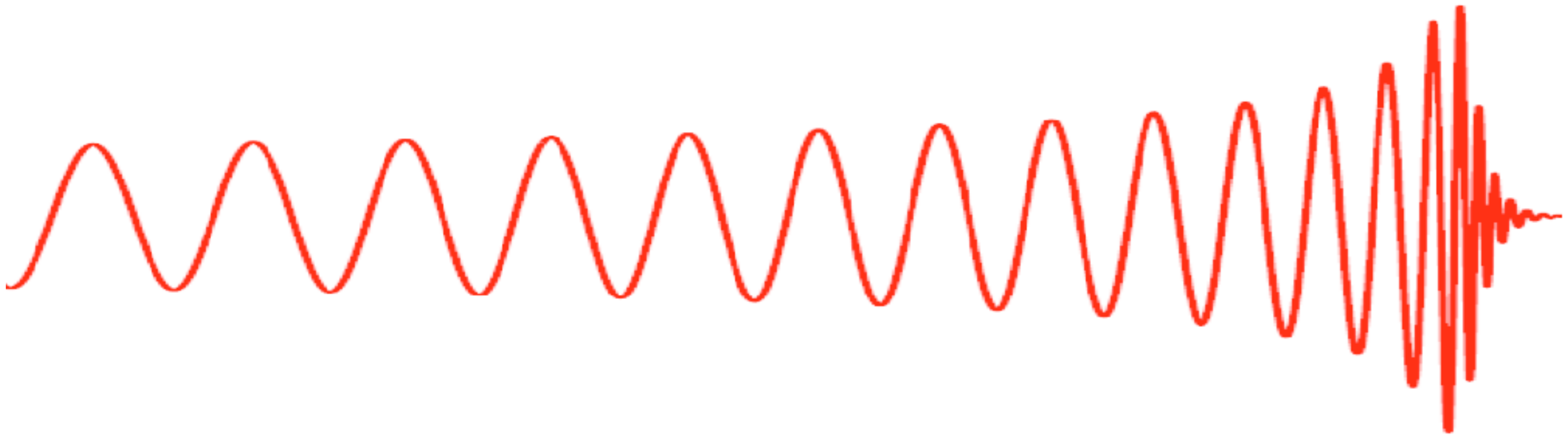


Airplane
Seismic
Seismic - Alignment
TCS
Laser Power Stab
h(t) = 0
Seismic –
Scattered light
Mysterious

# Glitches

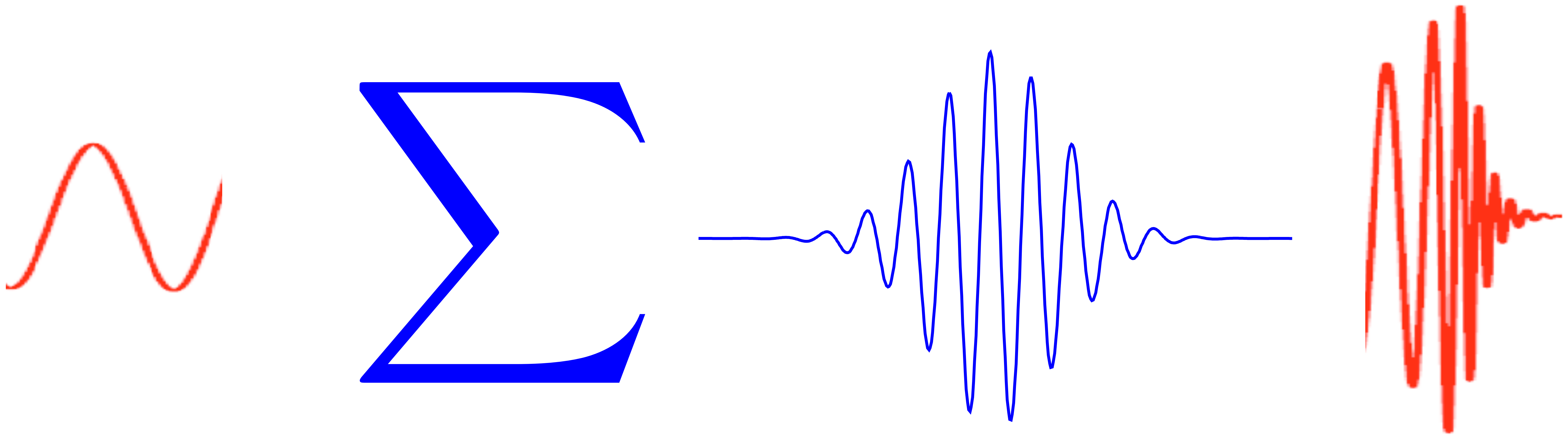

Model these too
Airplane
Seismic
Laser Power Stab
h(t) = 0
Seismic –
Scattered light
Mysterious

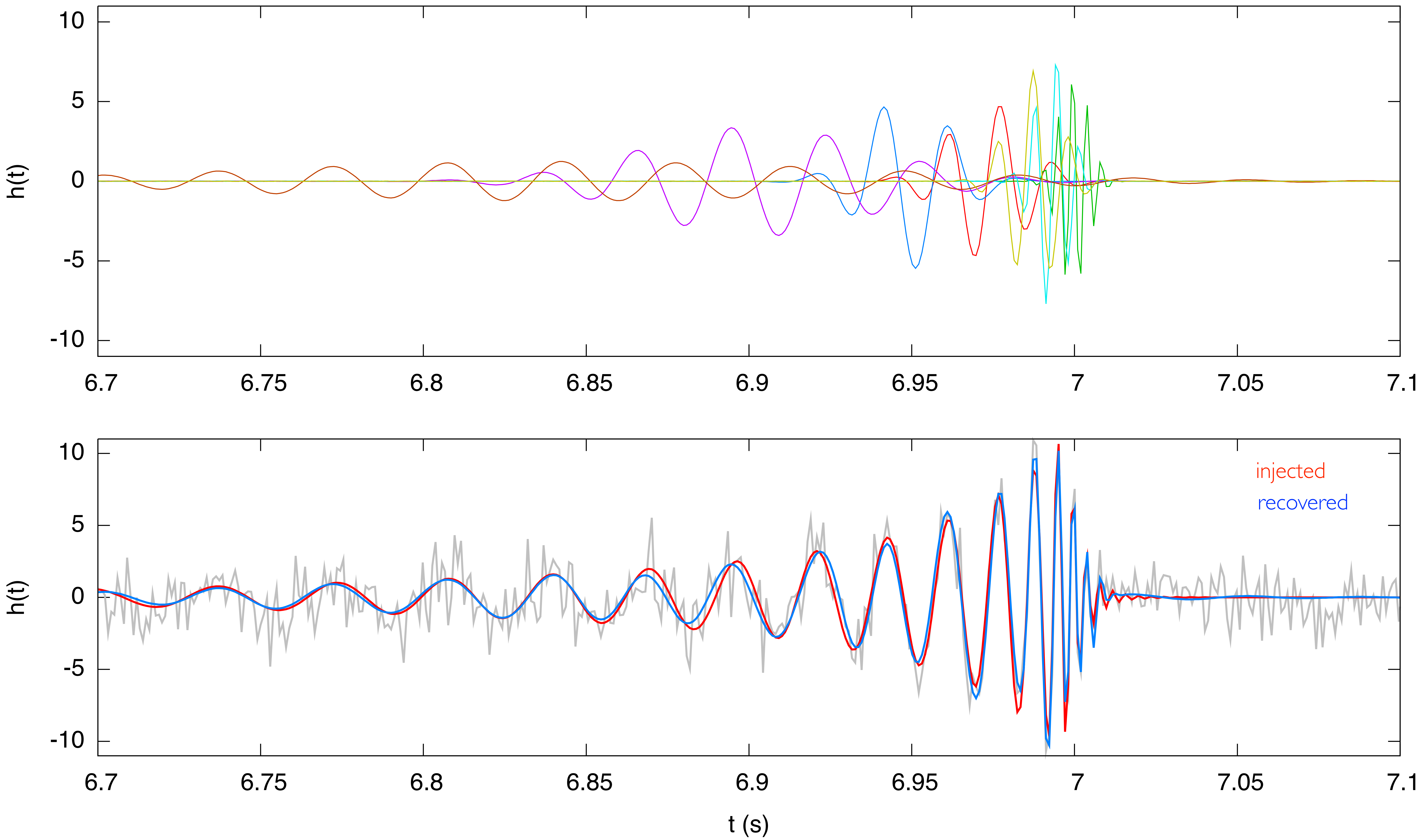
# Gravitational Waves



# Gravitational Waves

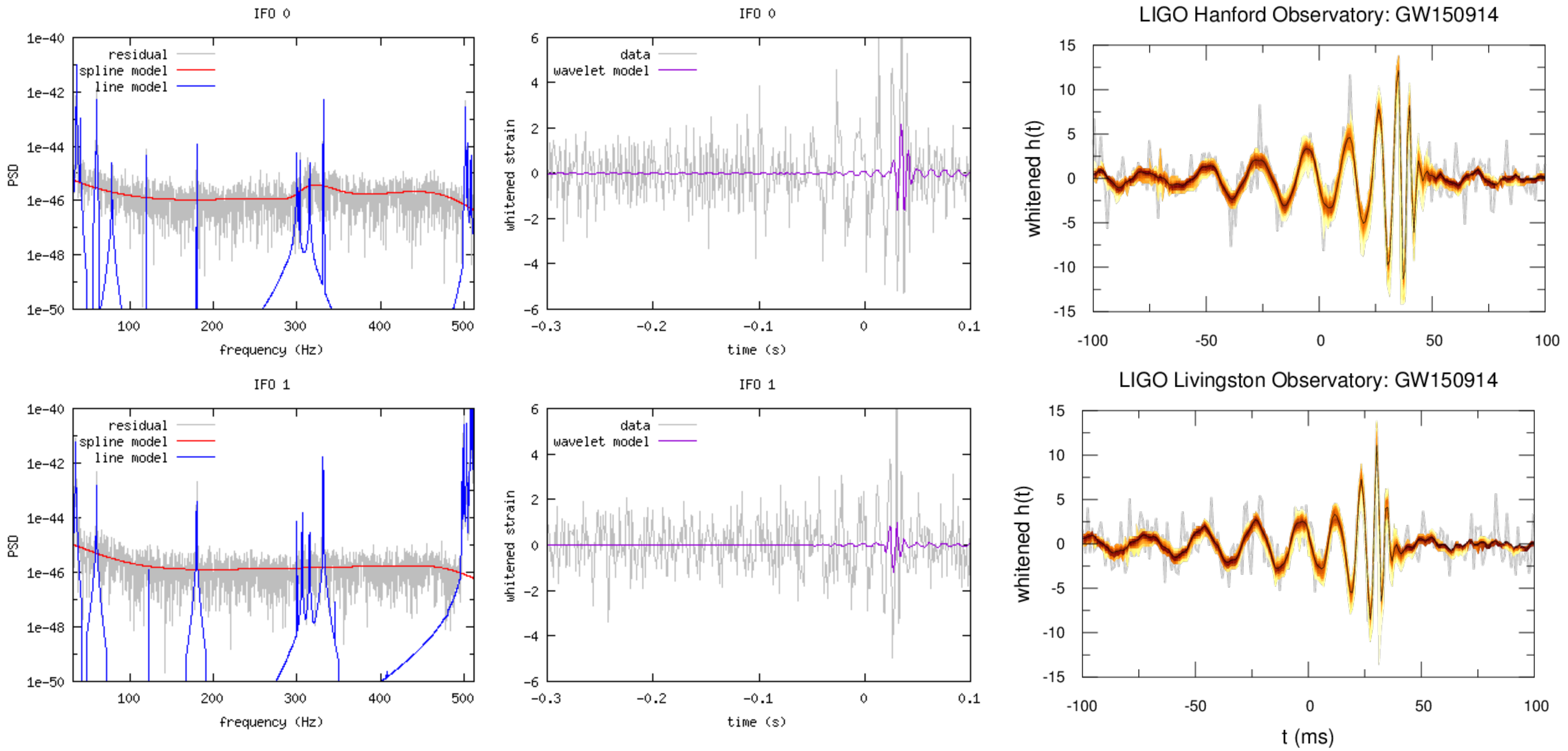
*and model these*



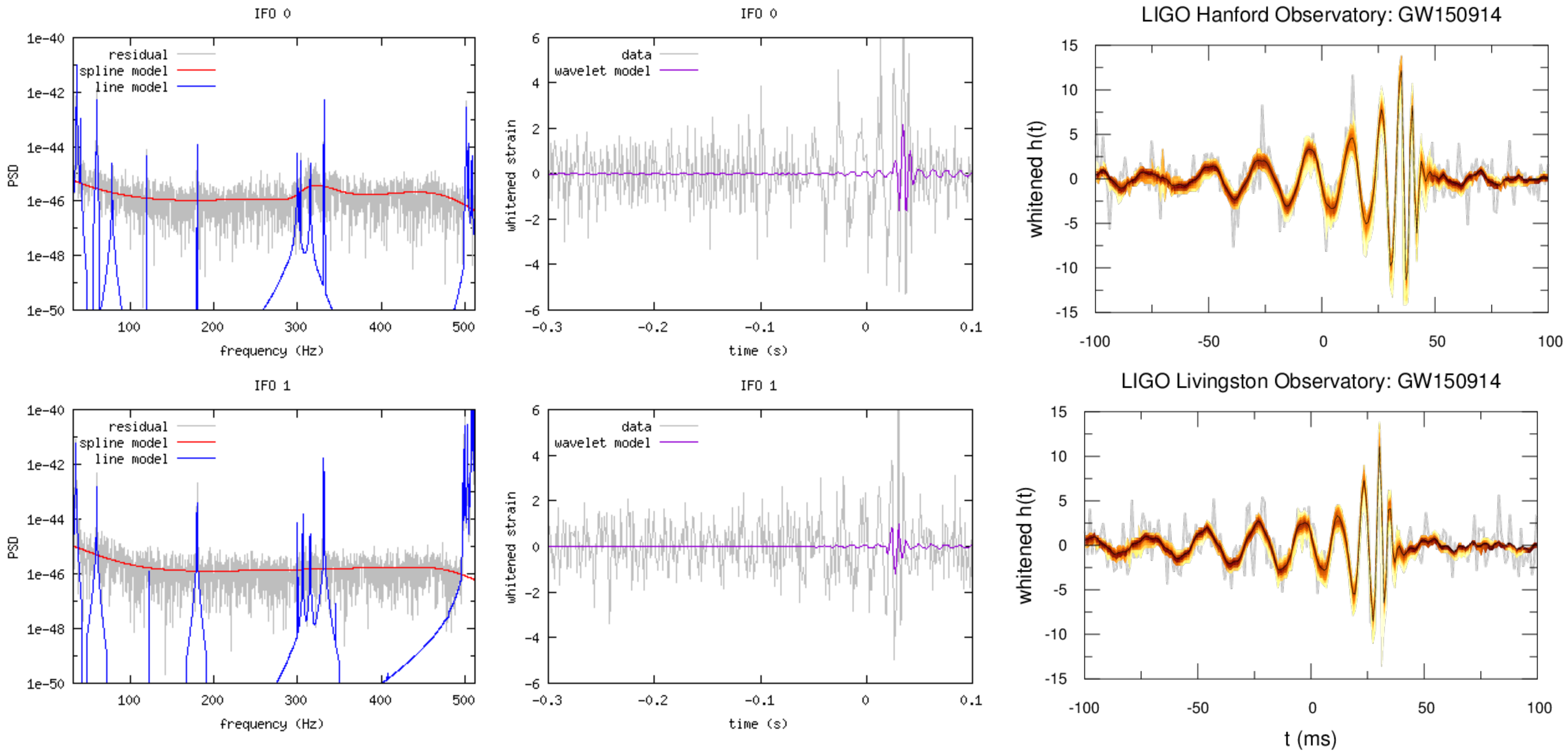


h(t)
10
5
0
-5
-10
6.7
6.75
6.8
6.85
6.9
6.95
7
7.05
7.1
injected
recovered
t (s)

# Reconstructing GW150914 with wavelets



# Reconstructing GW150914 with wavelets



# Bayesian Inference

Prior

$p(h|M)$

Likelihood

$p(d|h)$

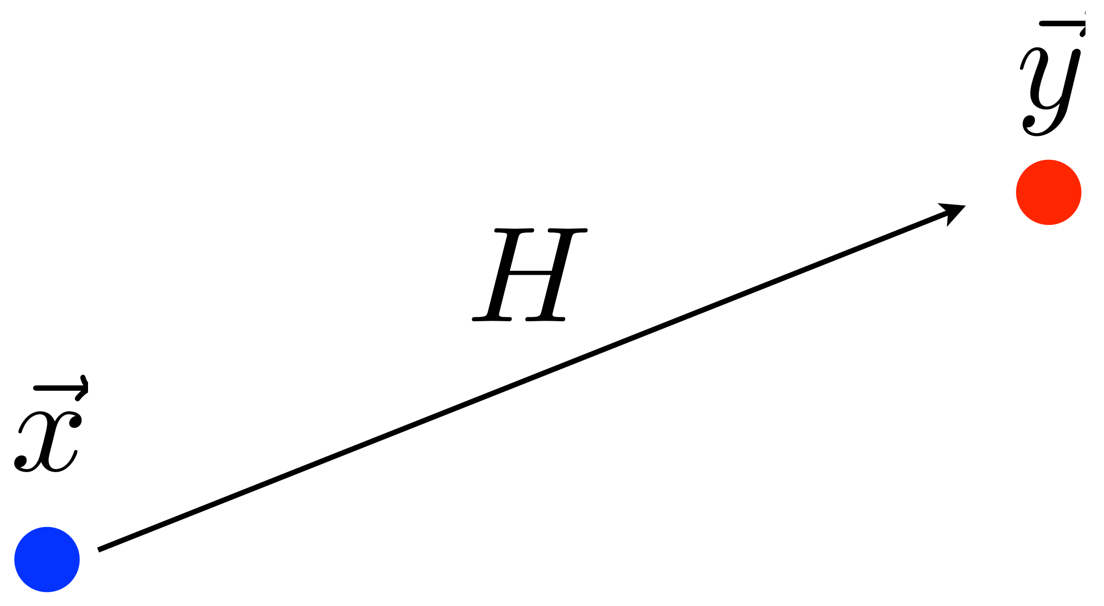

$p(h|d, M)$

Posterior

$p(d|M)$

Evidence

# Markov Chain Monte Carlo



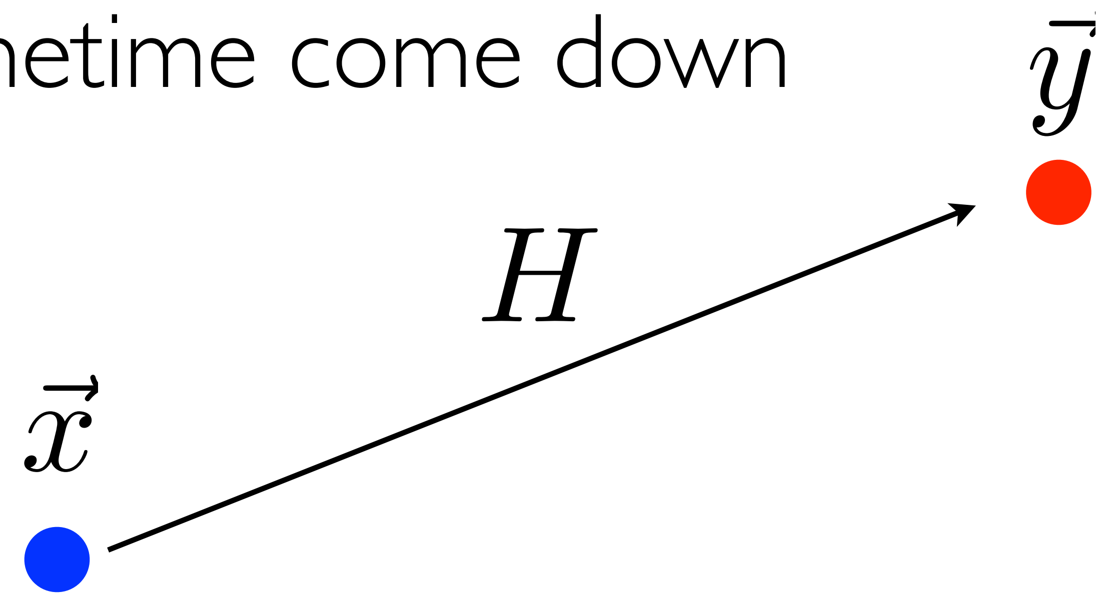

Yields PDF $p(\vec{x}|d)$ for parameters $\vec{x}$ given data $d$

$$H = \min\left(1, \frac{p(\vec{y})p(d|\vec{y})q(\vec{x}|\vec{y})}{p(\vec{x})p(d|\vec{x})q(\vec{y}|\vec{x})}\right)$$

Transition Probability
(Metropolis-Hastings)

# Markov Chain Monte Carlo

Always go up,
Sometime come down

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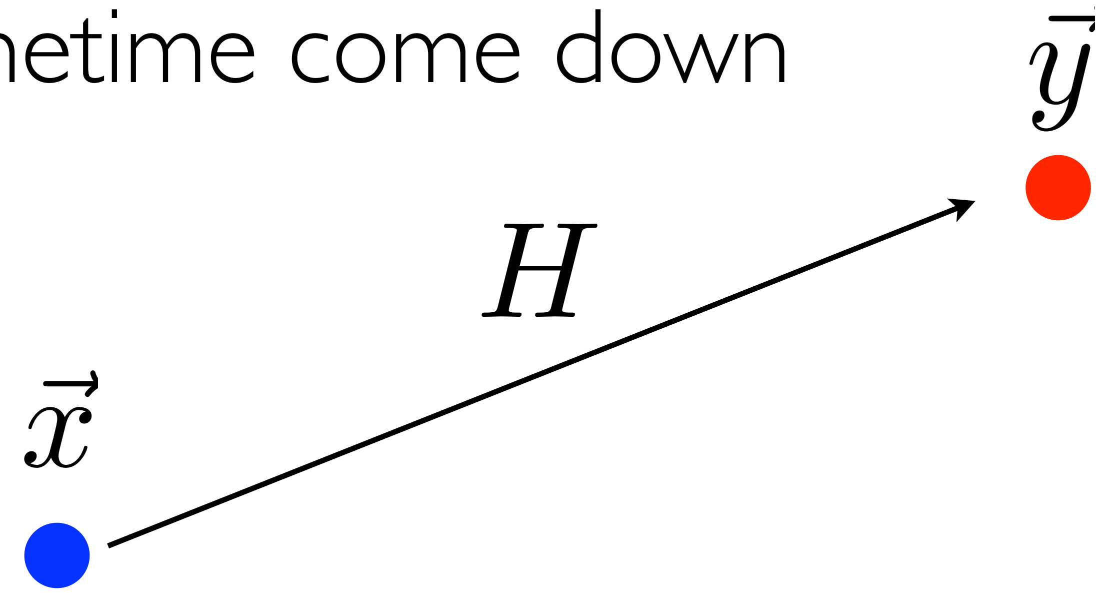


Yields PDF $p(\vec{x}|d)$ for parameters $\vec{x}$ given data $d$

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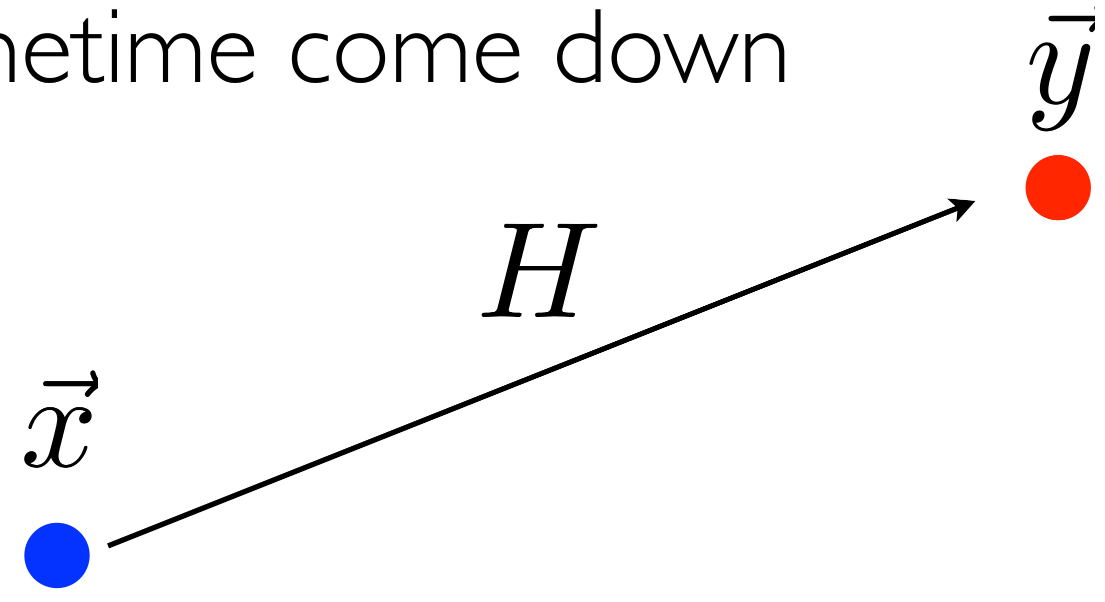
Prior

Transition Probability
(Metropolis-Hastings)

# Markov Chain Monte Carlo

Always go up,
Sometime come down

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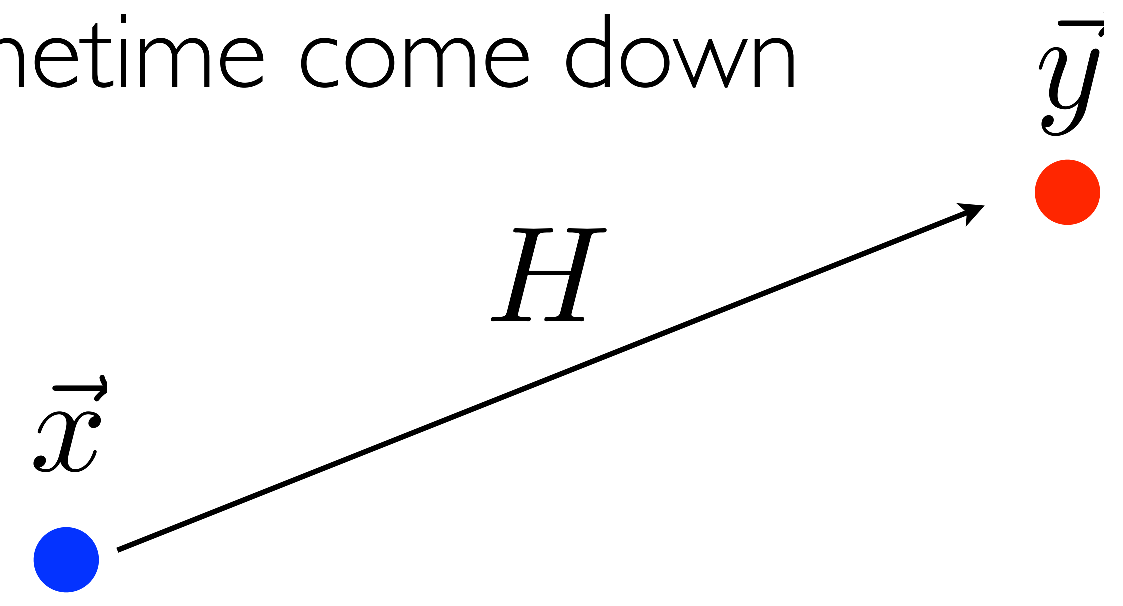
Prior

Likelihood

Transition Probability
(Metropolis-Hastings)

# Markov Chain Monte Carlo

Always go up,
Sometime come down




Yields PDF $p(\vec{x}|d)$ for parameters $\vec{x}$ given data $d$

$$H = \min\left(1, \frac{p(\vec{y})\,p(d|\vec{y})\,q(\vec{x}|\vec{y})}{p(\vec{x})\,p(d|\vec{x})\,q(\vec{y}|\vec{x})}\right)$$

Prior — Likelihood — Proposal

Transition Probability
(Metropolis-Hastings)

# Markov Chain Monte Carlo

The choice of jump proposal $q(\vec{y}|\vec{x})$ is key to convergence

Convergence to the target distribution has two facets:

- Burn-in (finding the dominant modes of the posterior)
- Mixing (exploring the dominant modes of the posterior)

The perfect proposal distribution is the posterior distribution itself, $q(\vec{y}|\vec{x}) = p(\vec{y}|d)$ , since then

$$\begin{aligned} H &= \min\left(1, \frac{p(\vec{y})p(d|\vec{y})p(\vec{x}|d)}{p(\vec{x})p(d|\vec{x})p(\vec{y}|d)}\right) \\ &= 1 \end{aligned}$$

But if we knew the posterior distribution in advance there would be no need for the MCMC procedure! Instead we seek ways to approximate the posterior.

# A MCMC Recipe

Ingredients:

Local posterior approximation

Global likelihood maps

Differential evolution proposals

Parallel tempering

Directions:

Mix all the proposals together. Check consistency by recovering the prior and diagonal PP plots.  Results are ready when distributions are stationary.

# Proposal Distributions

## Local posterior approximation

Quadratic approximation to the posterior using the augmented Fisher Information Matrix

$$q(\vec{y}|\vec{x}) = \frac{1}{\sqrt{\det(2\pi \mathbf{K}^{-1})}} e^{-\frac{1}{2} K_{ij}(x^i - y^i)(x^j - y^j)}$$

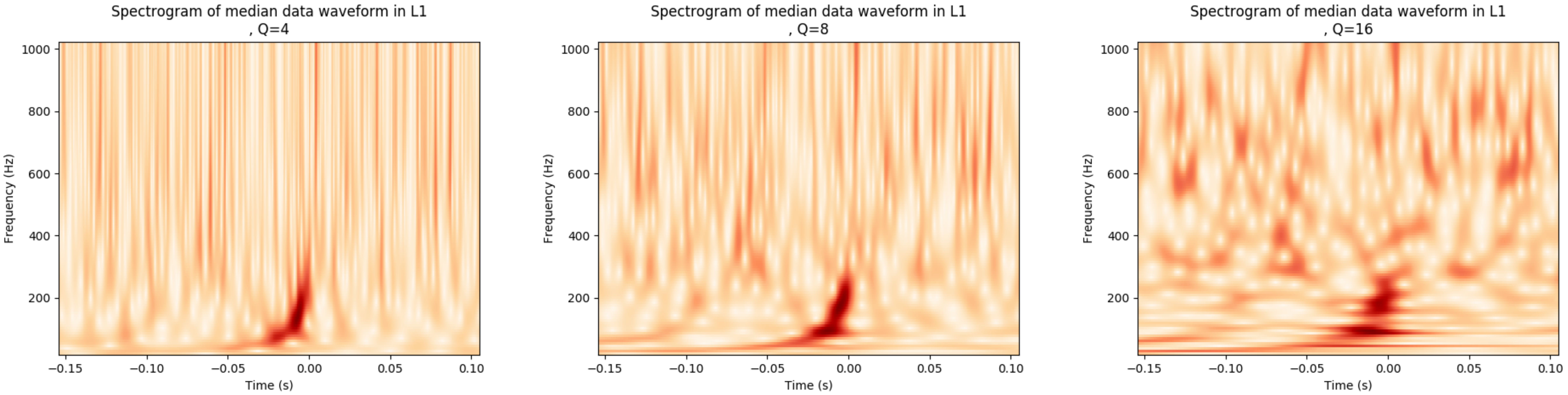
Propose jumps along eigendirections of **K**, scaled by eigenvalues

## Global likelihood maps

Use a Non-Markovian Pilot search (hill climbers, simulated annealing, genetic algorithms etc) to crudely map the posterior/likelihood and use this as a proposal distribution for a Markovian follow-up [Littenberg & Cornish, PRD 80, 063007, (2009)]

Time-frequency maps, Maximized likelihood maps

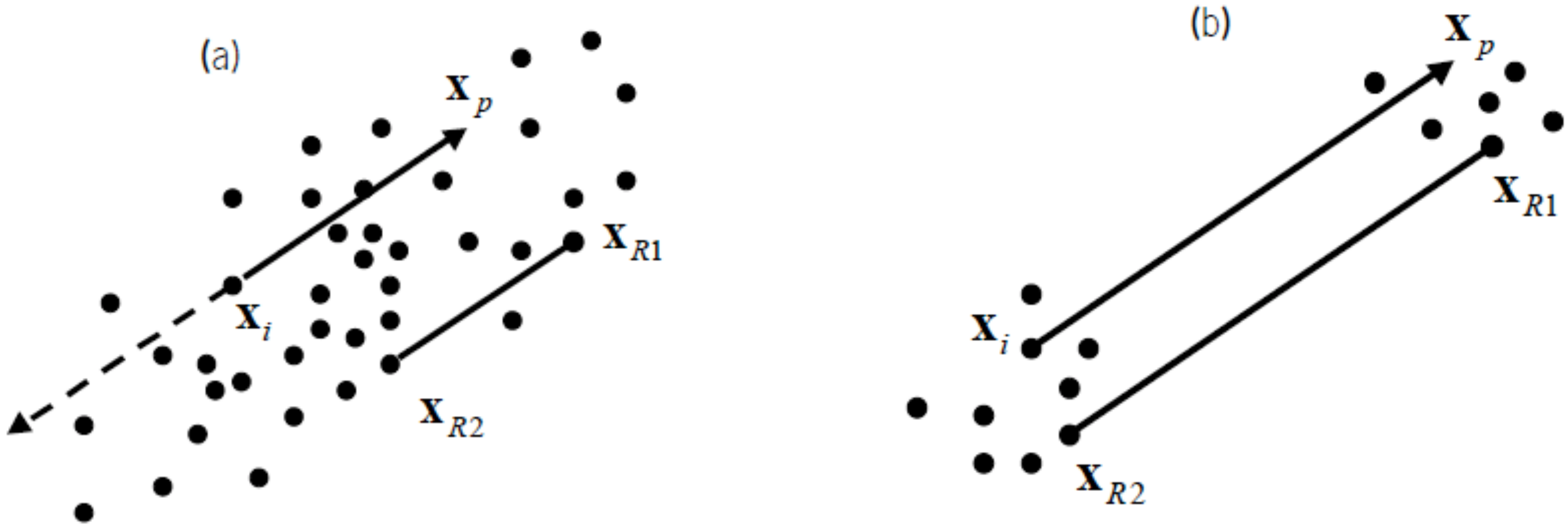
# BayesWave Global Map Proposal



# Proposal Distributions

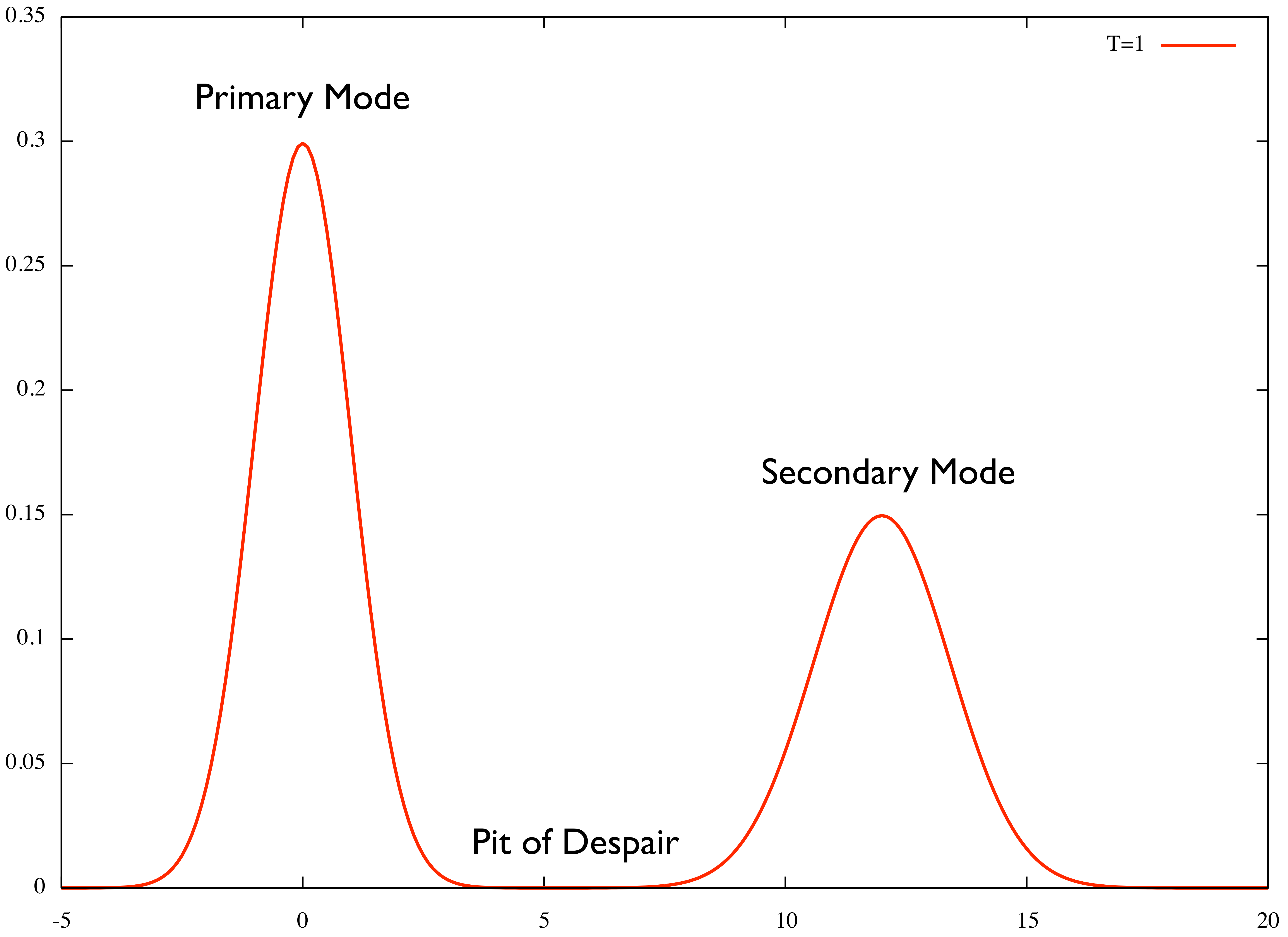
## Differential evolution

[Braak (2005)]



# Parallel Tempering

[Swendsen & Wang, 1986]




Ordinary MCMC techniques side-step the need to compute the evidence. PT uses multiple, coupled chains to improve mixing, and also allows the evidence to be computed.

Explore tempered posterior

$$\pi(\vec{\lambda}|\mathbf{d})_T = p(\mathbf{d}|\vec{\lambda})^{1/T} p(\vec{\lambda})$$

Compute model evidence

$$\log p(\mathbf{d}) = \int_0^1 \mathbb{E}[\log p(\mathbf{d}|\vec{\lambda})]_\beta \, d\beta$$

(Here $\beta = \frac{1}{T}$ )

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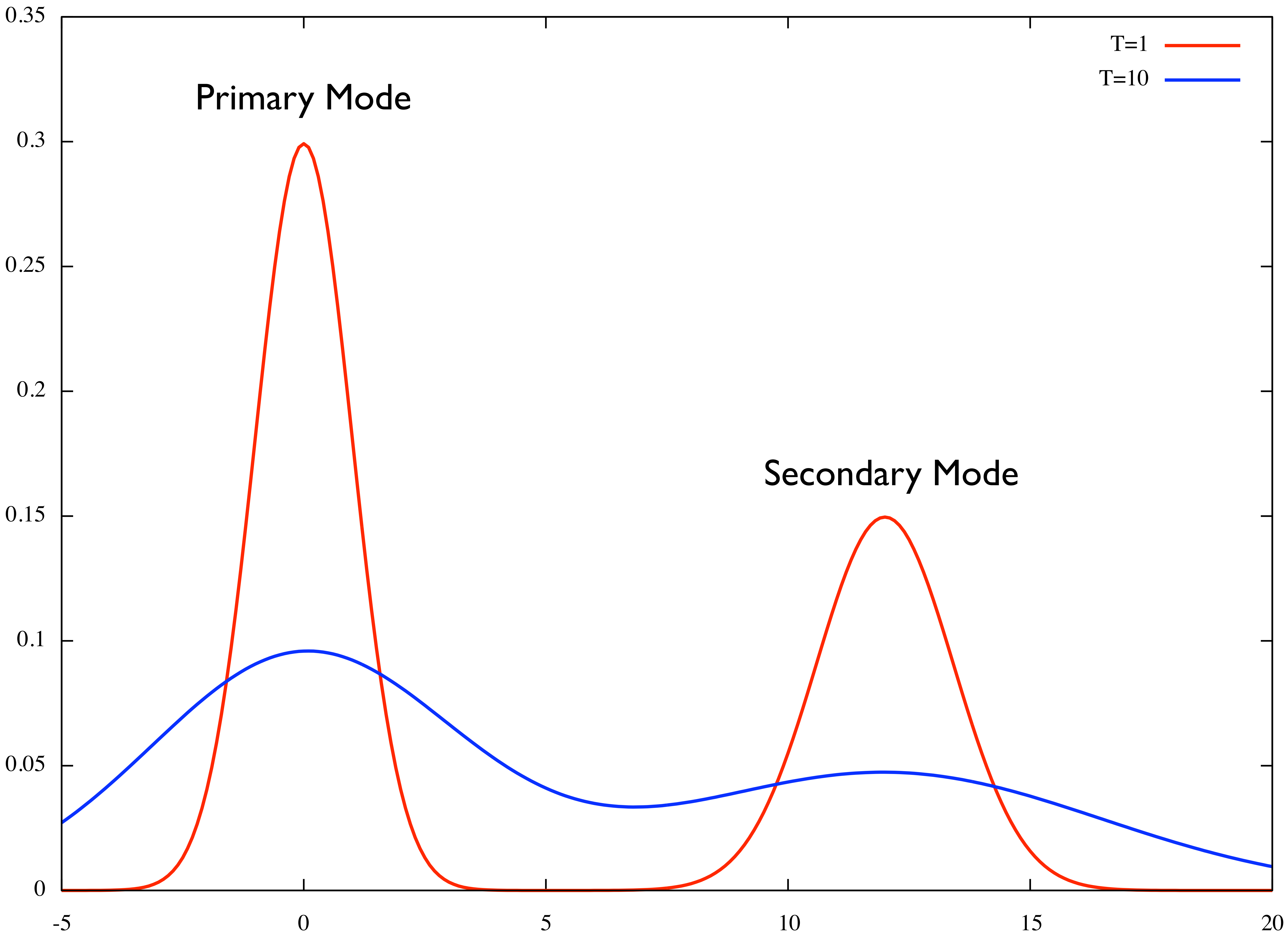


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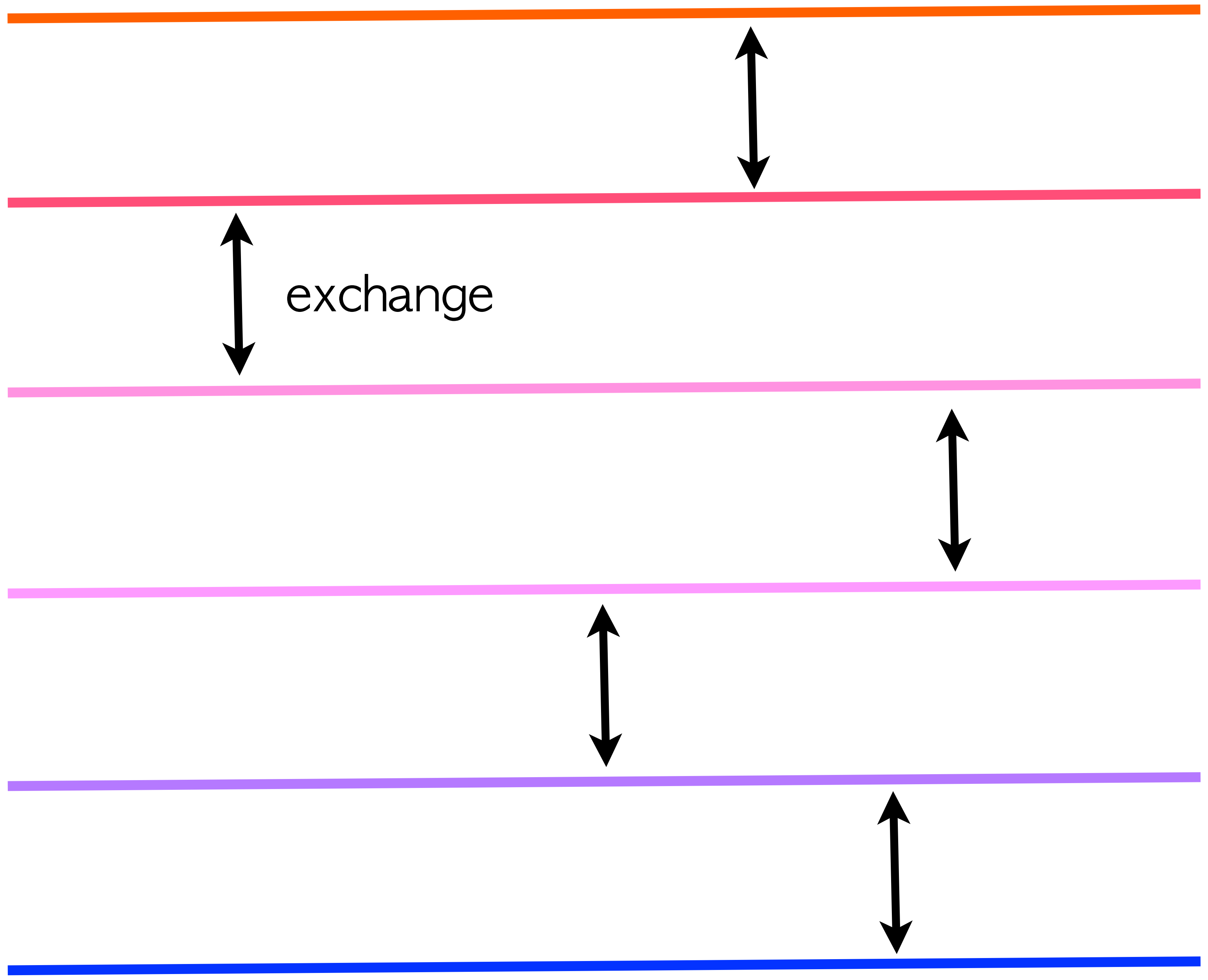
# Parallel Tempering

Wide exploration, poor "memory"




Limited exploration, good "memory"

Inter-chain transition probability

$$H = \min\left(1, \frac{\pi(\vec{\lambda}_{j+1}|\mathbf{d})_{T_j}\,\pi(\vec{\lambda}_j|\mathbf{d})_{T_{j+1}}}{\pi(\vec{\lambda}_j|\mathbf{d})_{T_j}\,\pi(\vec{\lambda}_{j+1}|\mathbf{d})_{T_{j+1}}}\right)$$

Good solutions sink down ladder and get stored

# Parallel Tempering - Temperature Spacing

$$\ln L_T = \ln(p(\mathbf{d}|\vec{\lambda})^{1/T}) = -\frac{1}{2T}(\mathbf{d} - \mathbf{h}(\vec{\lambda})|\mathbf{d} - \mathbf{h}(\vec{\lambda})) \approx -\frac{\Gamma_{ij}\Delta\lambda^i\Delta\lambda^j}{2T}$$

Coordinate transformation:

$$\ln L_T \approx -\frac{\delta_{ij}\theta^i\theta^j}{2T} \quad \Rightarrow \quad p(\ln L_T) = \frac{(-\ln L)^{D/2-1}}{\Gamma(\frac{D}{2})T^{D/2}}\, e^{\ln L/T}$$

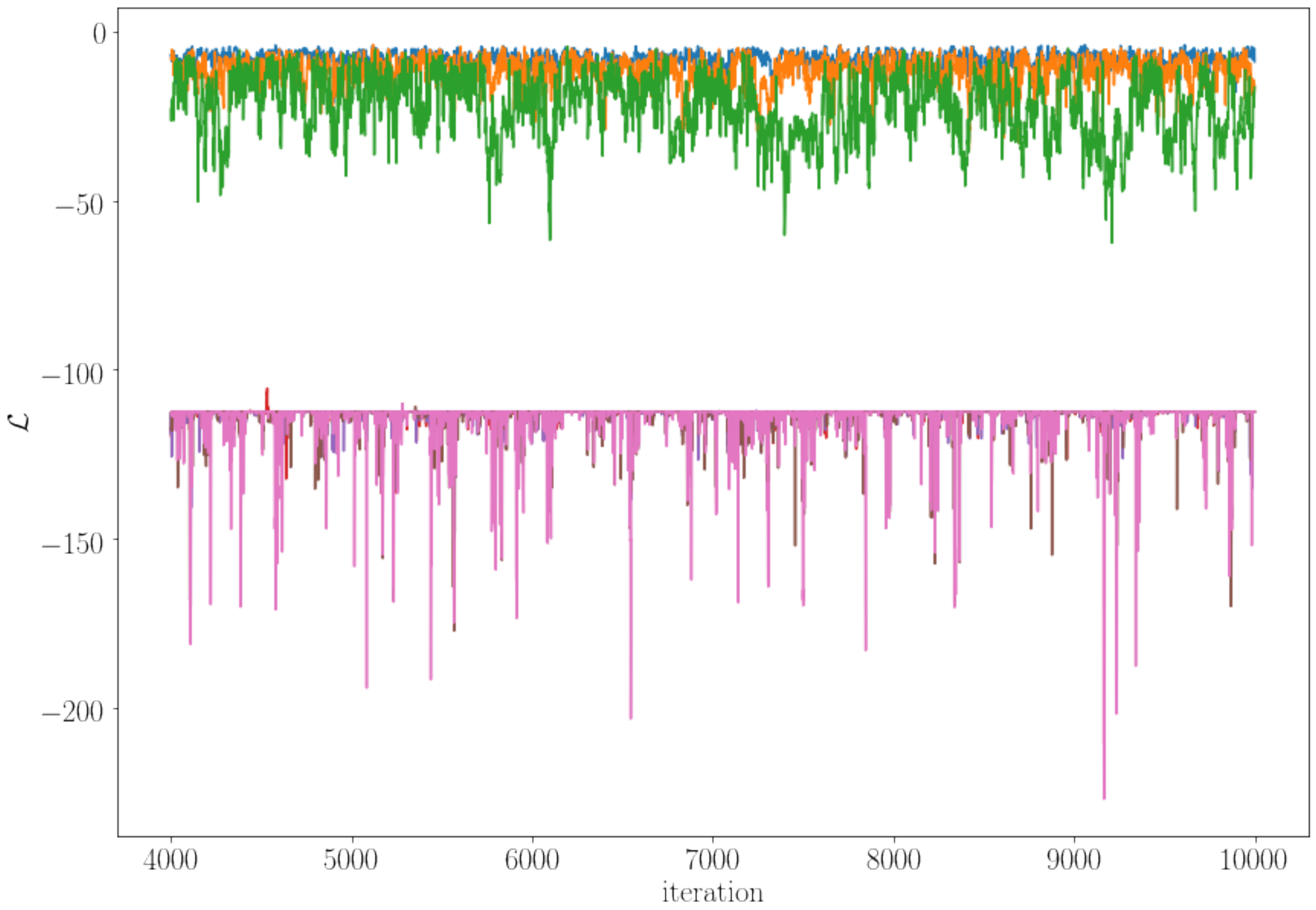
$$\mathrm{E}[\ln L_T] = -\frac{DT}{2} \qquad \mathrm{Var}[\ln L_T] = \frac{DT^2}{2}$$

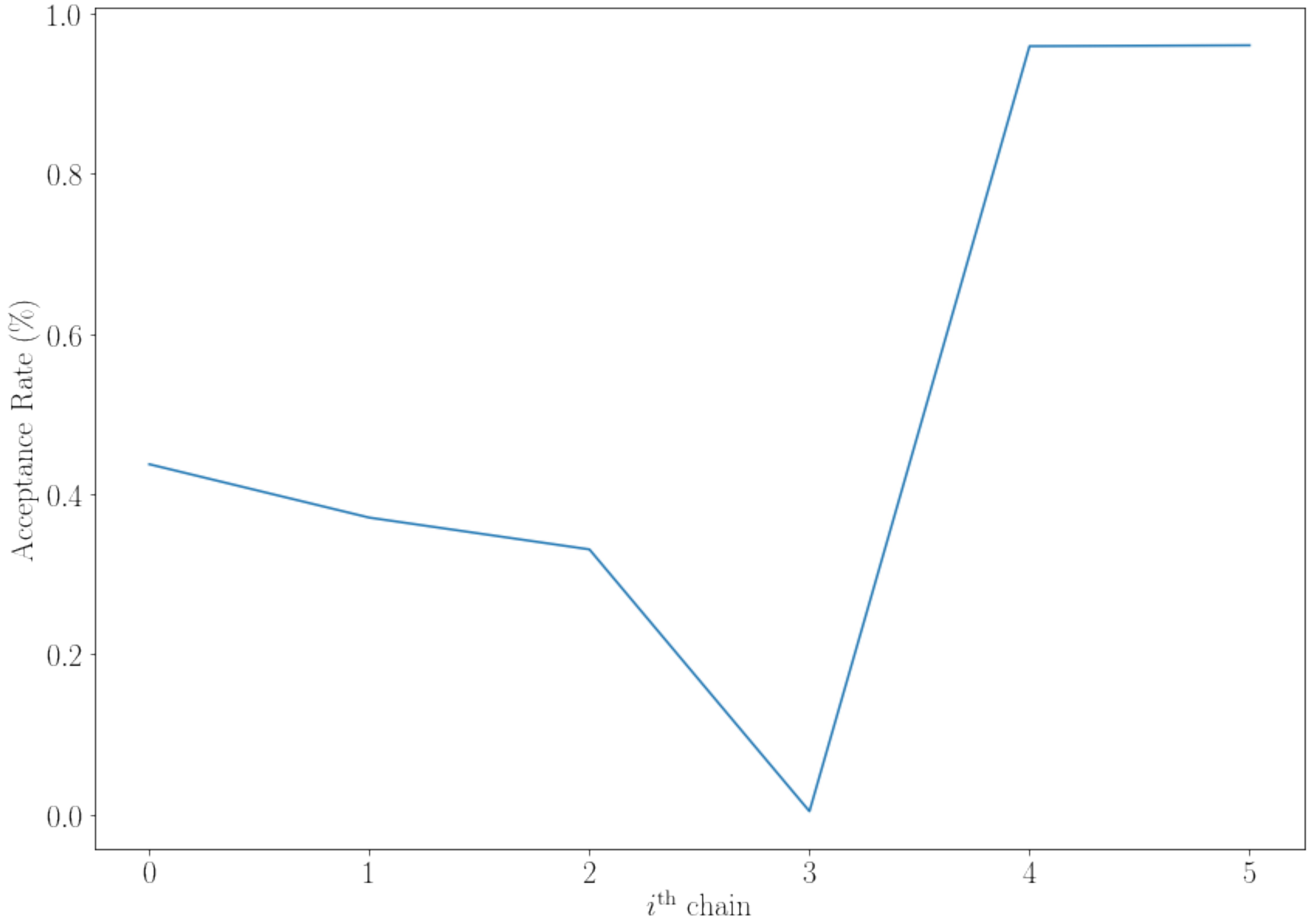
Scaled distance between chains:

$$\frac{\mathrm{E}[\ln L_{T_i}] - \mathrm{E}[\ln L_{T_{i+1}}]}{\sqrt{\mathrm{Var}[\ln L_{T_i}]}} = \frac{\Delta T}{T}\sqrt{\frac{D}{2}}$$

$\Rightarrow$ Uniform acceptance of exchange for logarithmically spaced temperature ladder. Larger dimension models require closer temperature spacing.

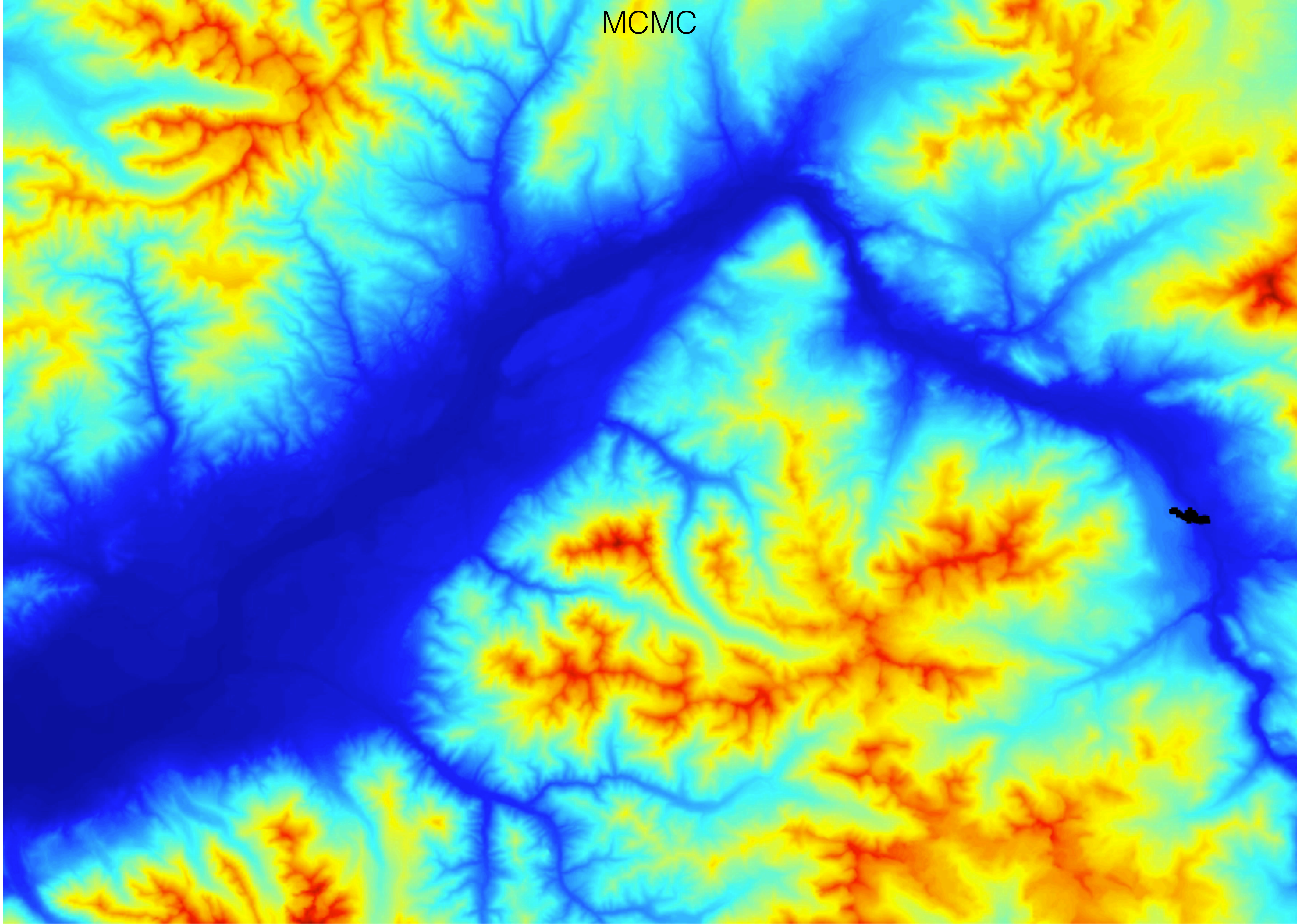
# Parallel Tempering



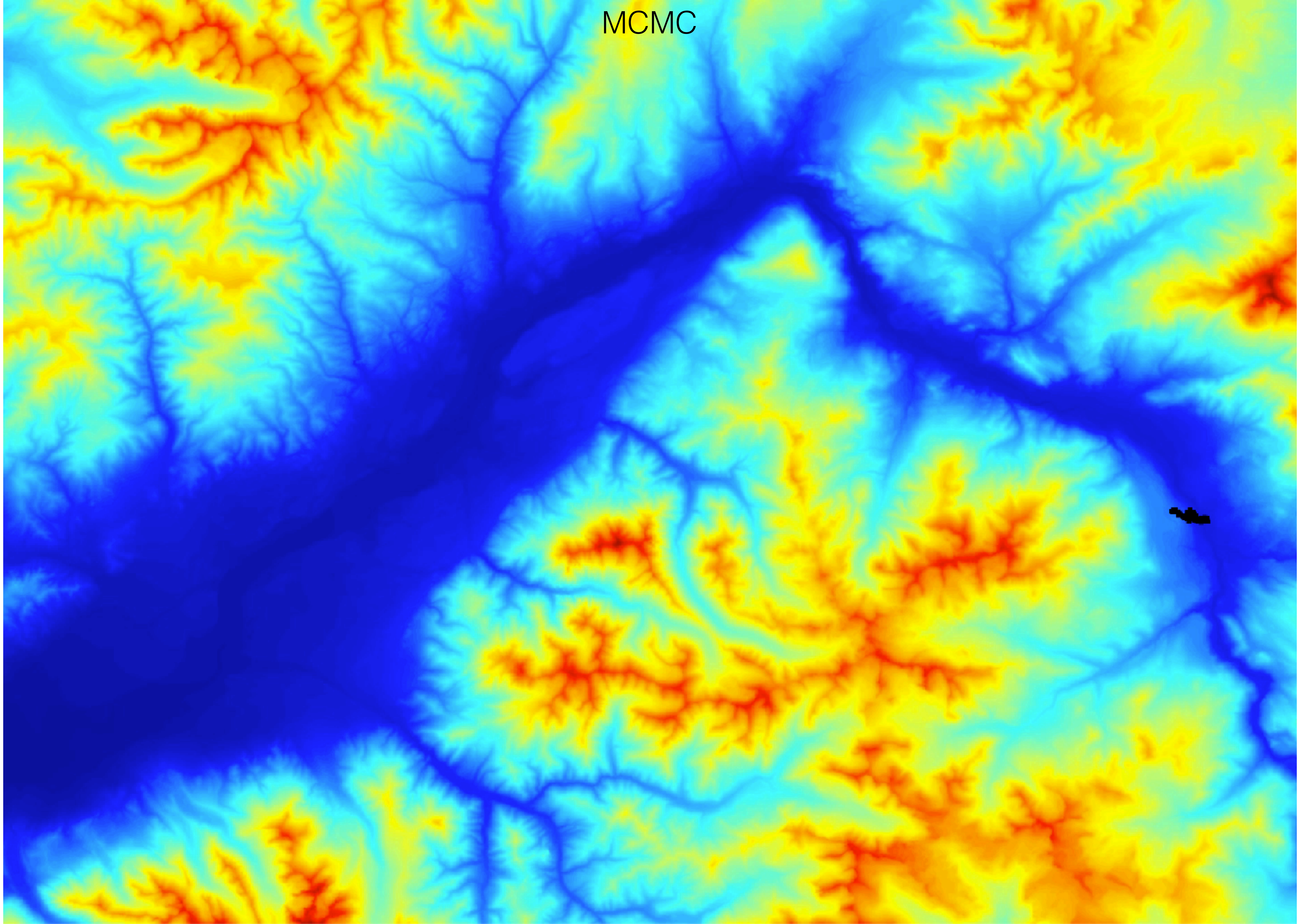




Need to use dynamic temperature spacing: [Vousden, Farr, Mandel, MNRAS **455** 1919 (2016)]

MCMC



MCMC



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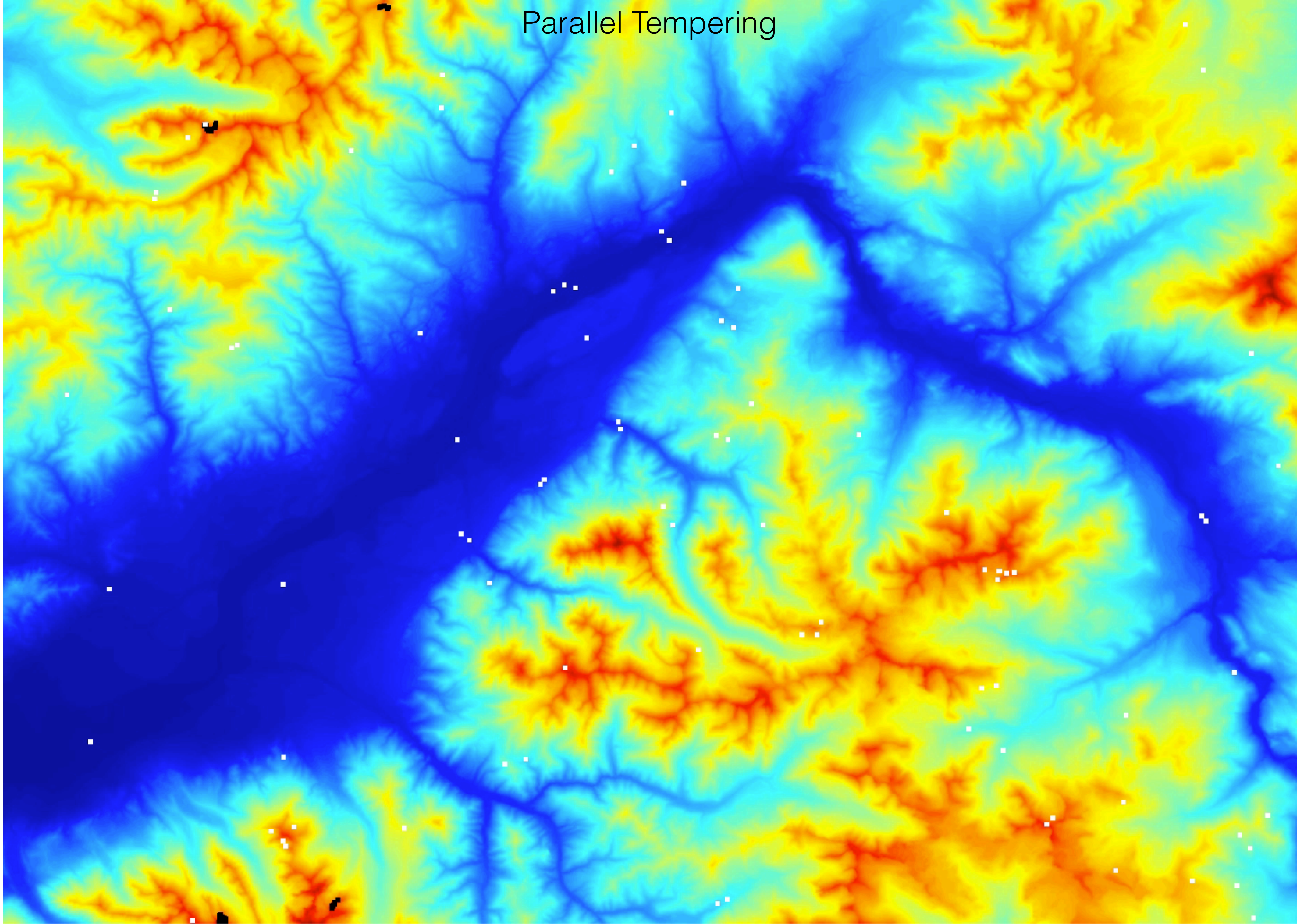
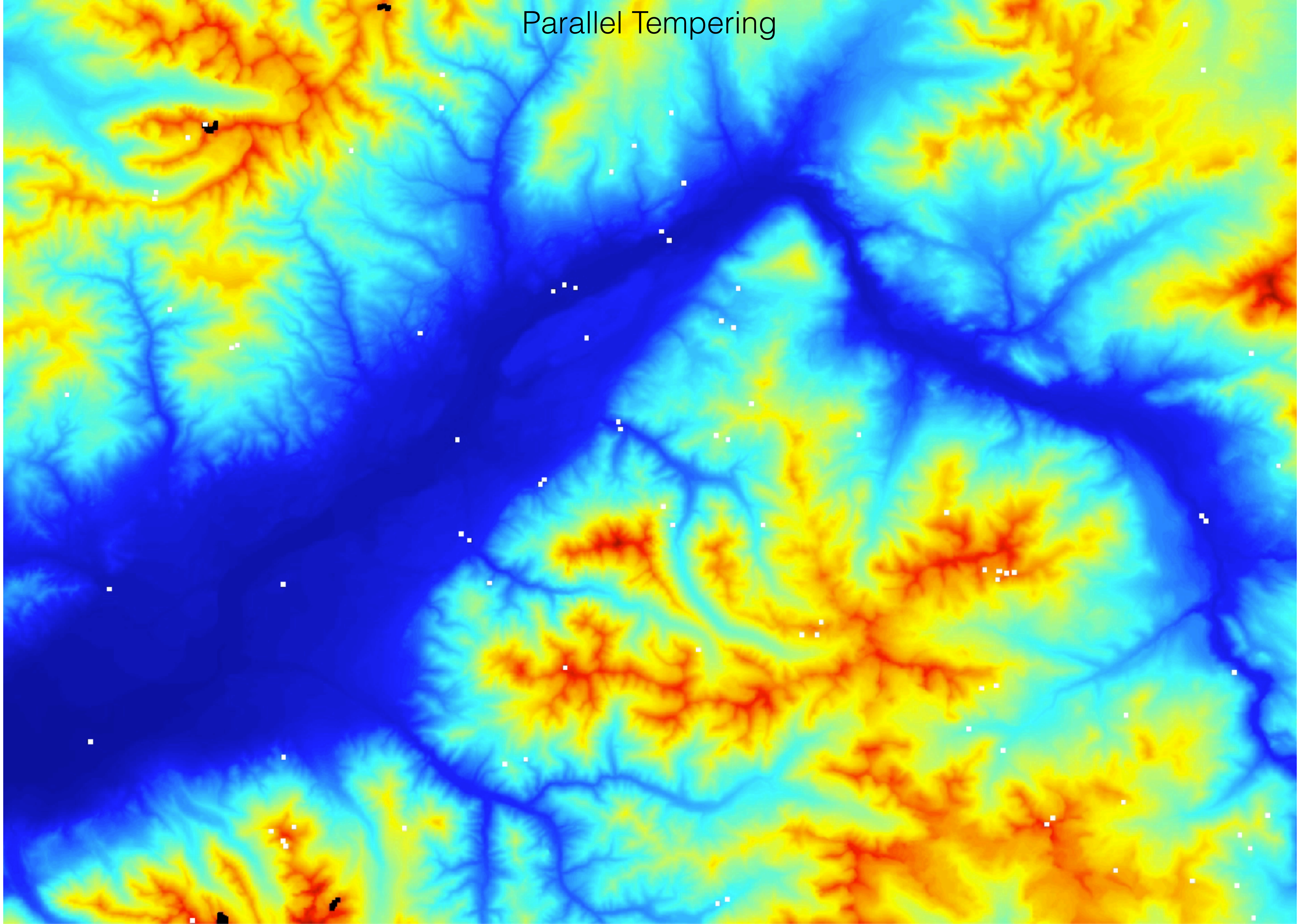


## Parallel Tempering



# Solving for everything

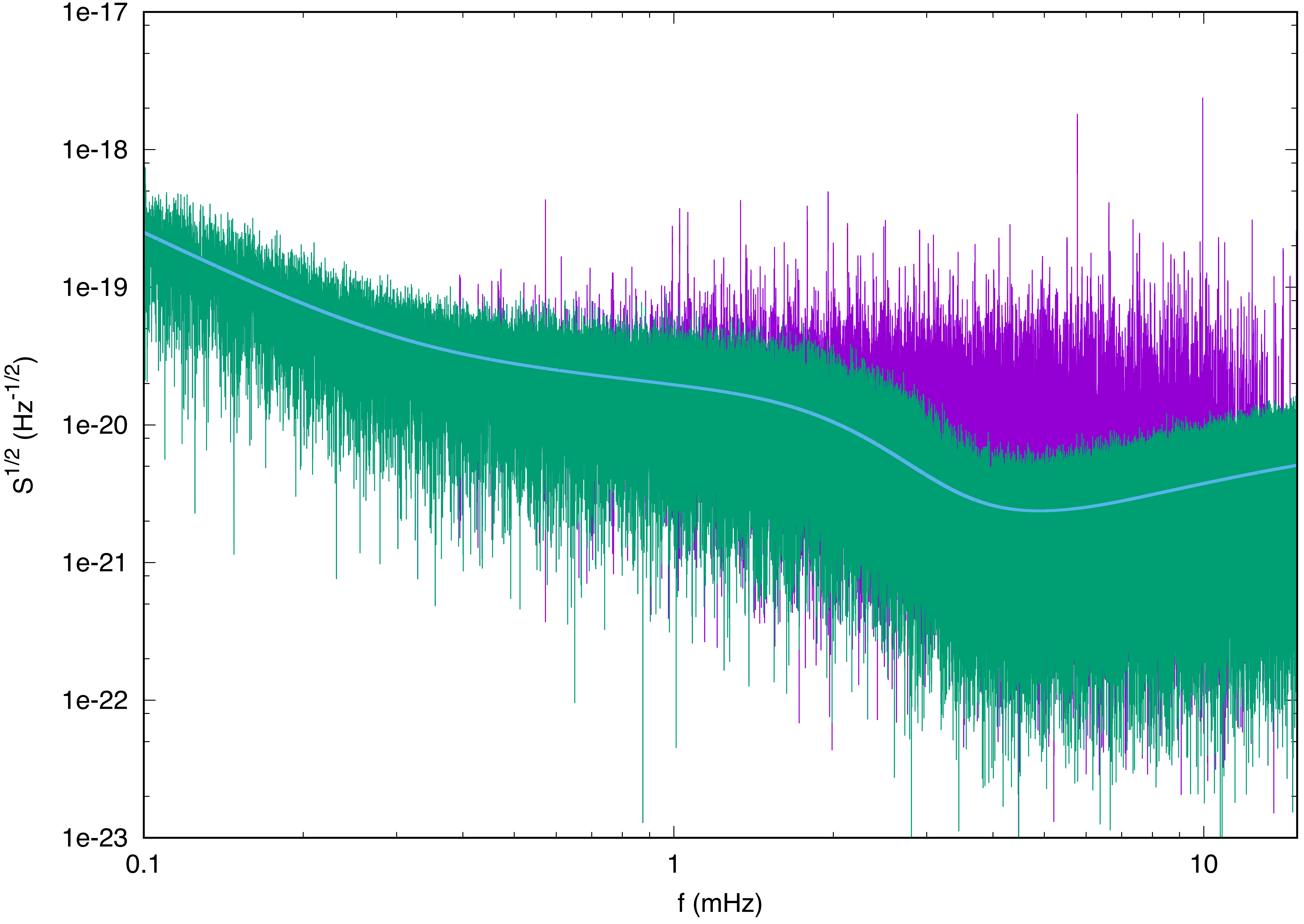
- Cab update one source, or even one parameter, at a time
- Model dimension will grow to D ~ 500K
- Cost scales as some polynomial of the model dimension
- Highly parallel. Can collect samples from thousands of chains
- Parallel tempering ideal for distributed computing - chain swaps can be done asynchronously

# Return to the galactic binary problem

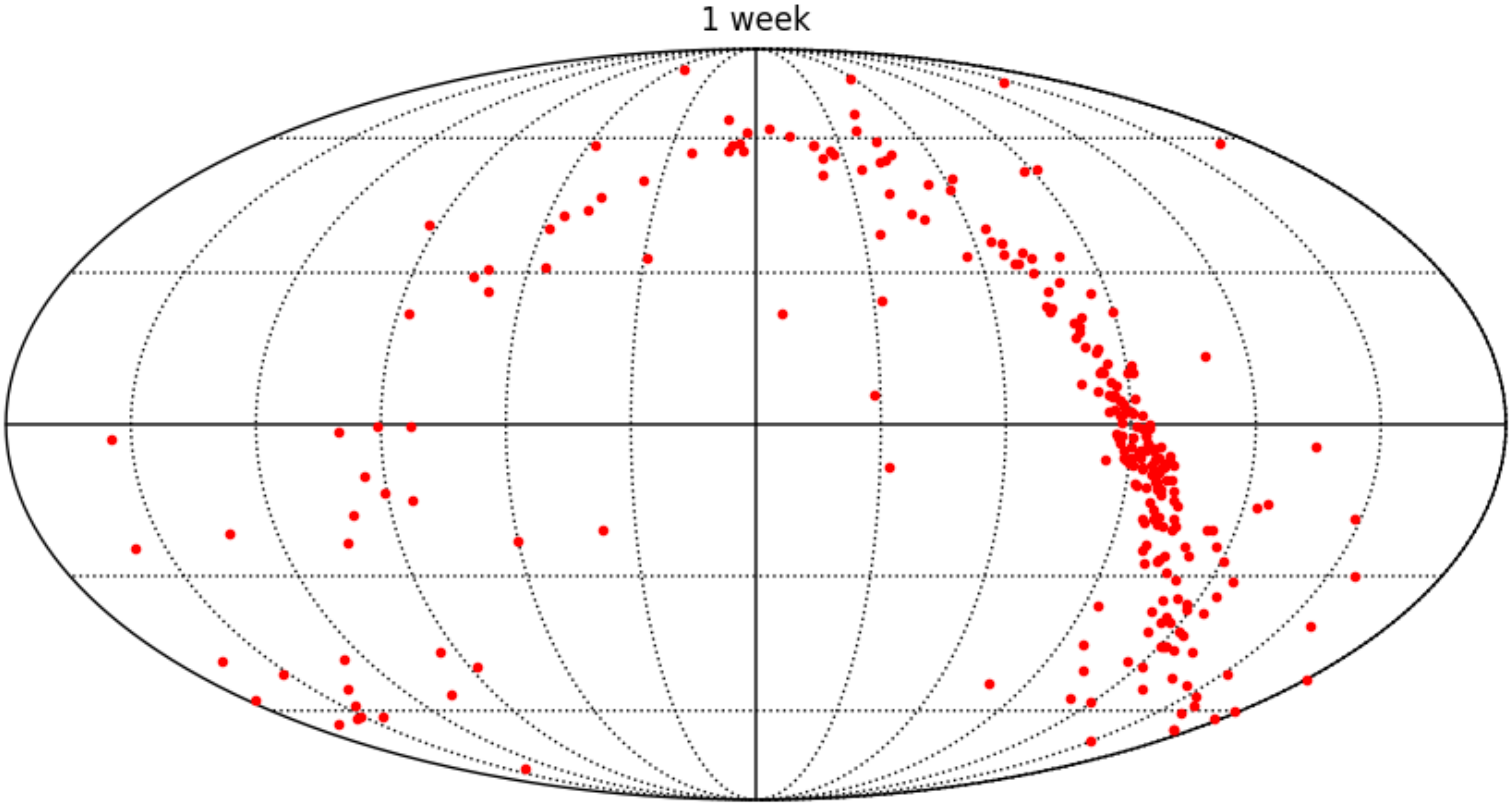
Travis Robson, Neil Cornish & Tyson Littenberg

- Uses BayesWave code base (originally developed from LISA GB experience)
- Simultaneous solution for noise and signals
- Time evolving approach = weekly increments
- F-statistic based likelihood map as global proposal

# Galaxy + Instrument noise spectrum, 1 year



# LISA galaxy catalog over time



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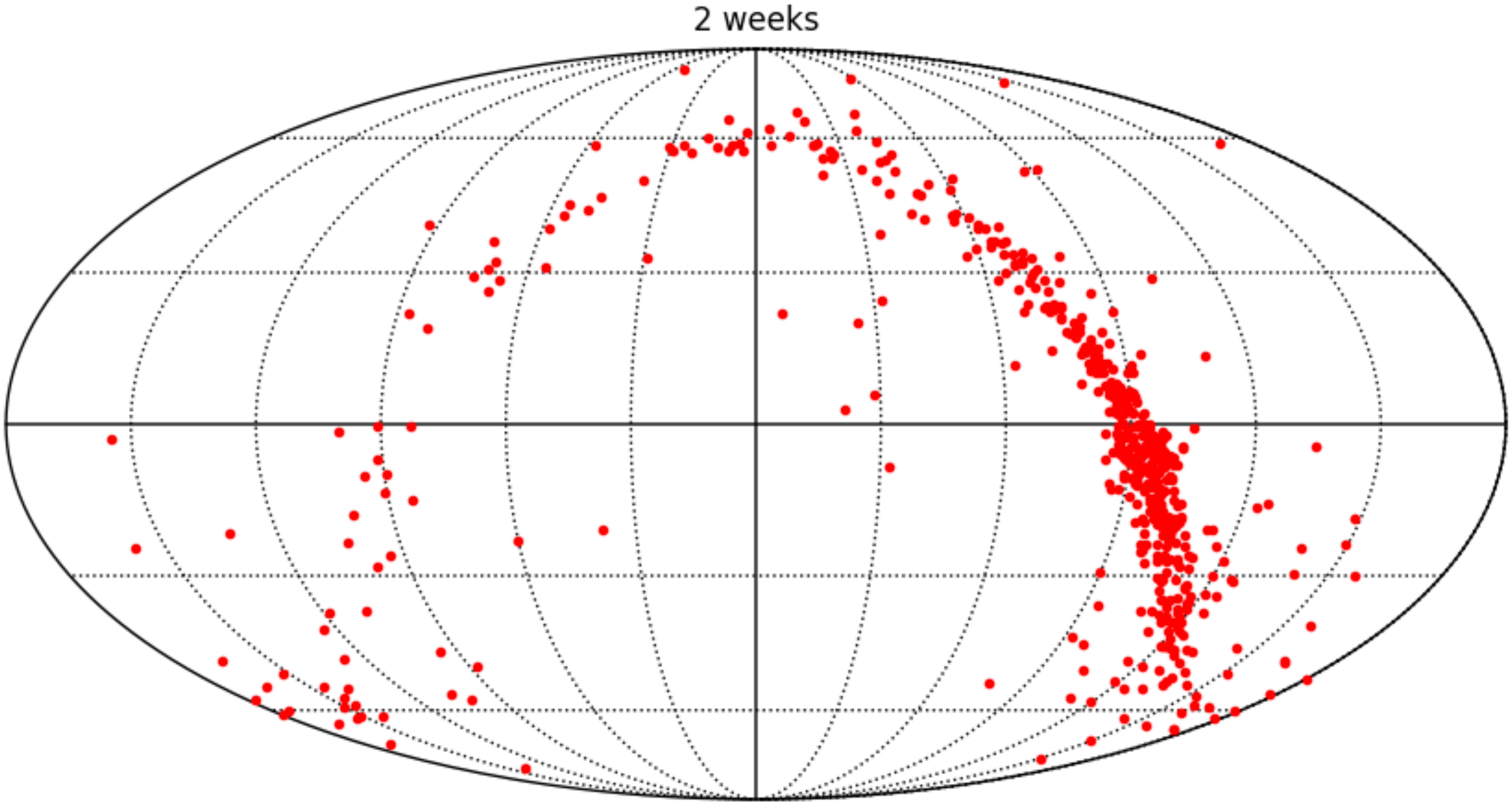

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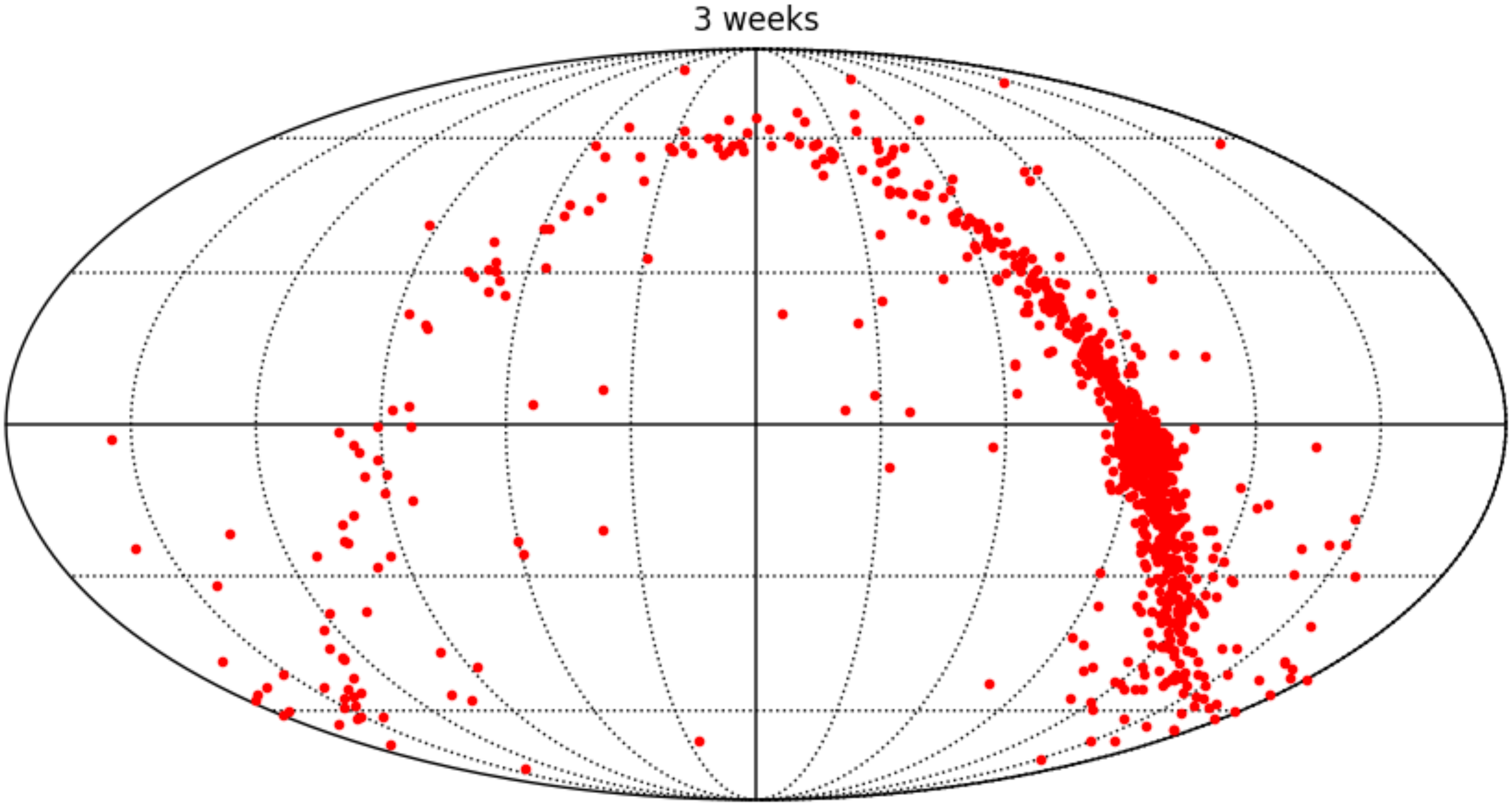

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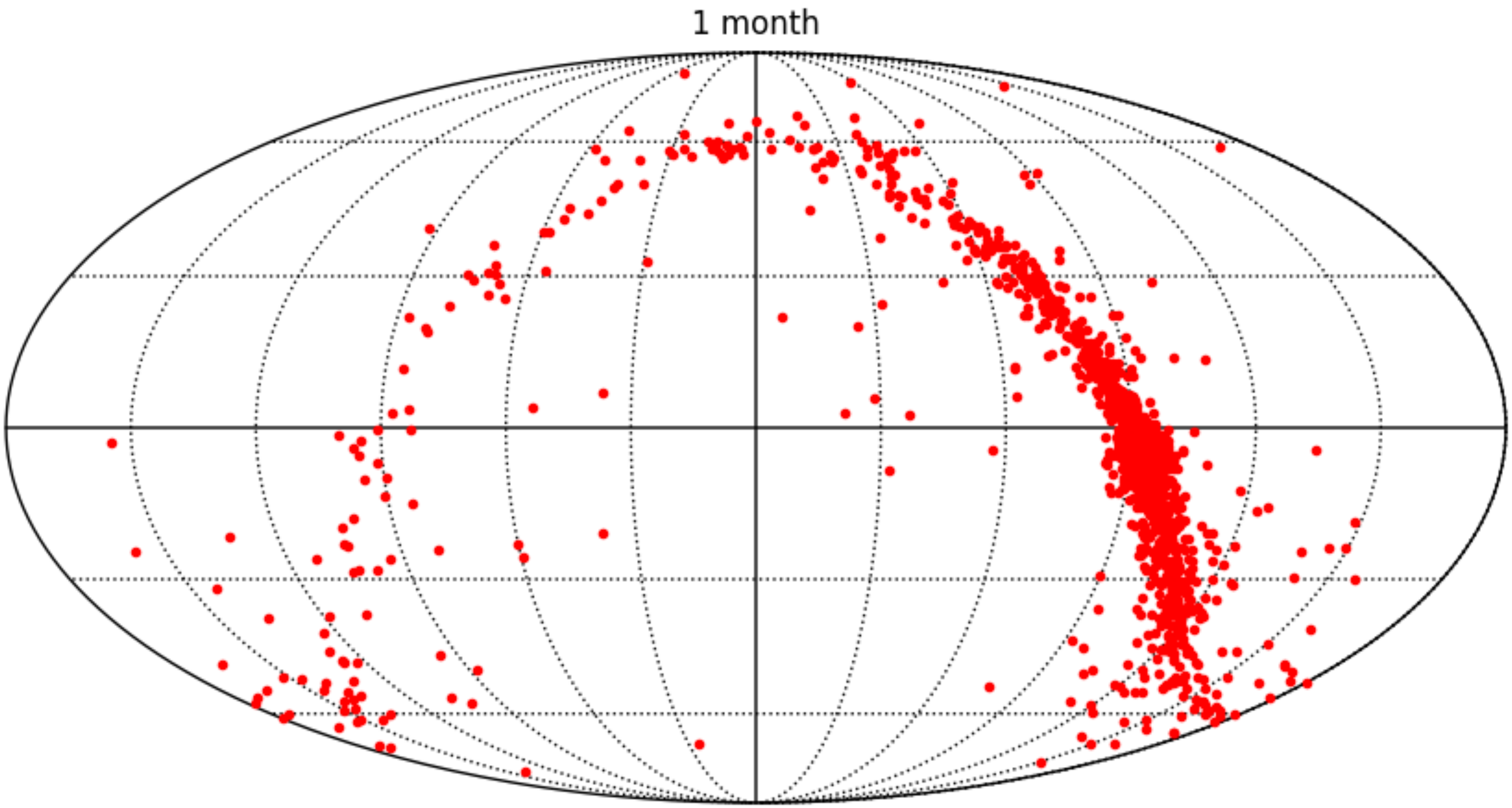
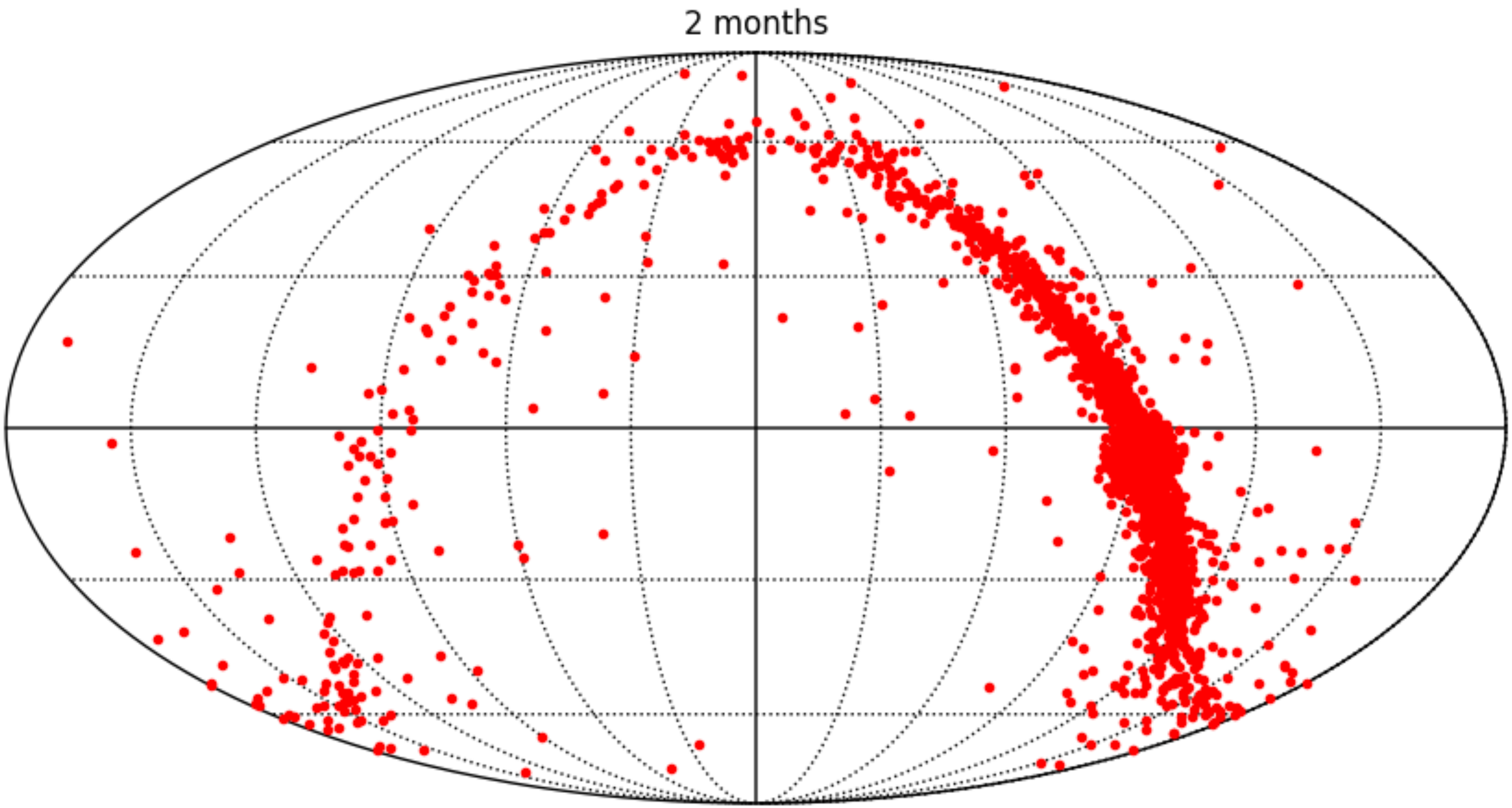


# LISA galaxy catalog over time



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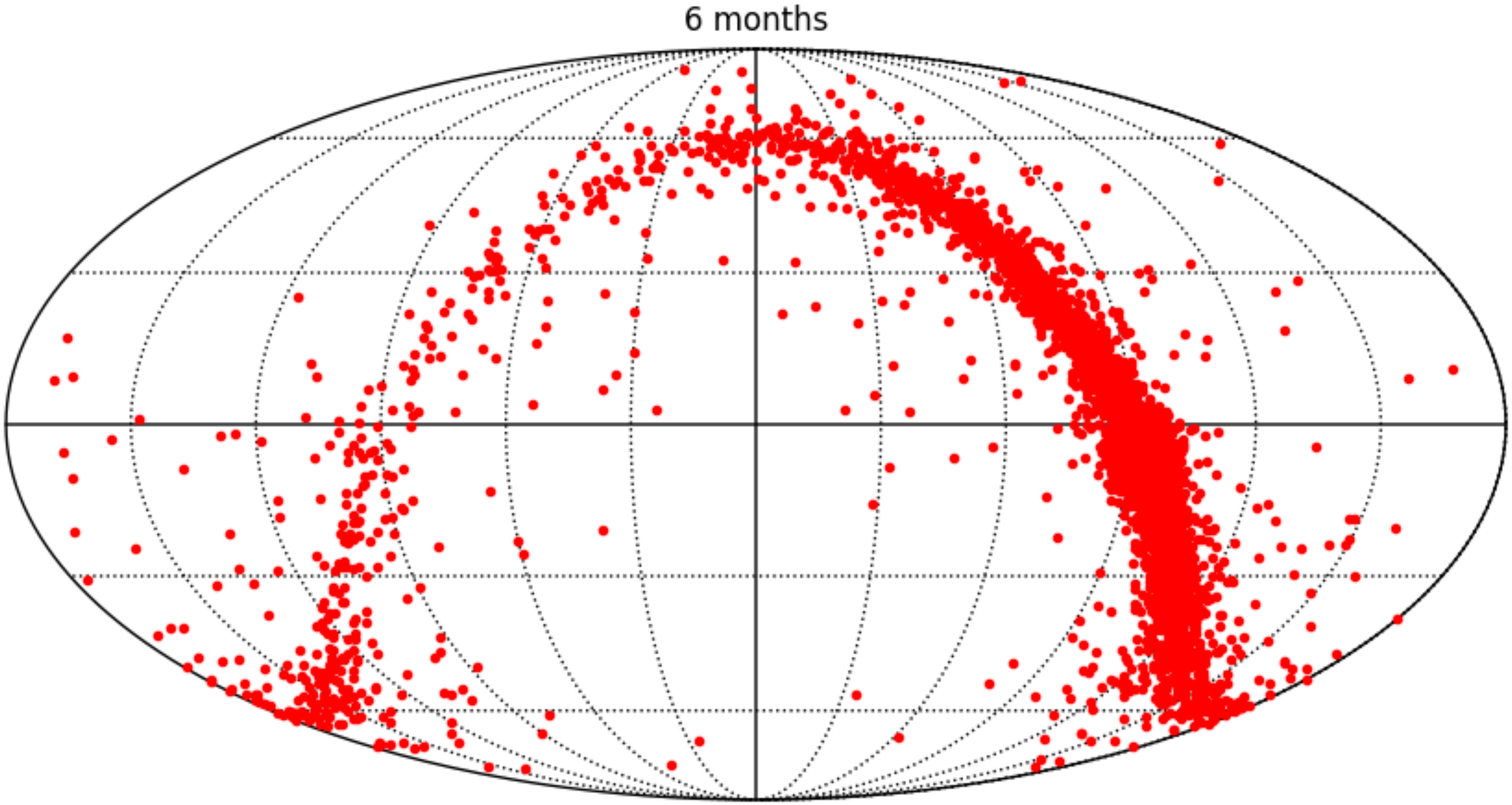

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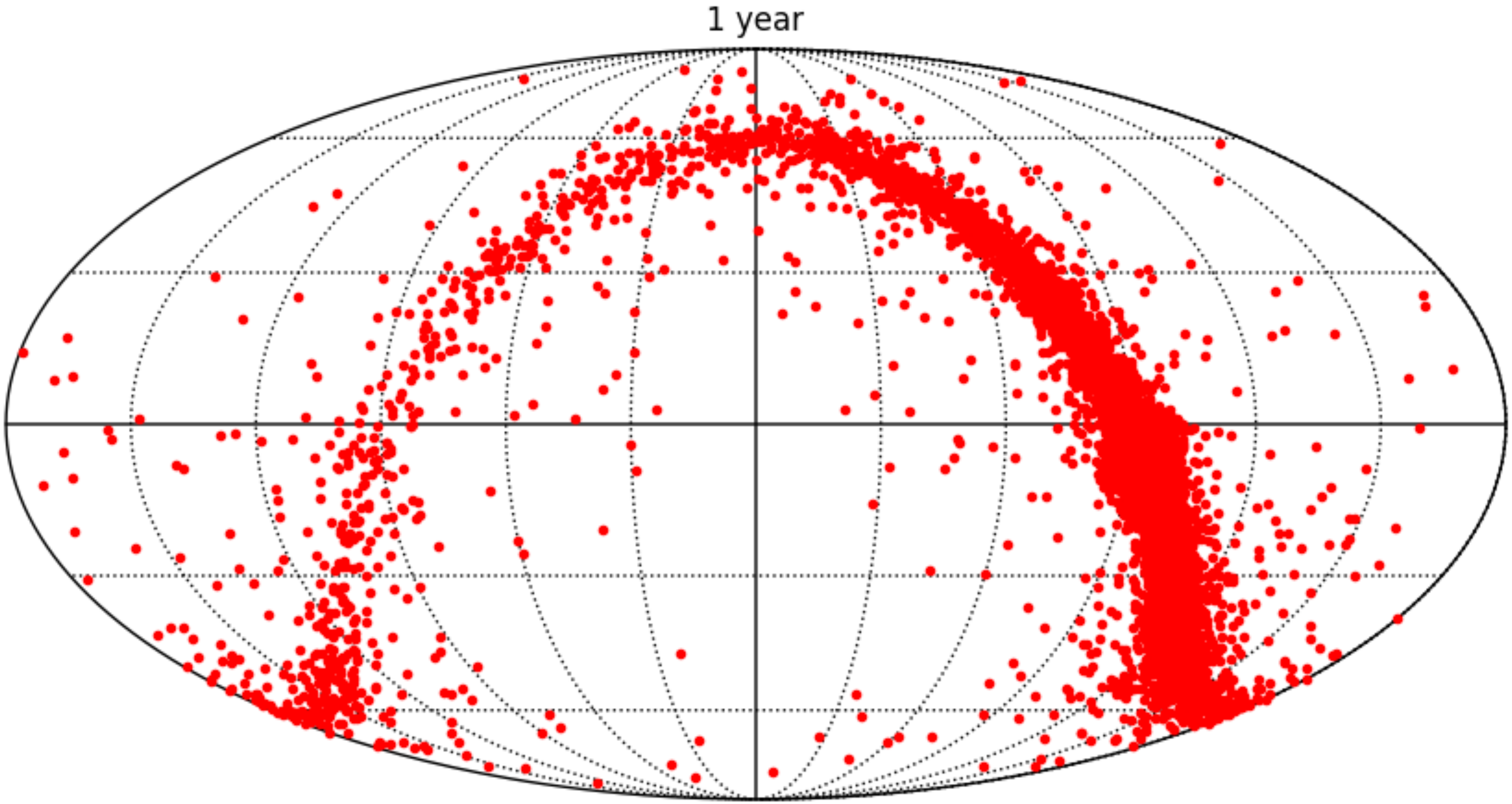

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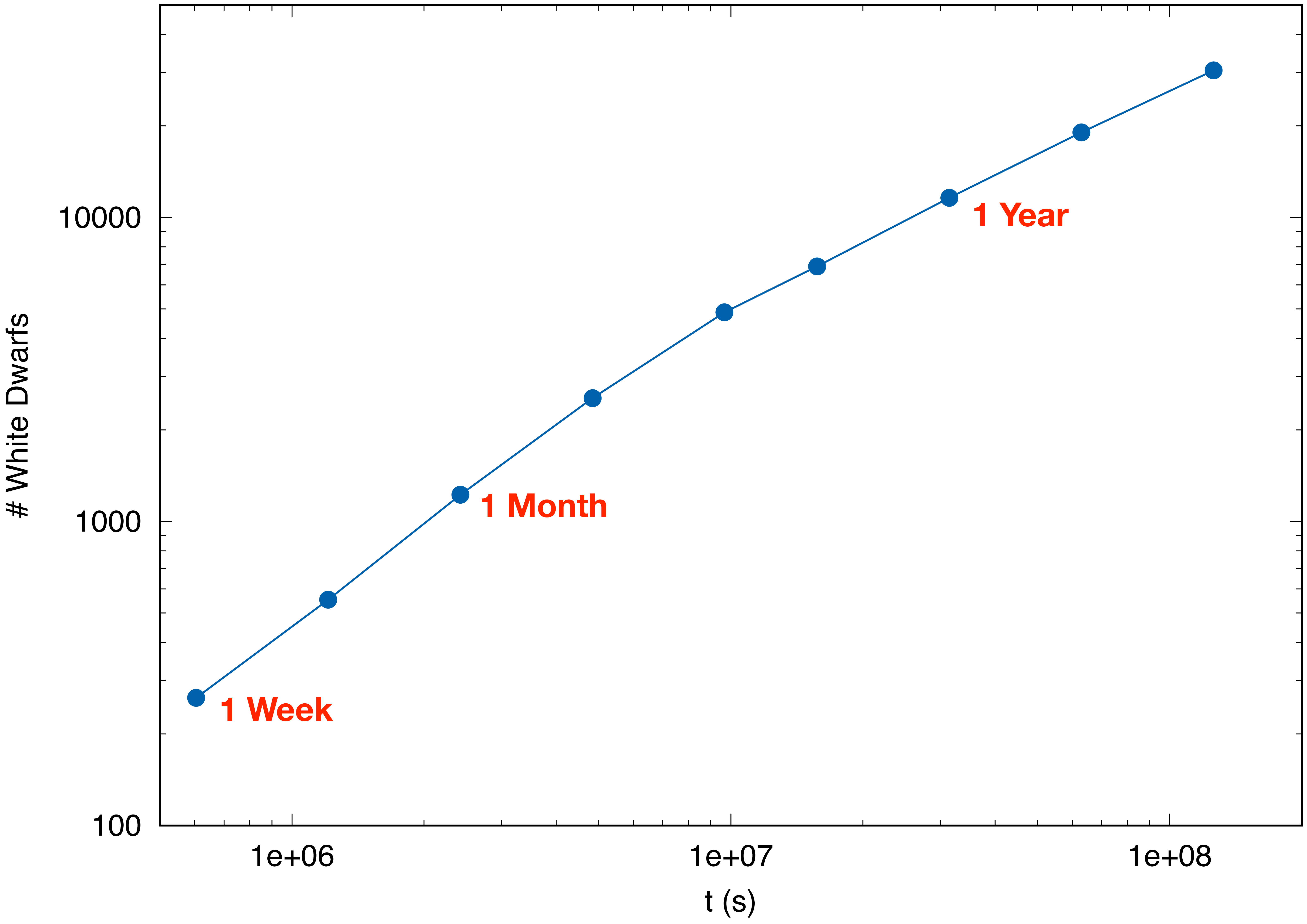


# Galactic Binaries: F-statistic Likelihood Maps

$$h(t) = \sum_{k=1}^{4} a_k(A, \psi, \iota, \psi_0)\, \hat{h}^k(t, f_0, \dot{f}_0, \theta, \phi)$$

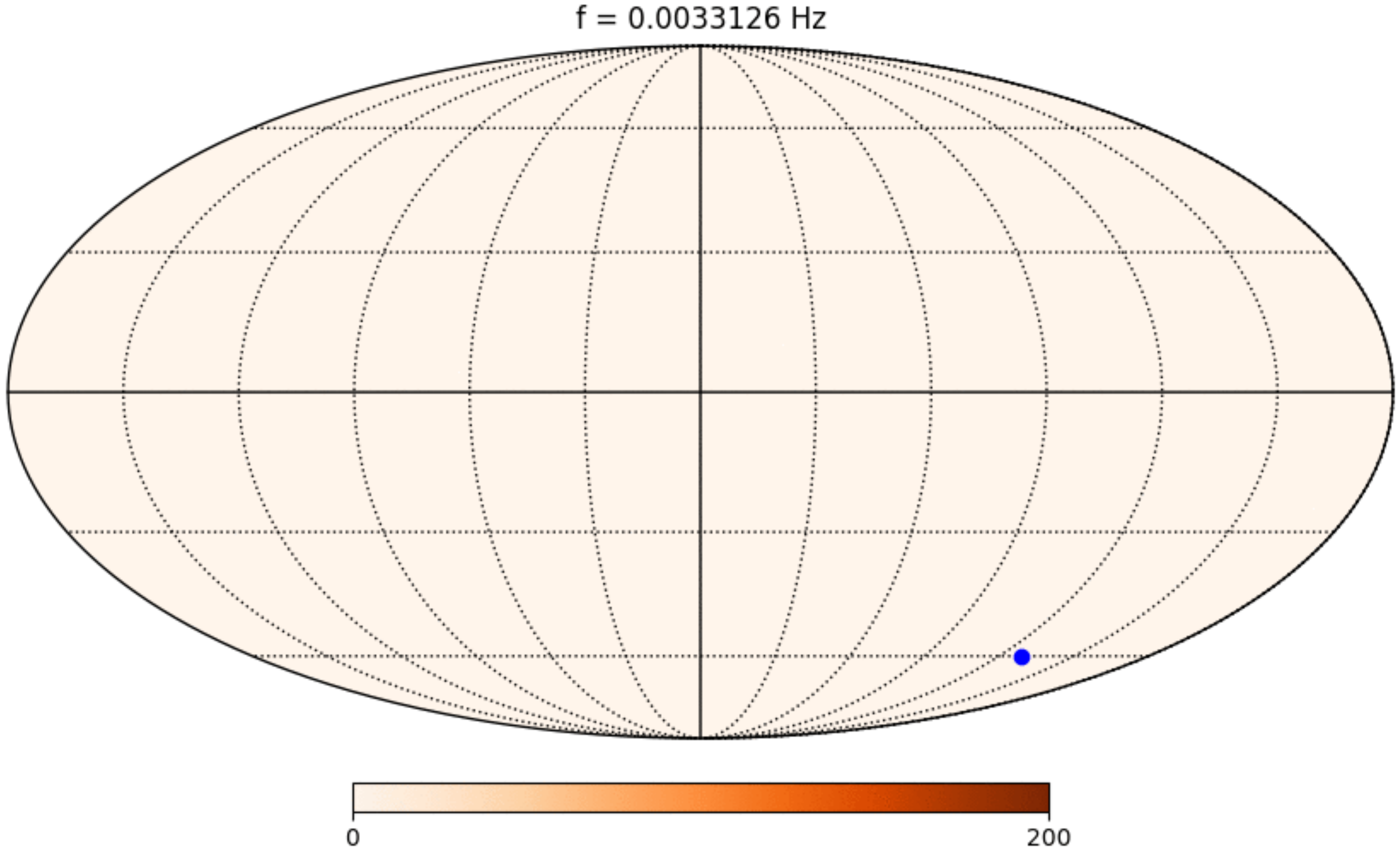
[Jaranowski, Krolak & Schutz, Phys. Rev. D58 063001 (1998)]

$$\mathcal{F} = \max_{(A,\psi,\iota,\psi_0)} \left[ (d|h) - \frac{1}{2}(h|h) \right] = \frac{1}{2}(\hat{h}^i|\hat{h}^j)^{-1}(d|\hat{h}^i)(d|\hat{h}^j)$$

Use maps of $\mathcal{F}(f_0, \dot{f}_0, \theta, \phi)$ as a global proposal

Ignores massive black hole mergers, EMRIs etc. Some overlap with LIGO BHs

# F-statistic map. Single Galactic Binary



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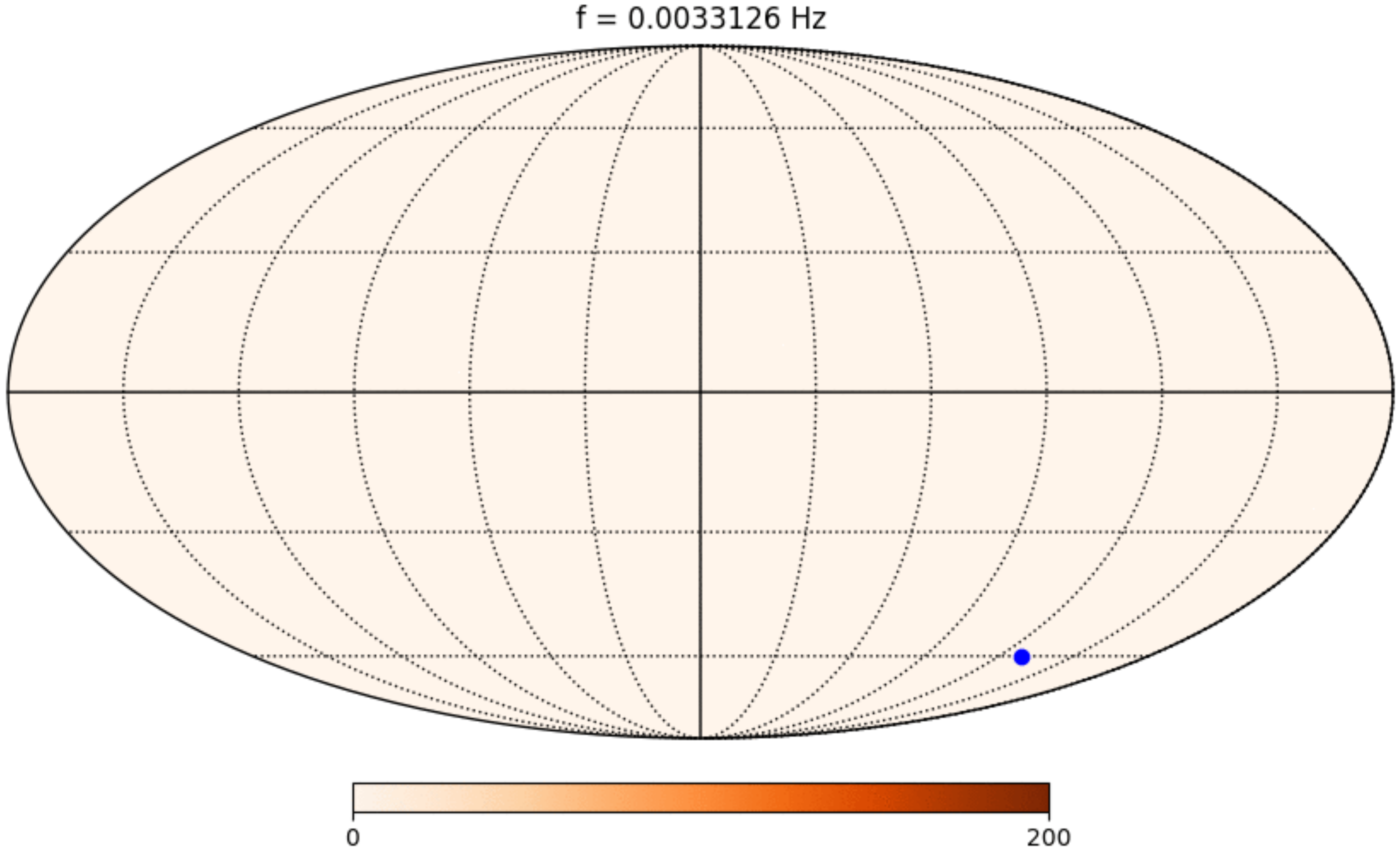
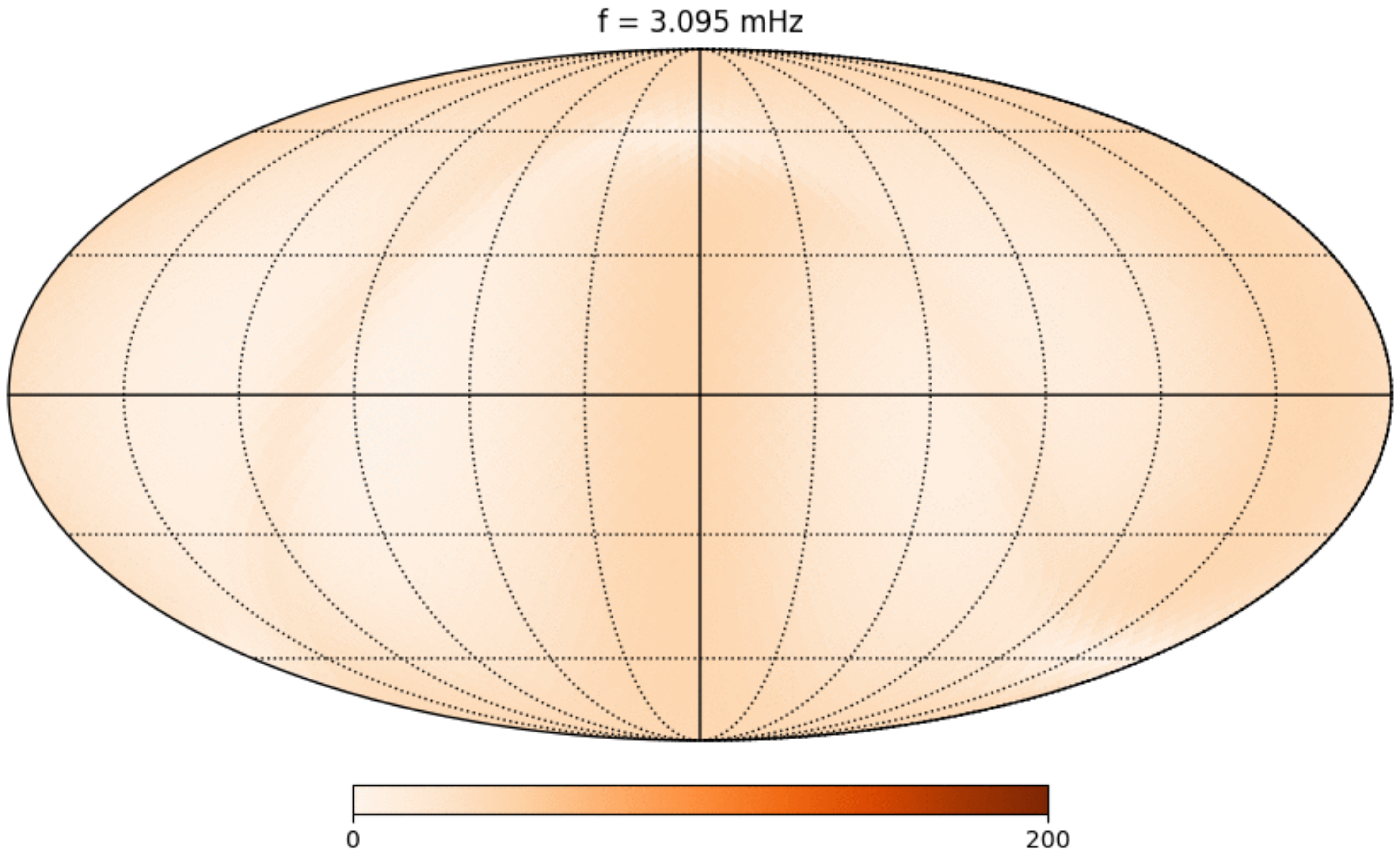
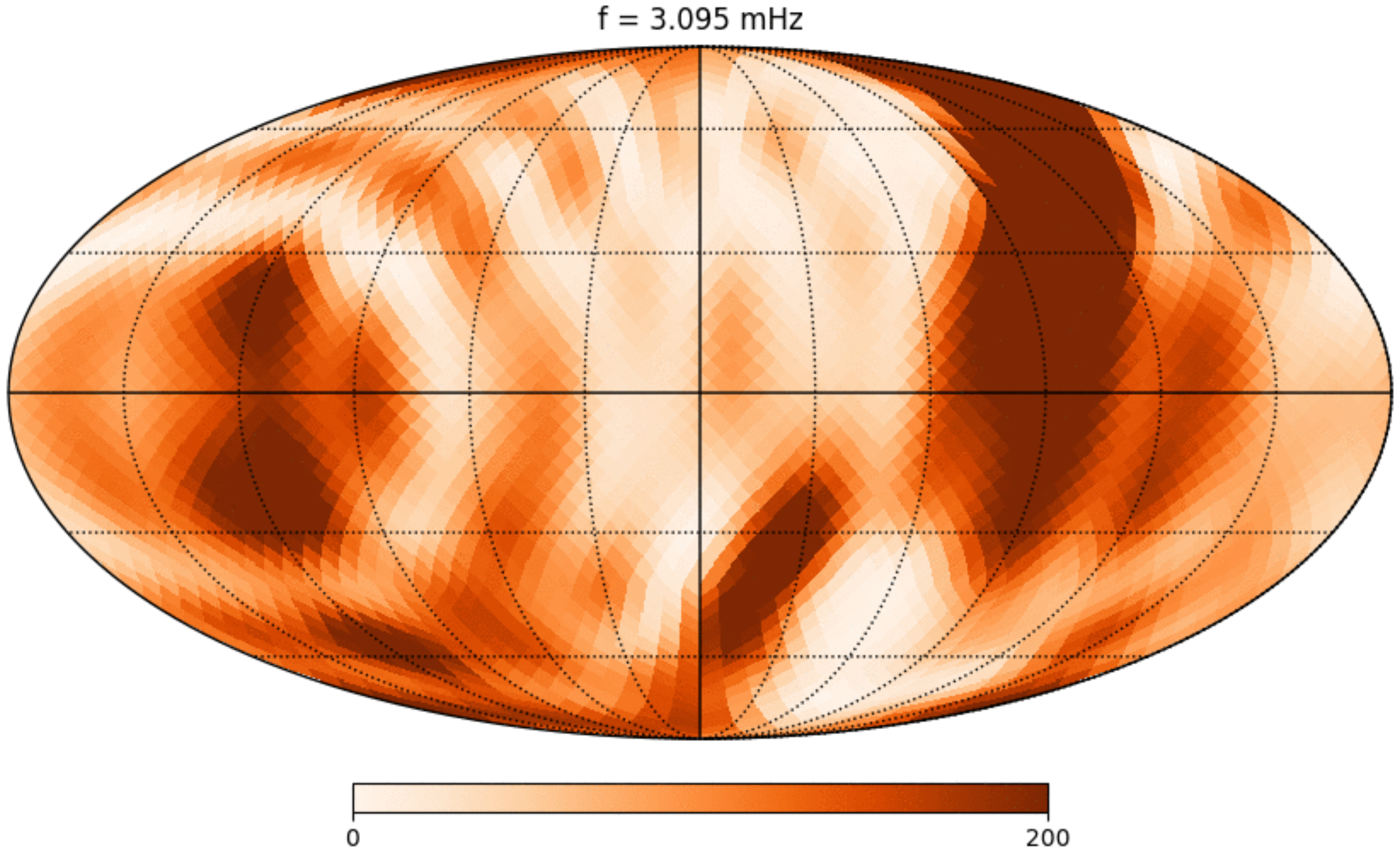


# F-statistic maps: Full galaxy near 3 mHz




1 - month




1 - year

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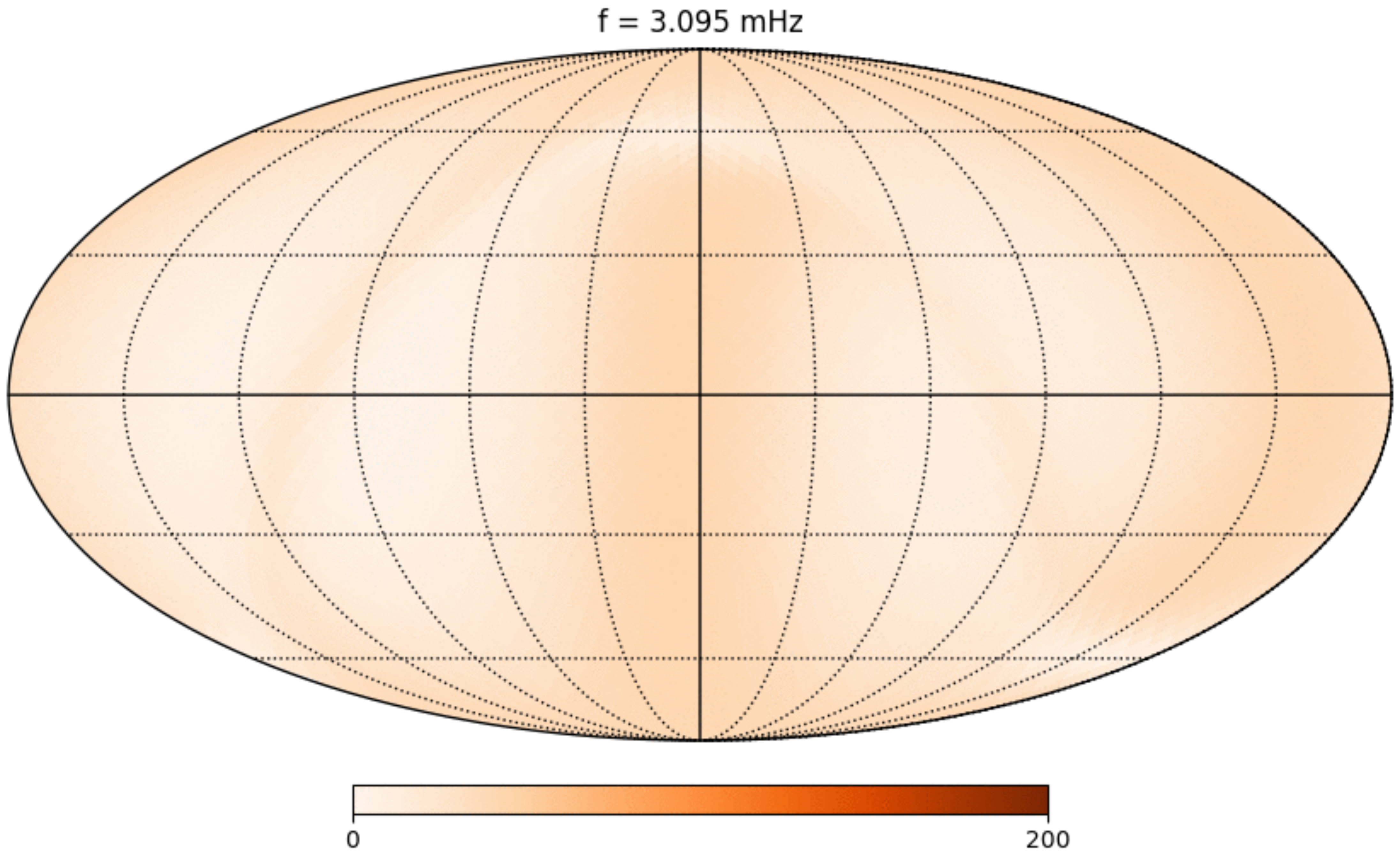


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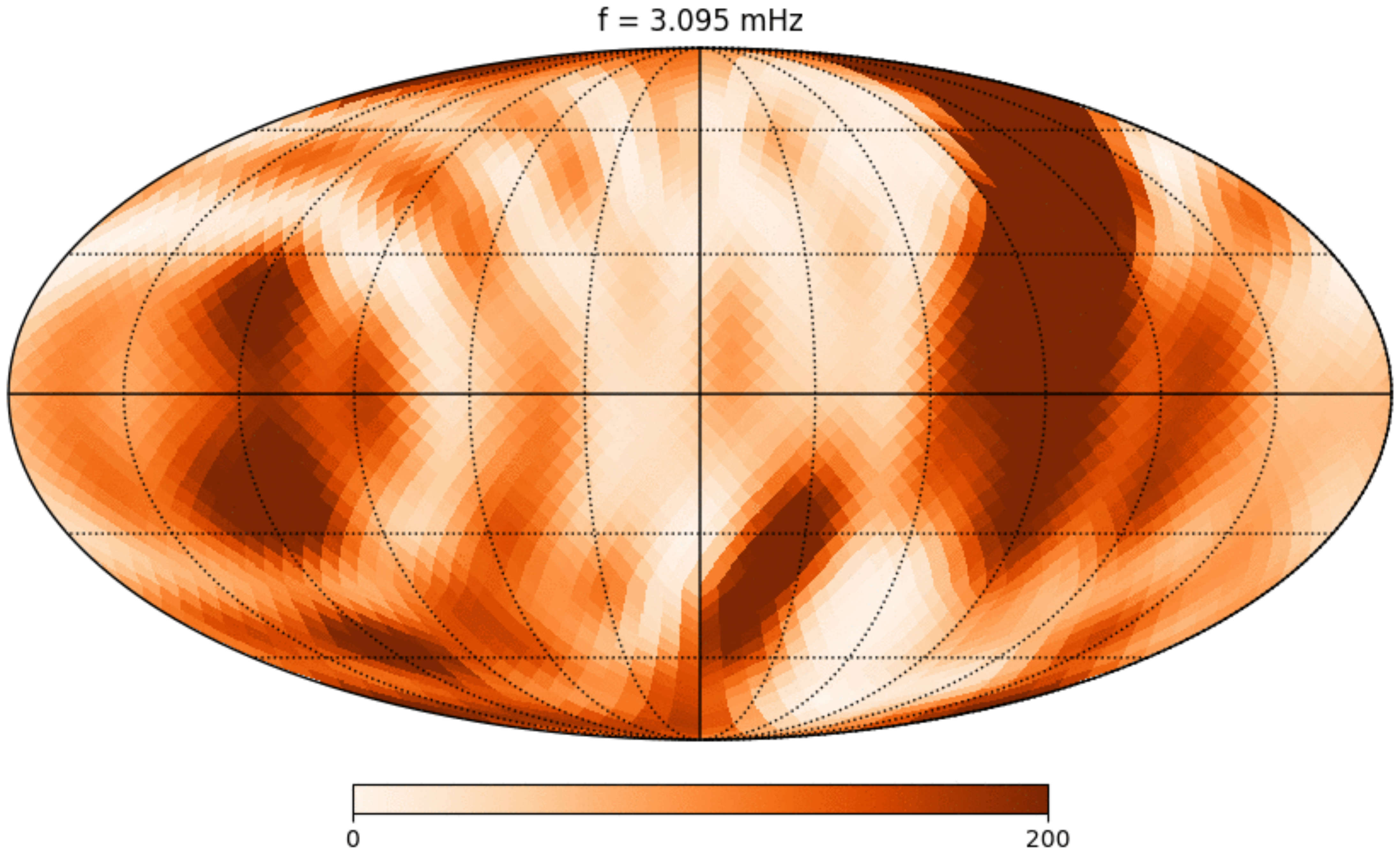
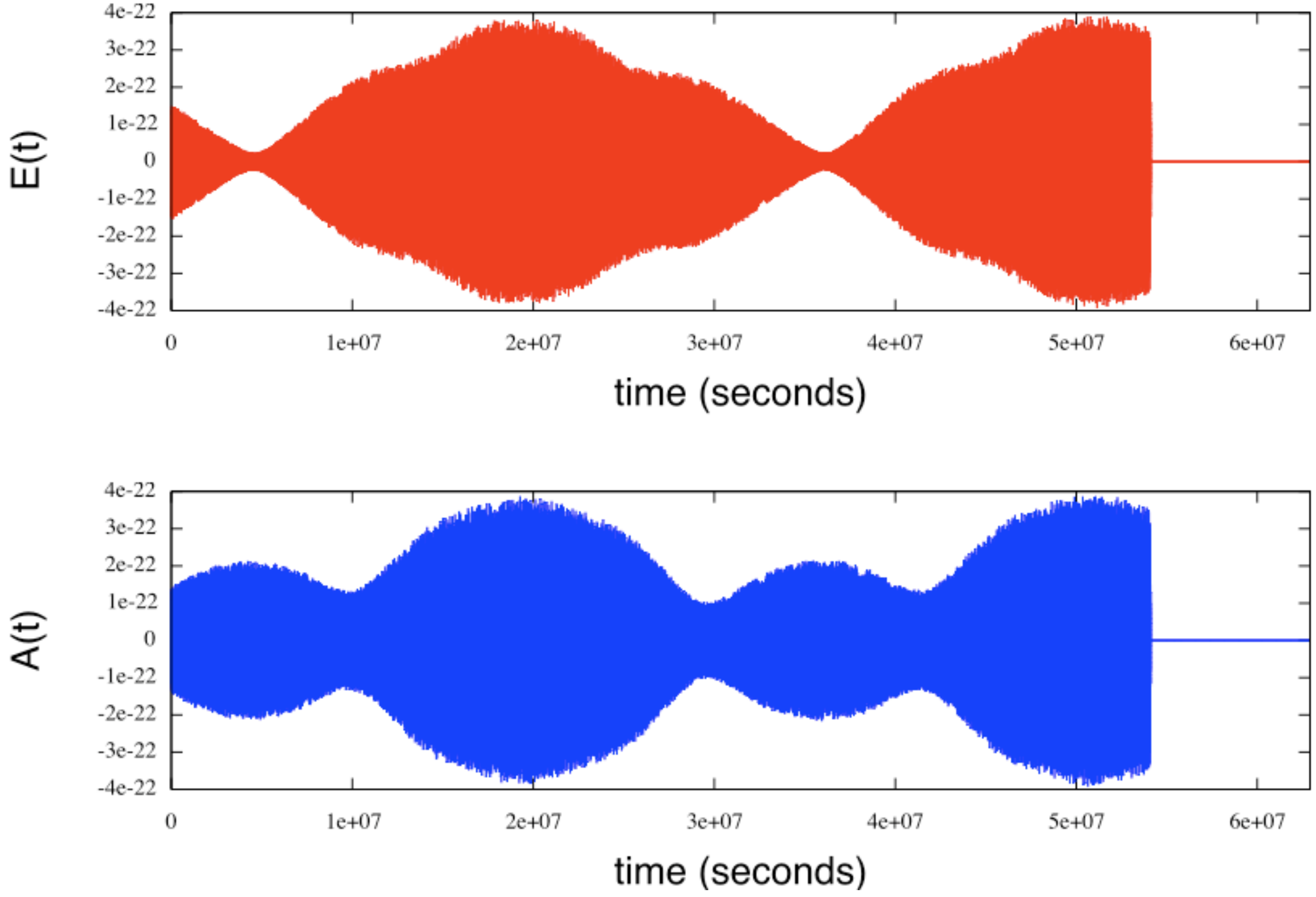



1 - year

# Extreme Mass Ratio Inspirals



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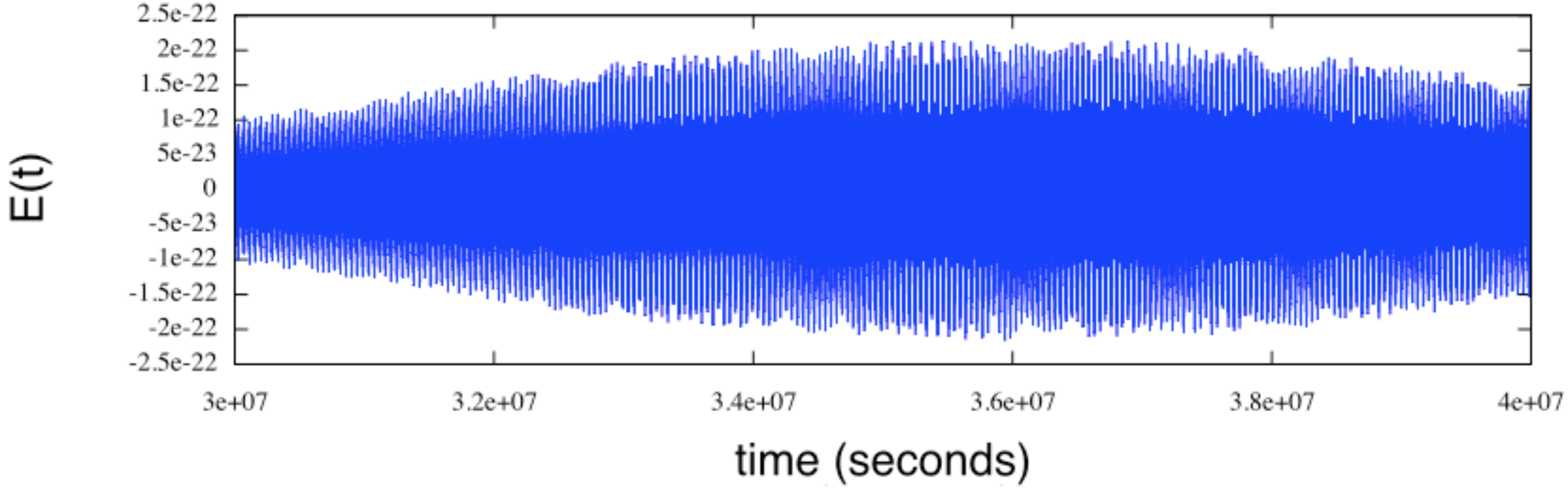


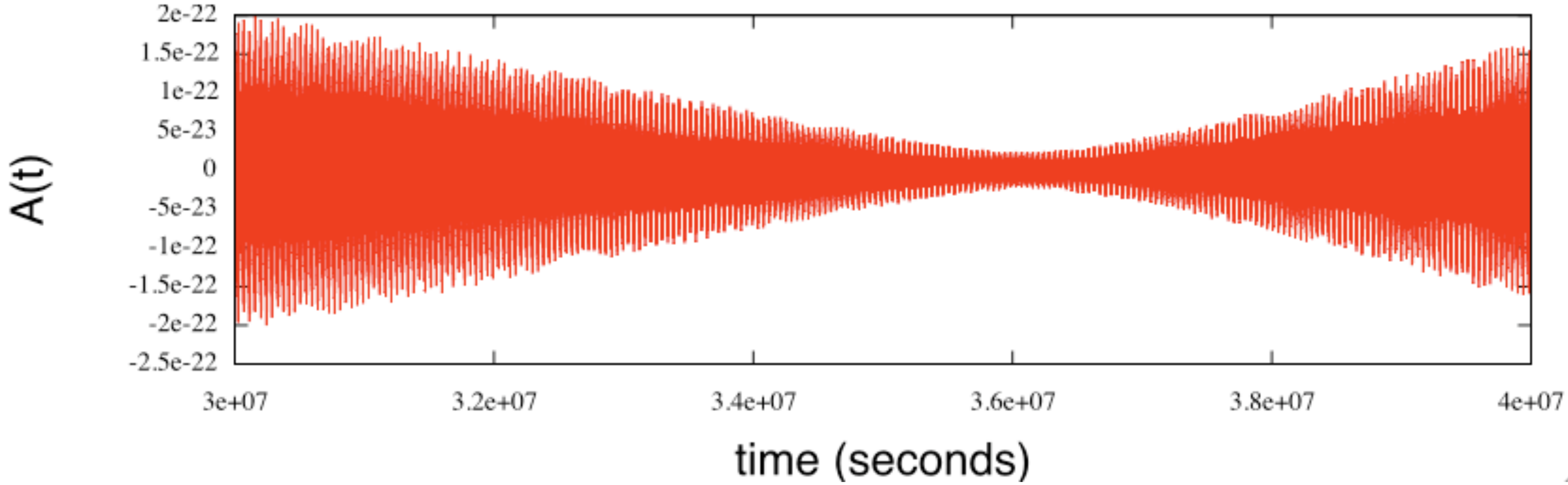

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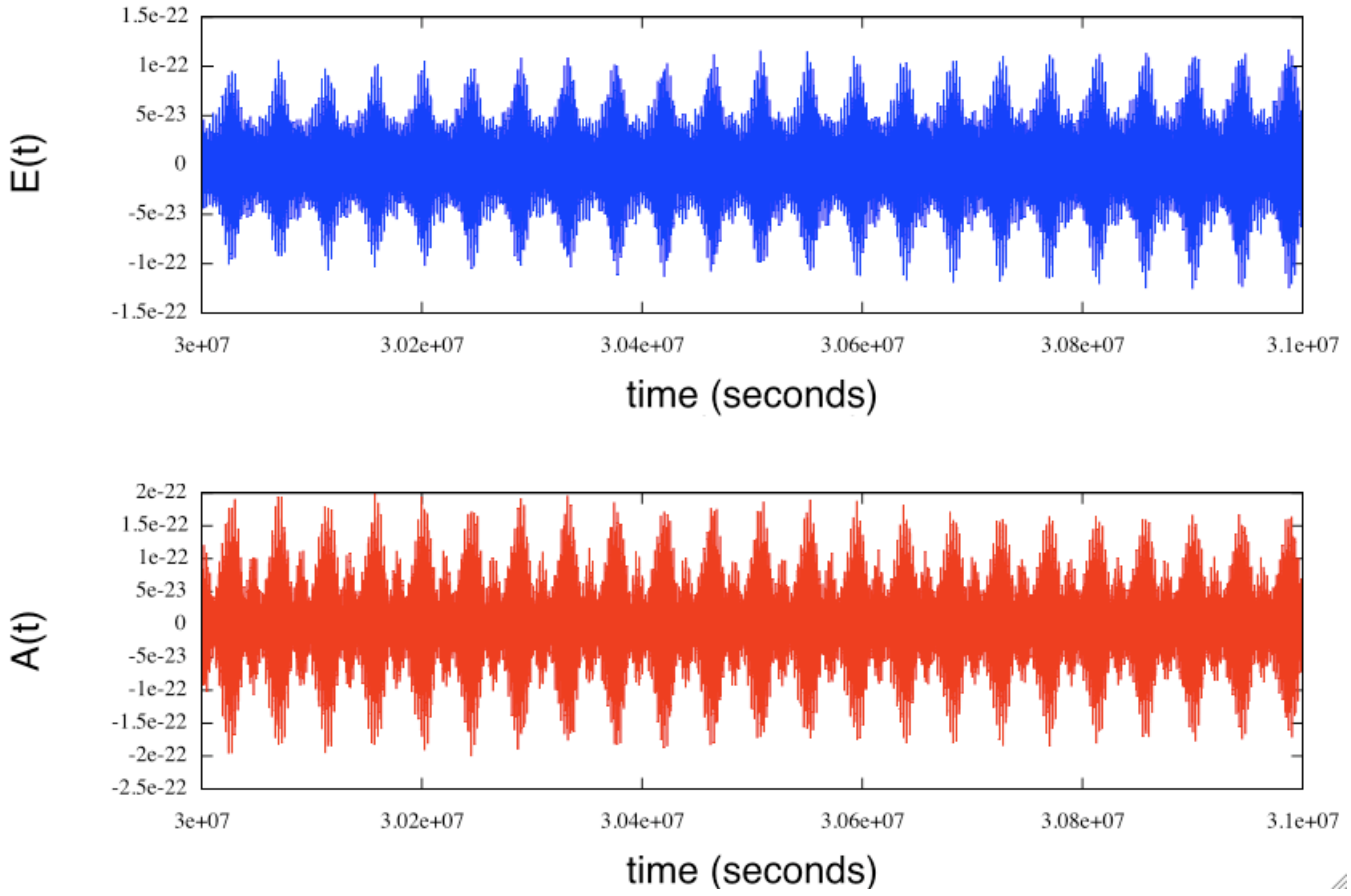

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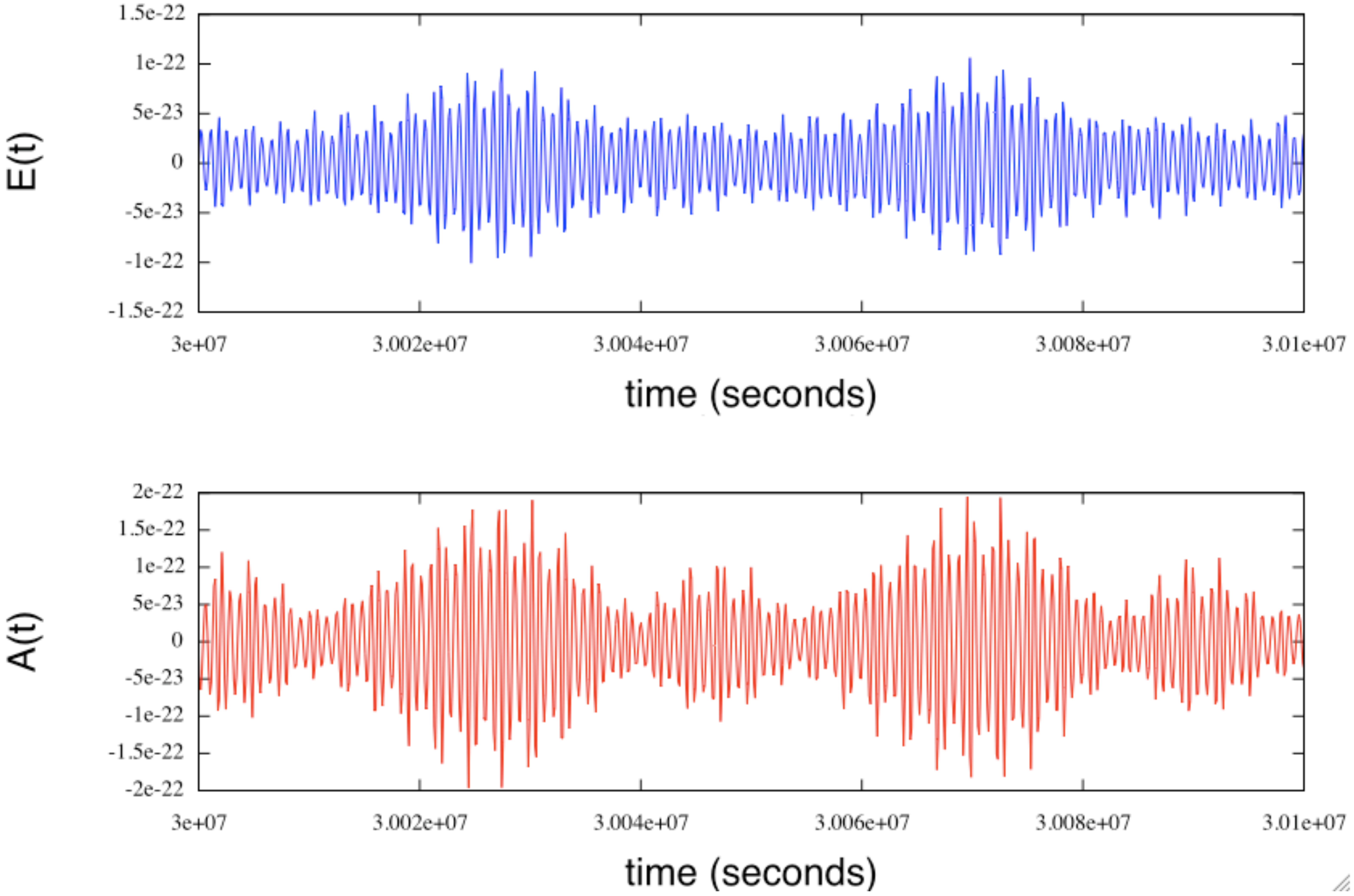

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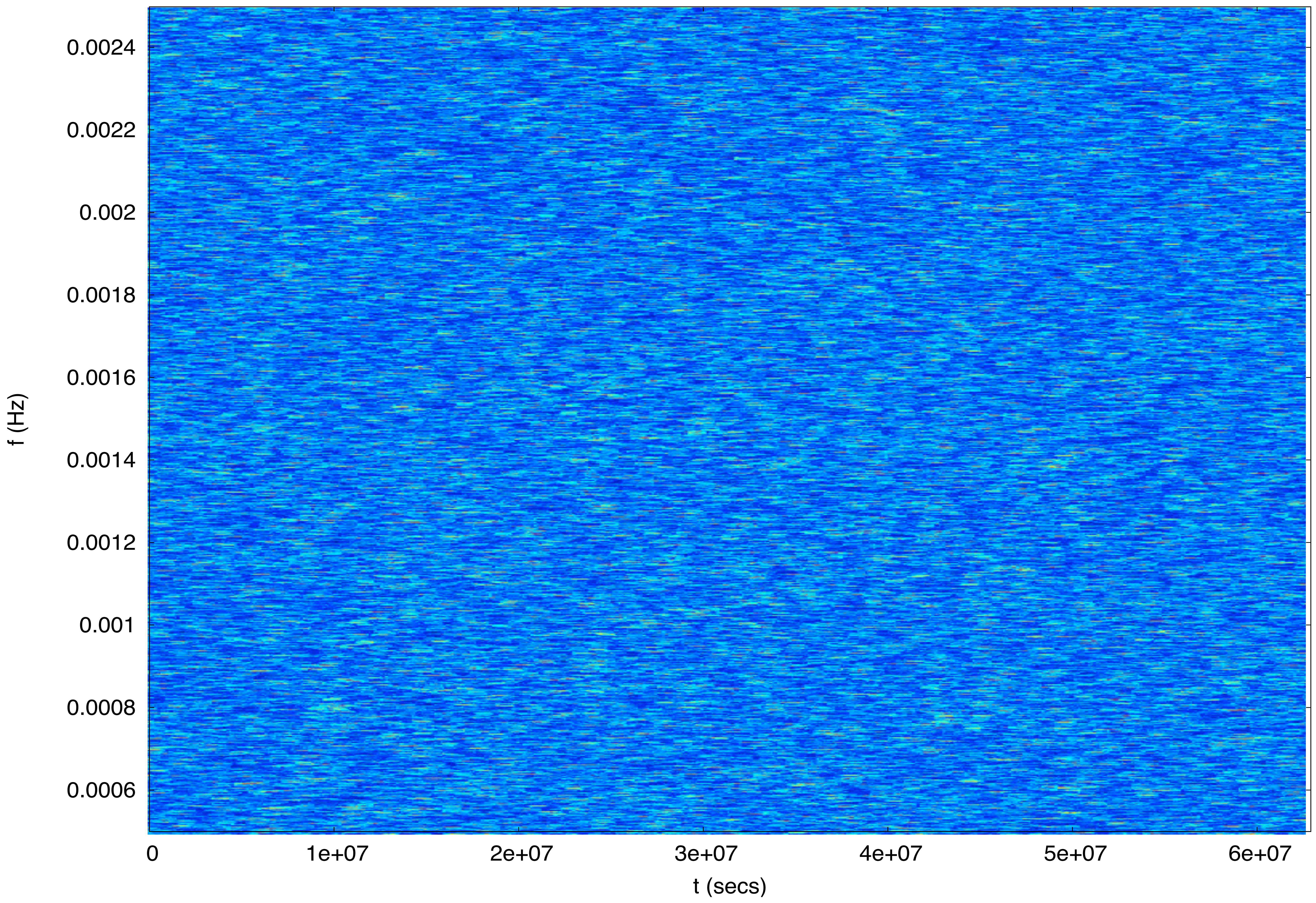

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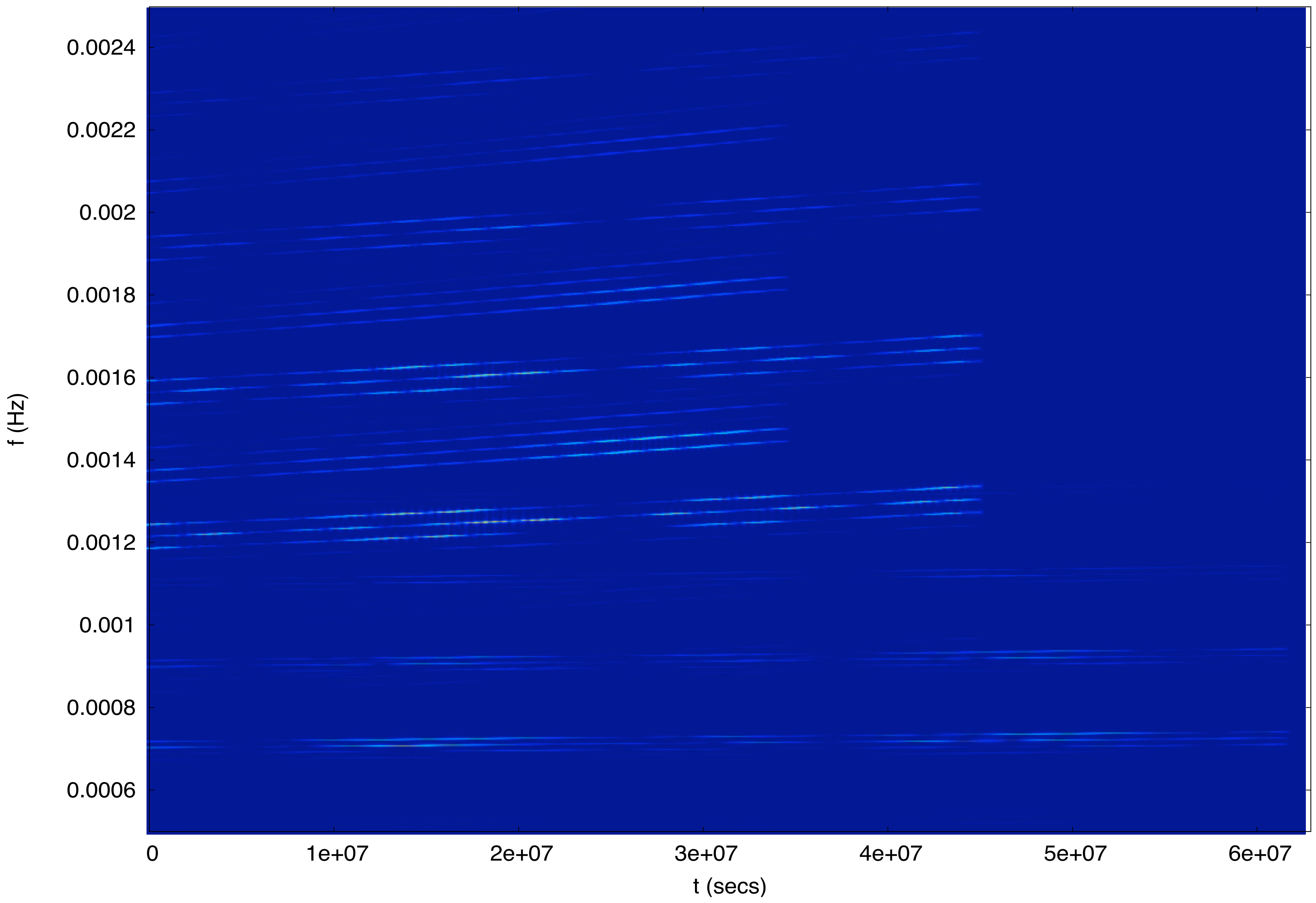

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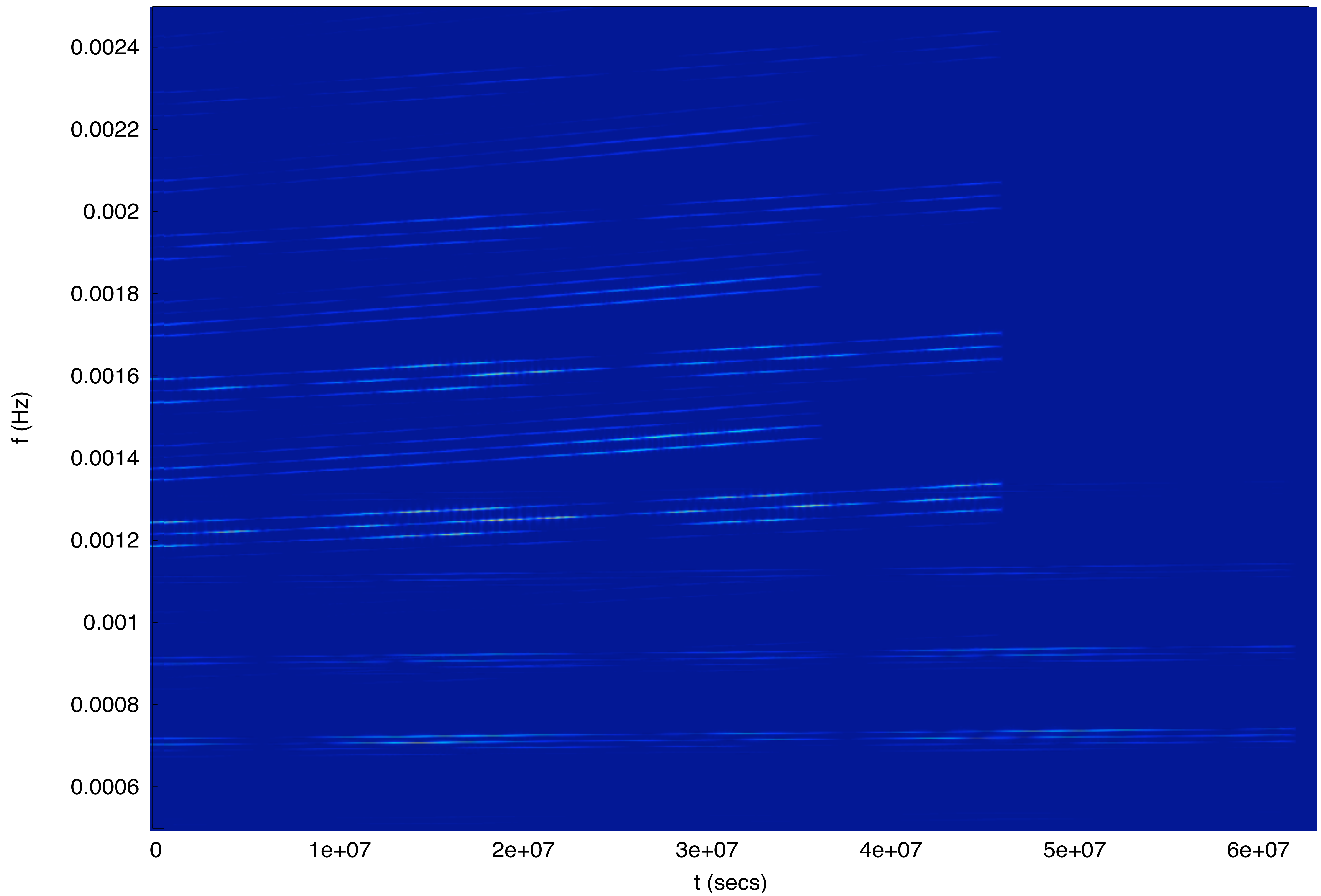
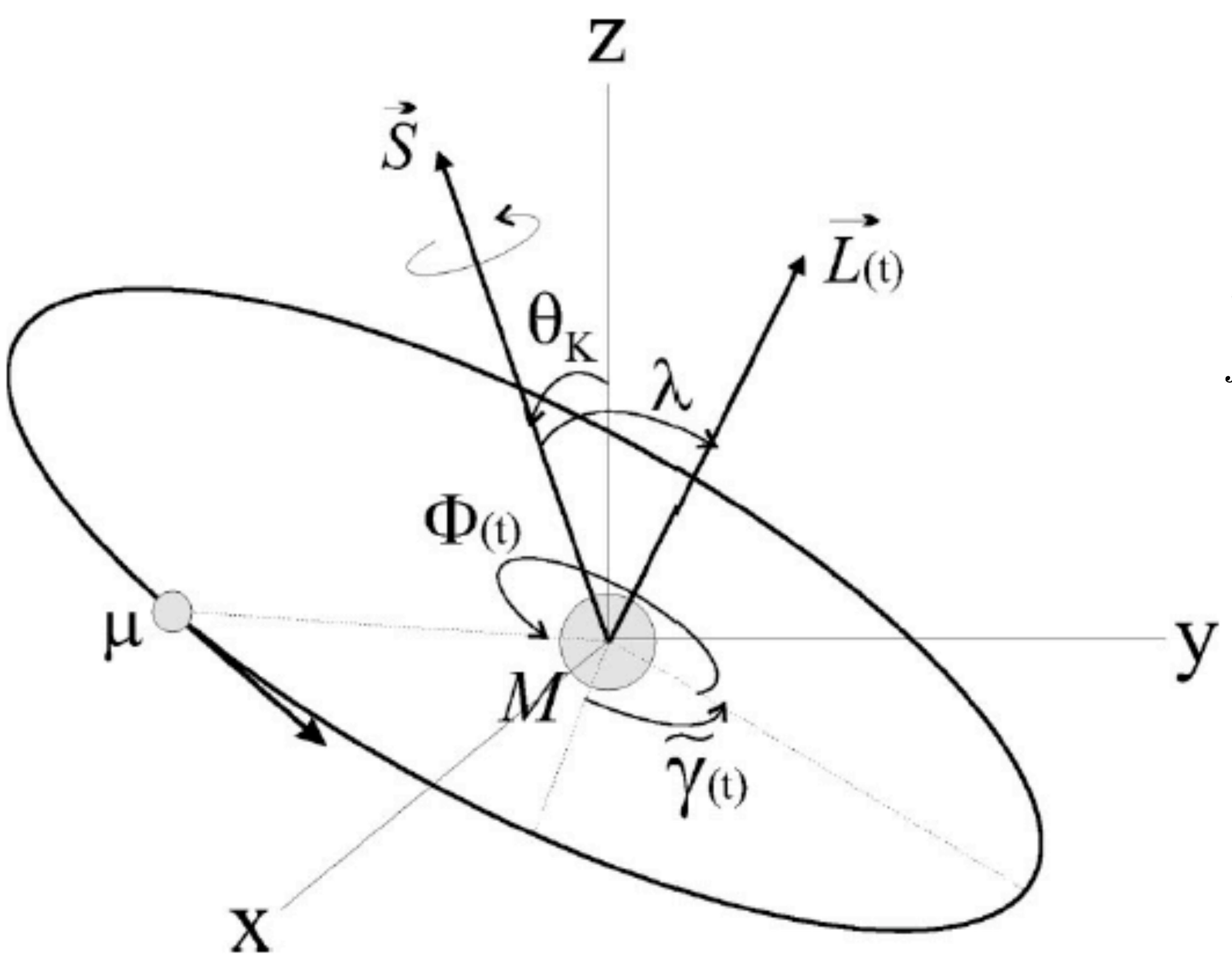


# EMRI Harmonics

$$f_{nmk} = n\nu + mf_{\tilde{\gamma}} + kf_{\alpha}$$

BC Kludge $\quad m = 2 \quad k \in [-2, 2]$




$\nu$ = azimuthal orbital frequency

$f_{\tilde{\gamma}}$ = periholon precession frequency

$f_{\alpha}$ = orbital plane precession frequency

[Barack & Cutler, Phys.Rev. D**69** 082005 (2004)]

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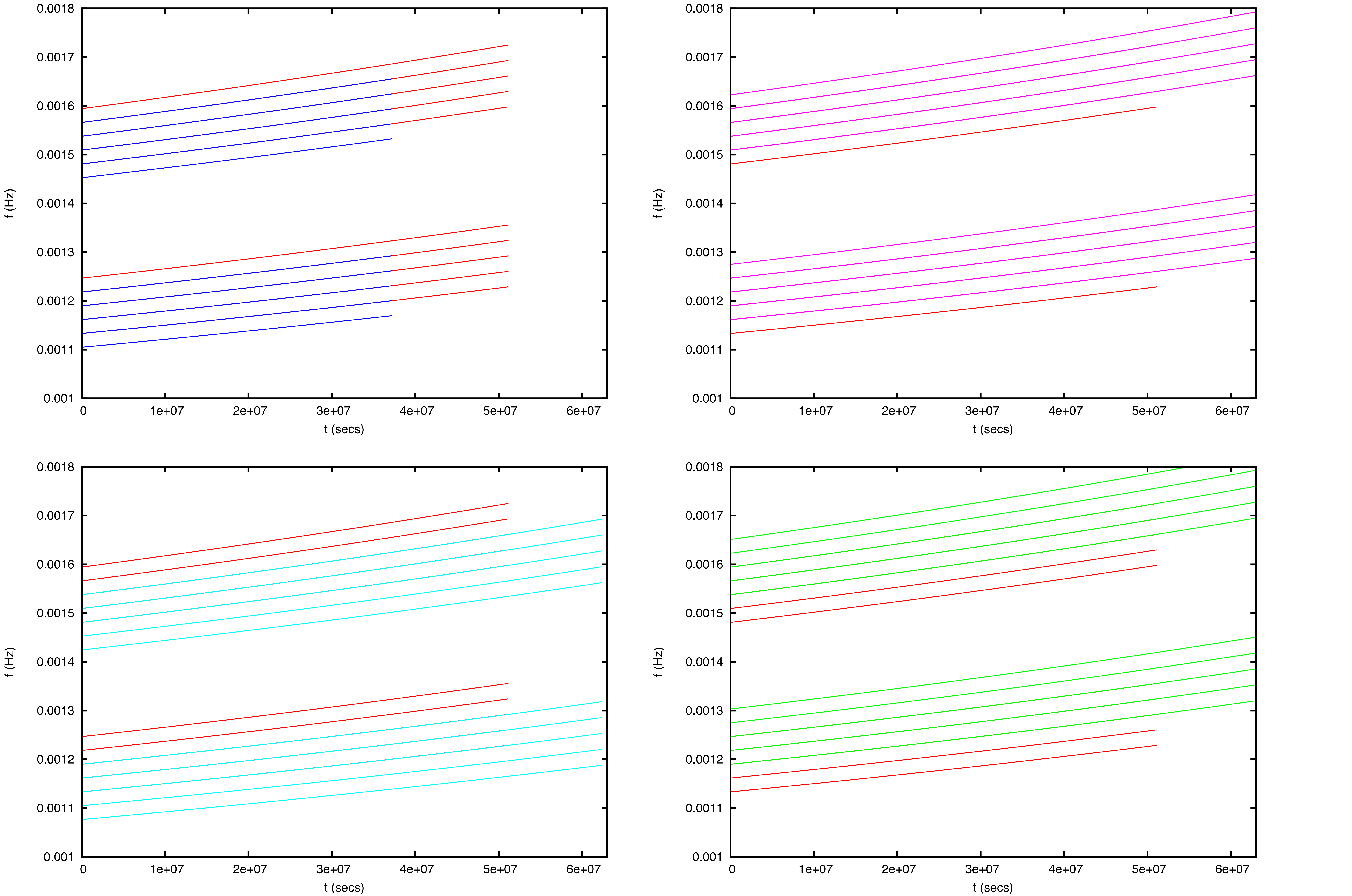
$$\begin{aligned} f_{\tilde{\gamma}} &= 3\nu(2\pi\nu M)^{2/3}(1-e^2)^{-1}\left[1 + \frac{1}{4}(2\pi\nu M)^{2/3}(1-e^2)^{-1}(26-15e^2)\right] \\ &\quad -6\nu\cos\lambda(S/M^2)(2\pi M\nu)(1-e^2)^{-3/2} \end{aligned}$$

$$f_{\alpha} = 2\nu(S/M^2)(2\pi M\nu)(1-e^2)^{-3/2}$$

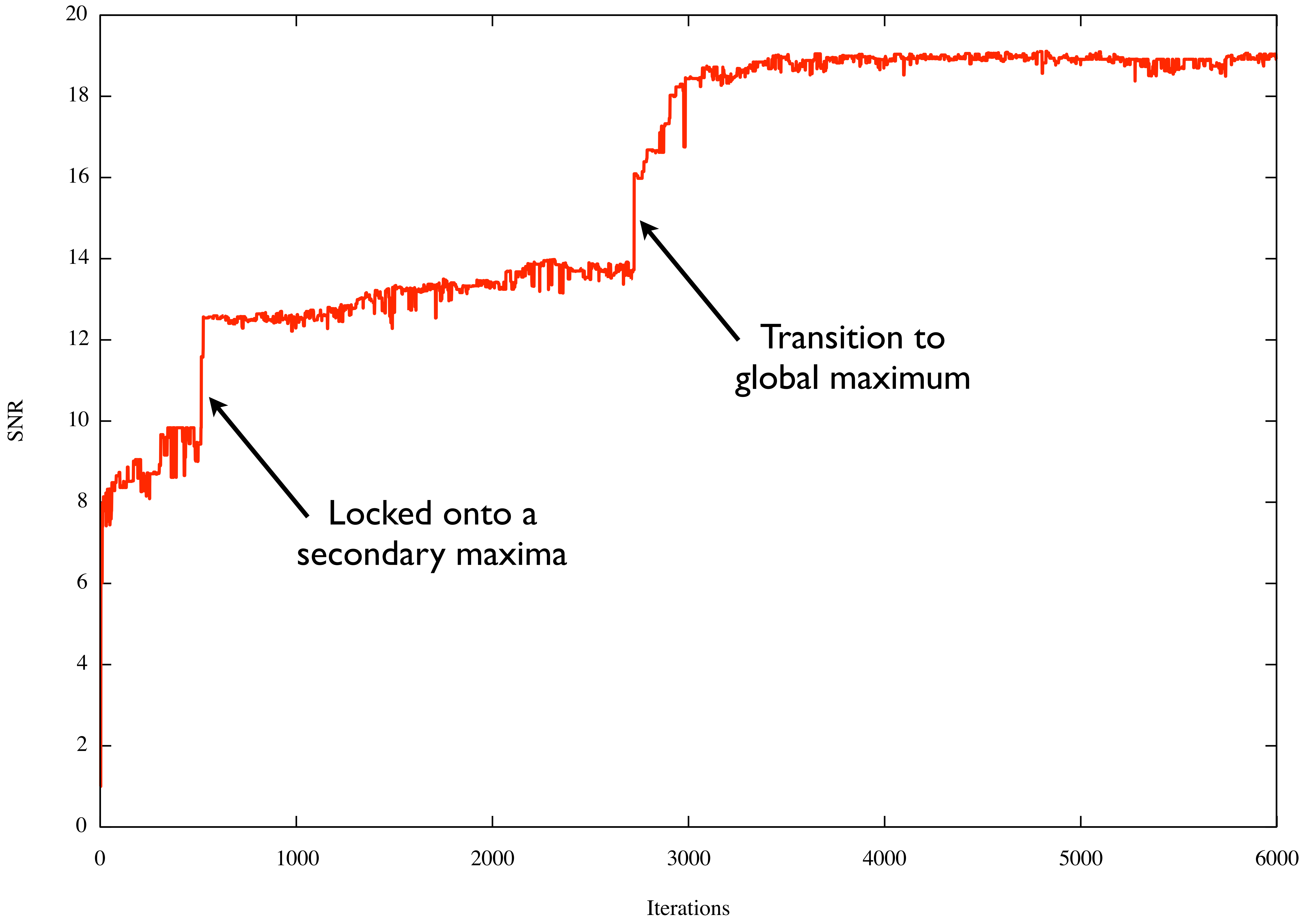
harmonic mismatch

$$f_{n2k} = f_{n'2k'}$$
$$\dot{f}_{n2k} = \dot{f}_{n'2k'}$$

# Sideband Secondaries

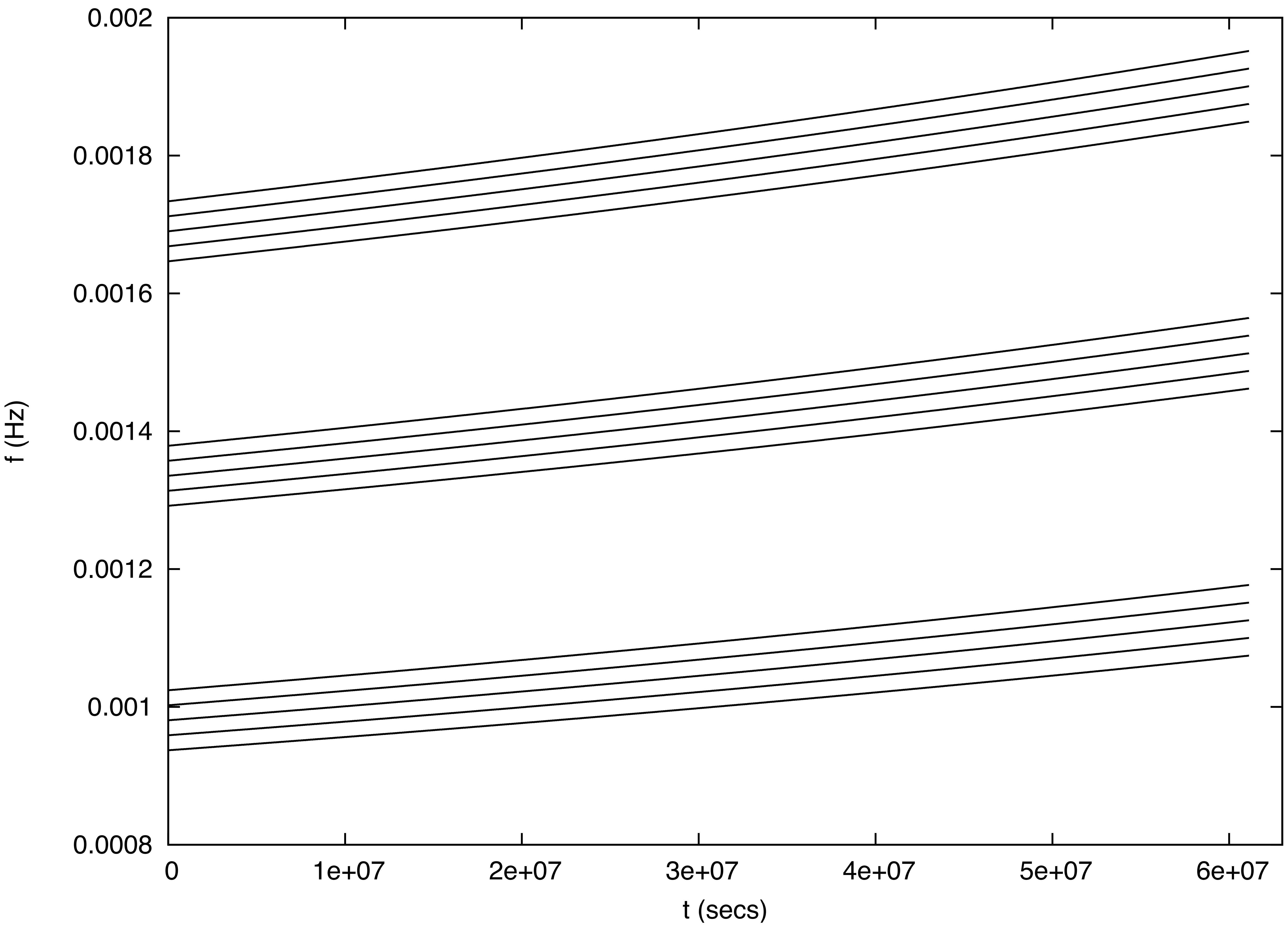


# Detecting Extreme Mass Ratio Inspirals



# Analytic Maximization

$$\{t_p,\ \phi_0,\ D_L\}$$



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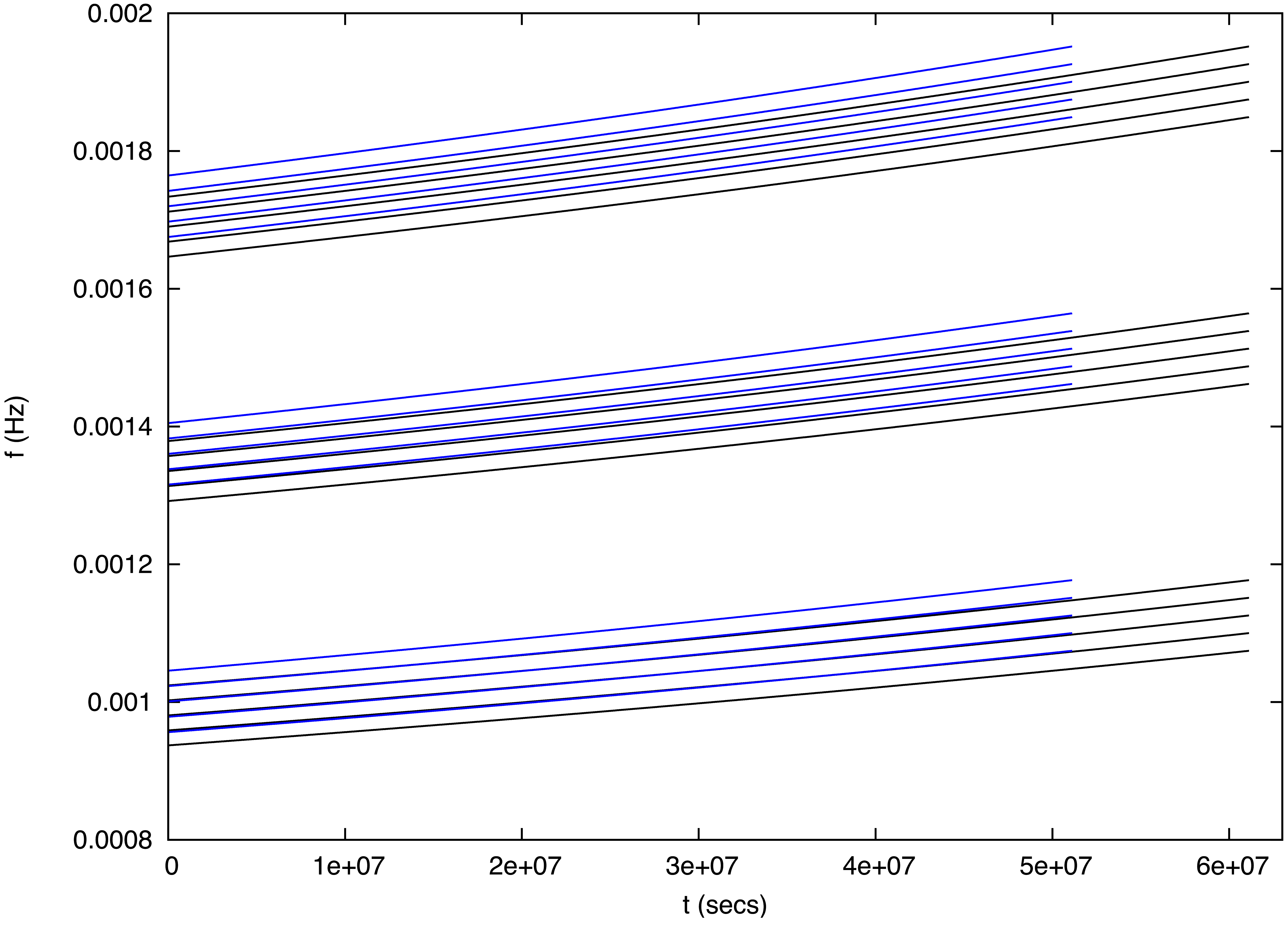

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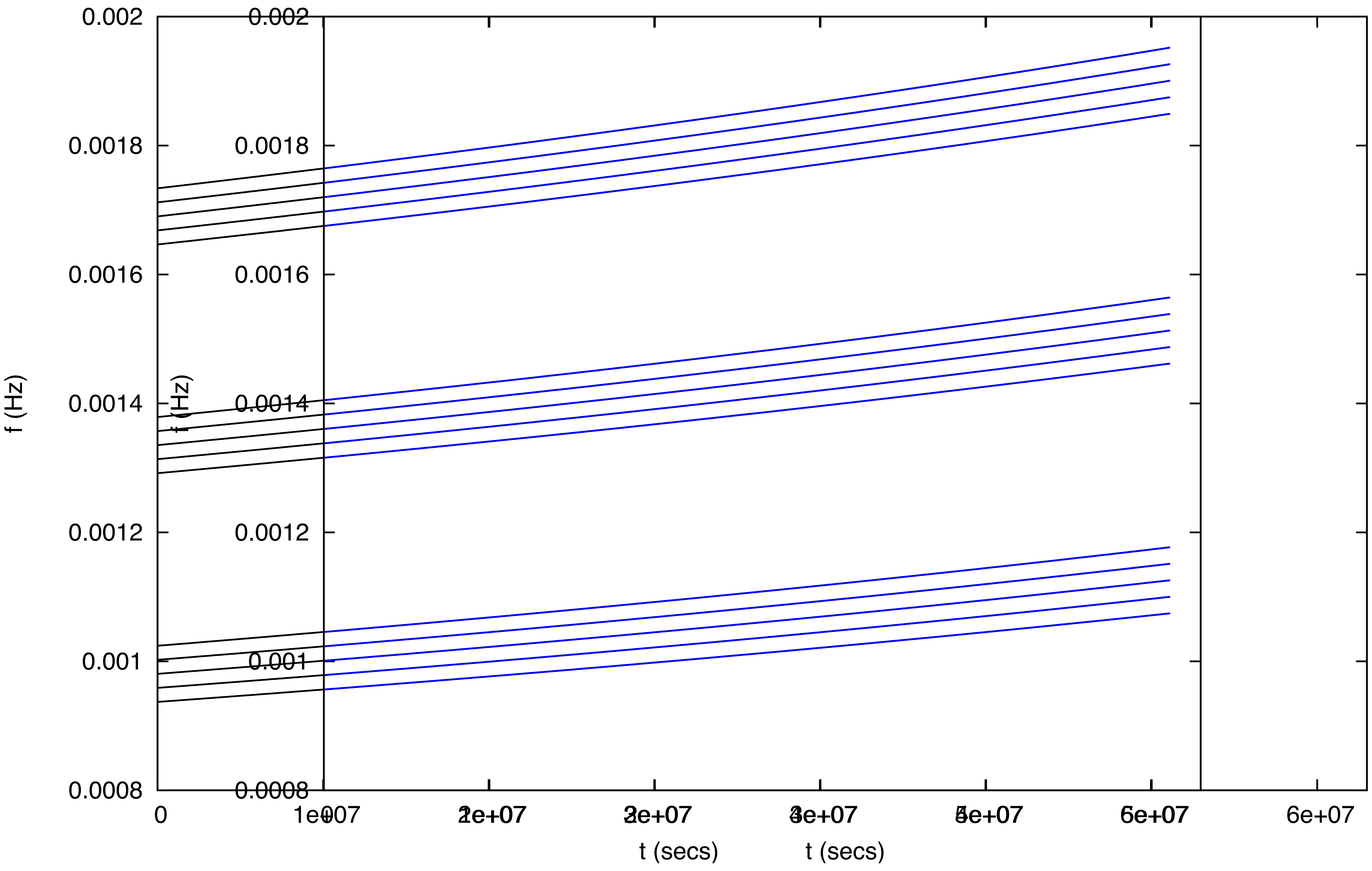
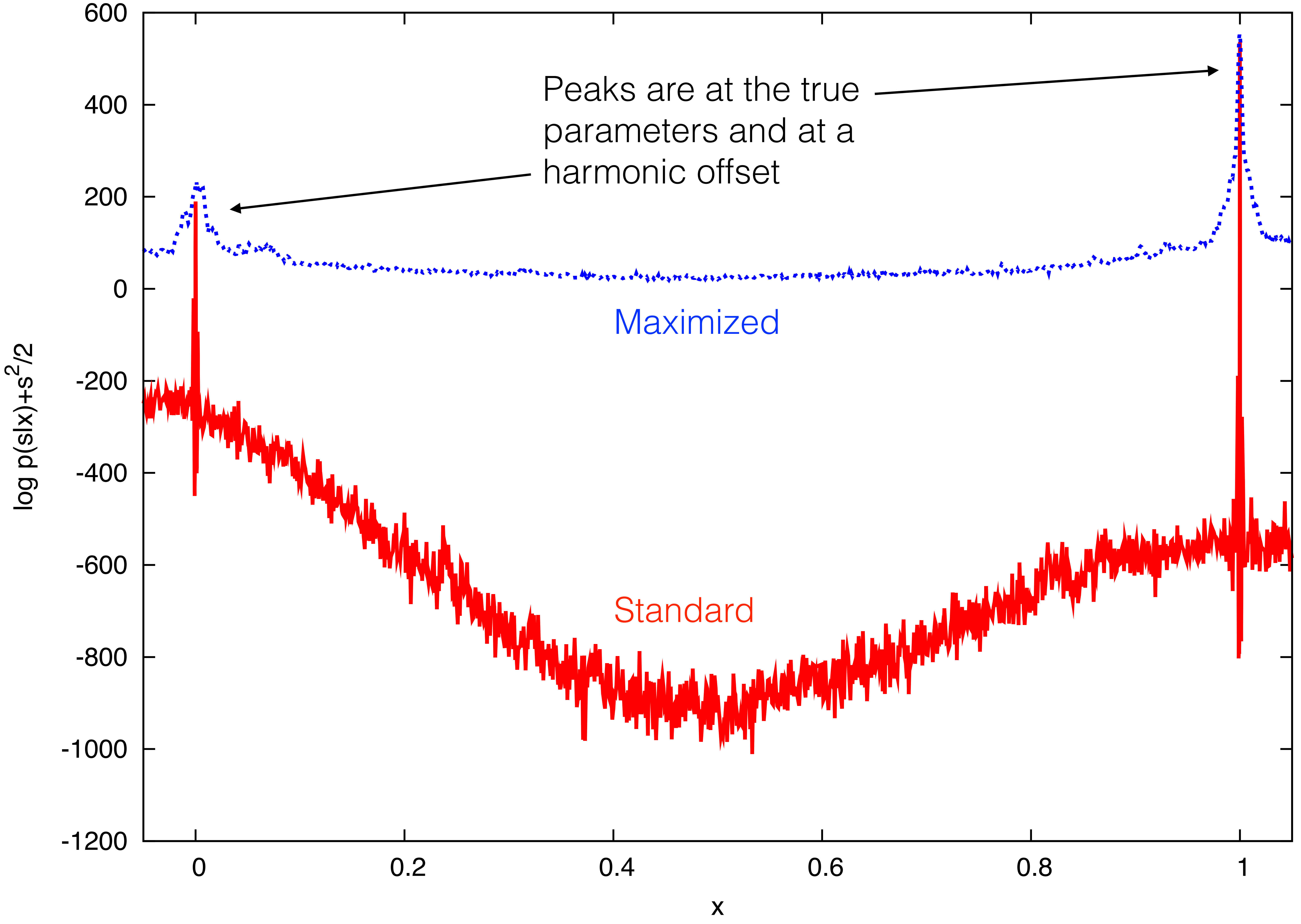


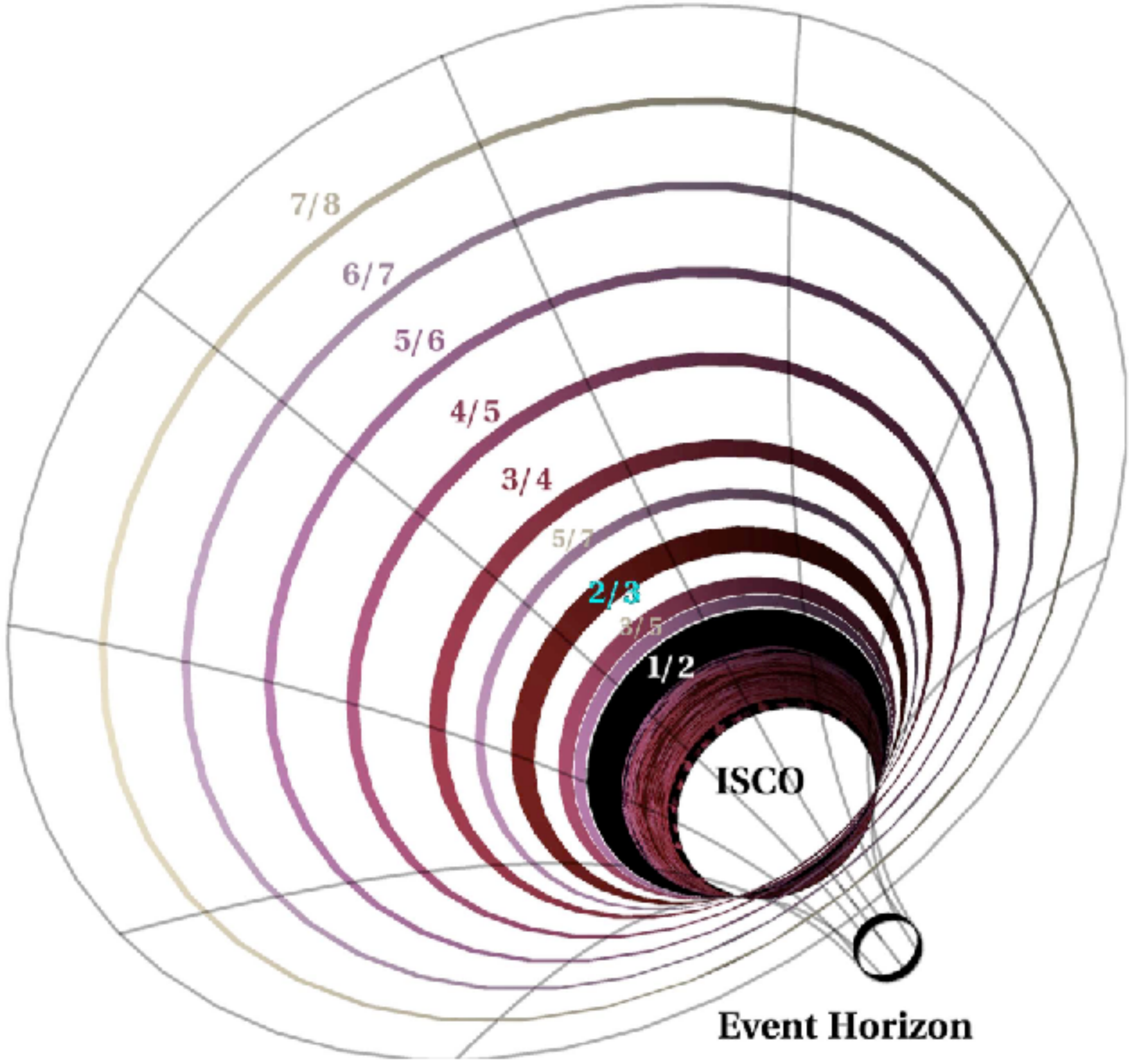
# Analytic Maximization of the Likelihood



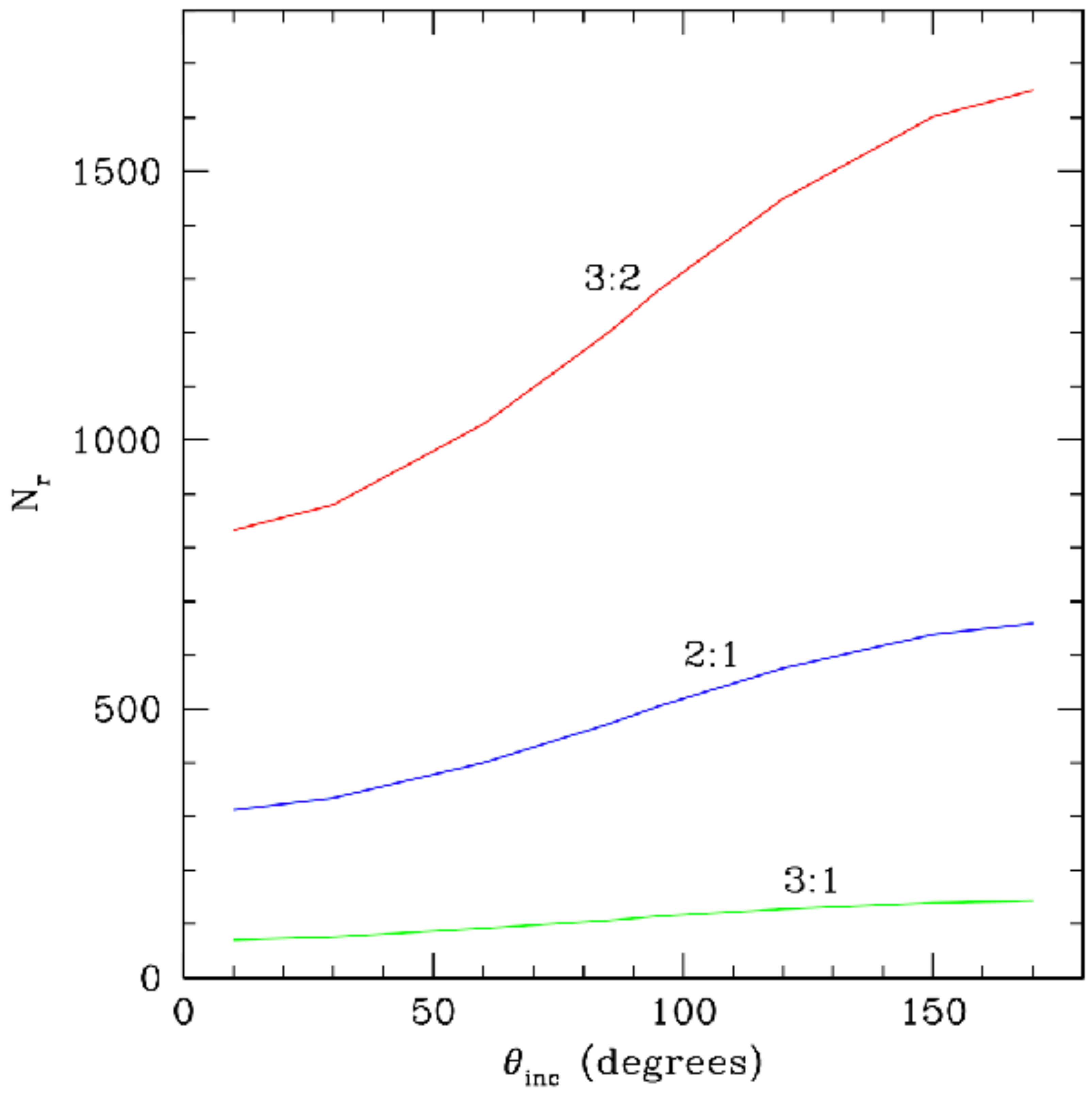

[Cornish, Class. Quant. Grav. **28**, 094016 (2011)]

# Extras

# Other challenges for EMRIs: Resonances

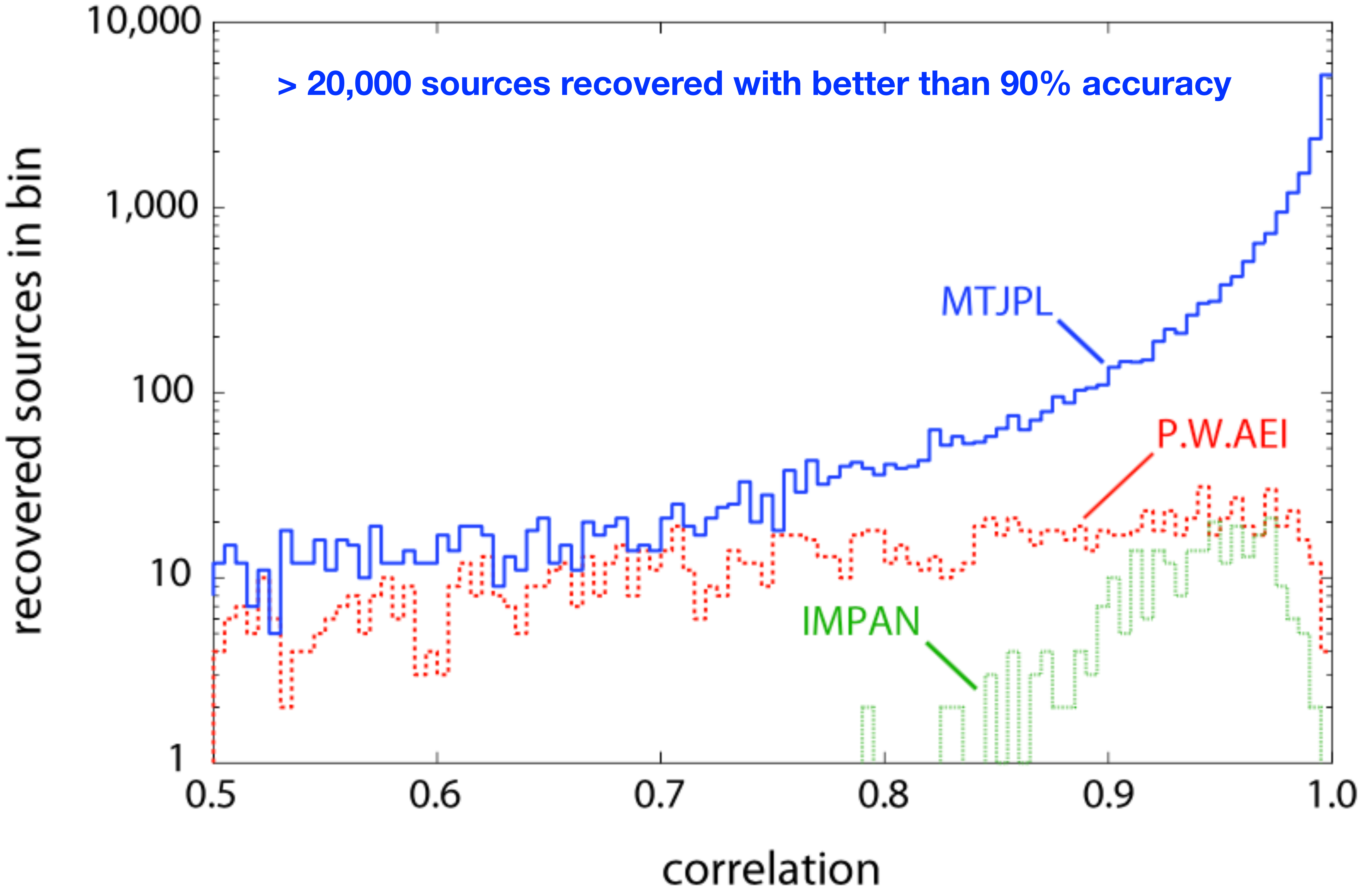



(Brink, Geyer, Hinderer 13)

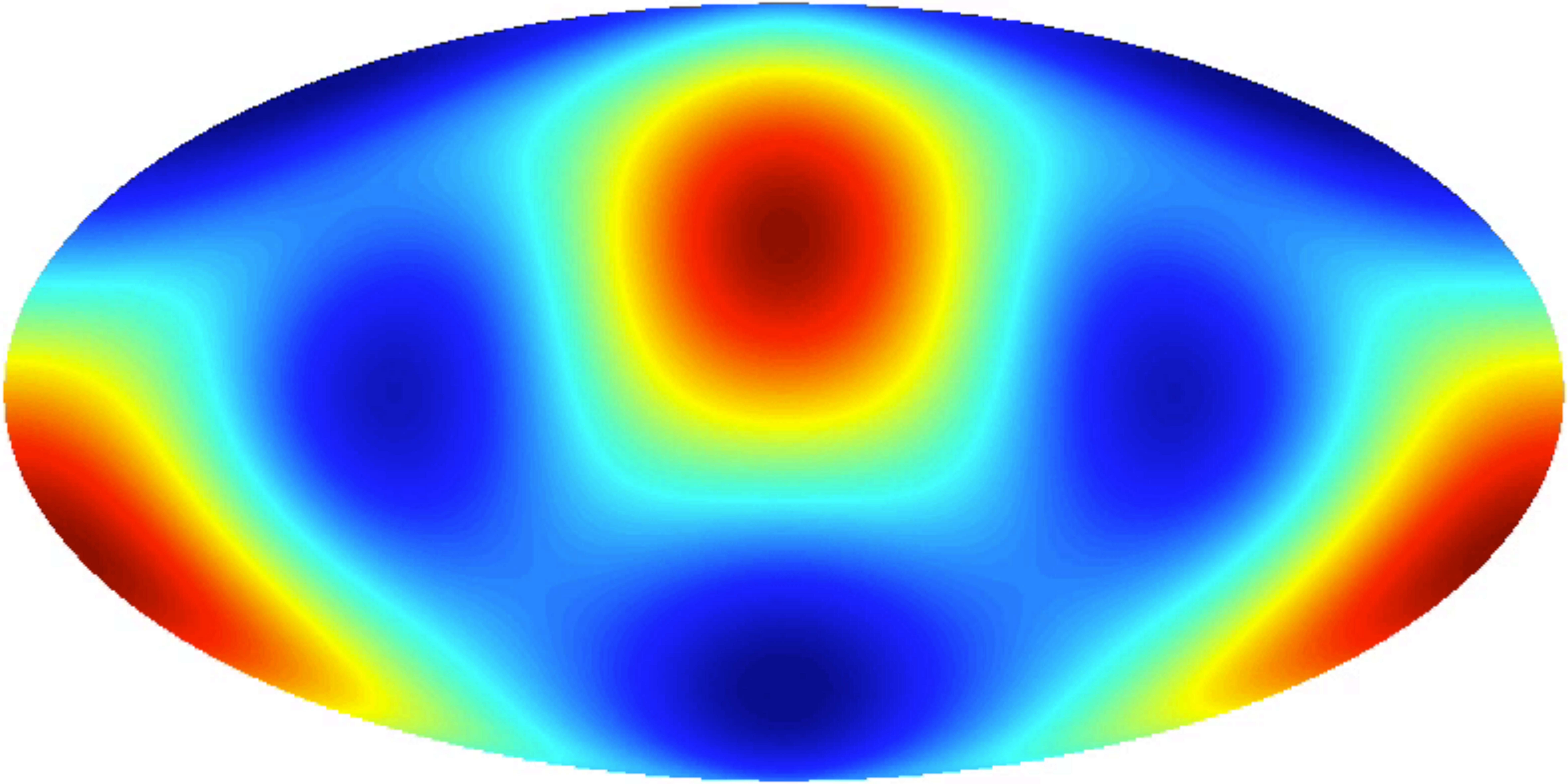



(Ruangsri, Hughes 14)

# Galactic Binaries in past Mock LISA Data Challenges



# Major Challenge: White Dwarf Foreground



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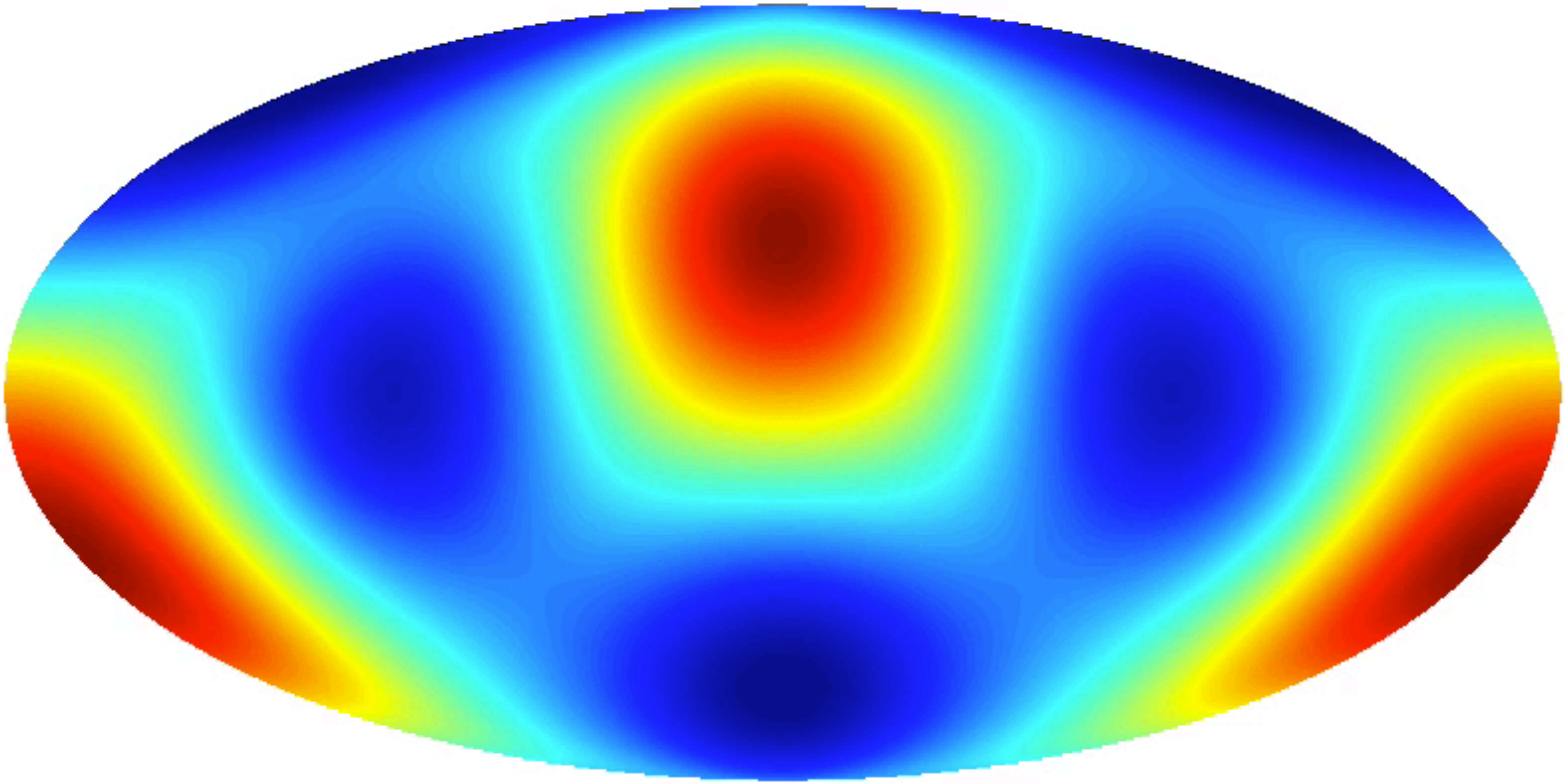

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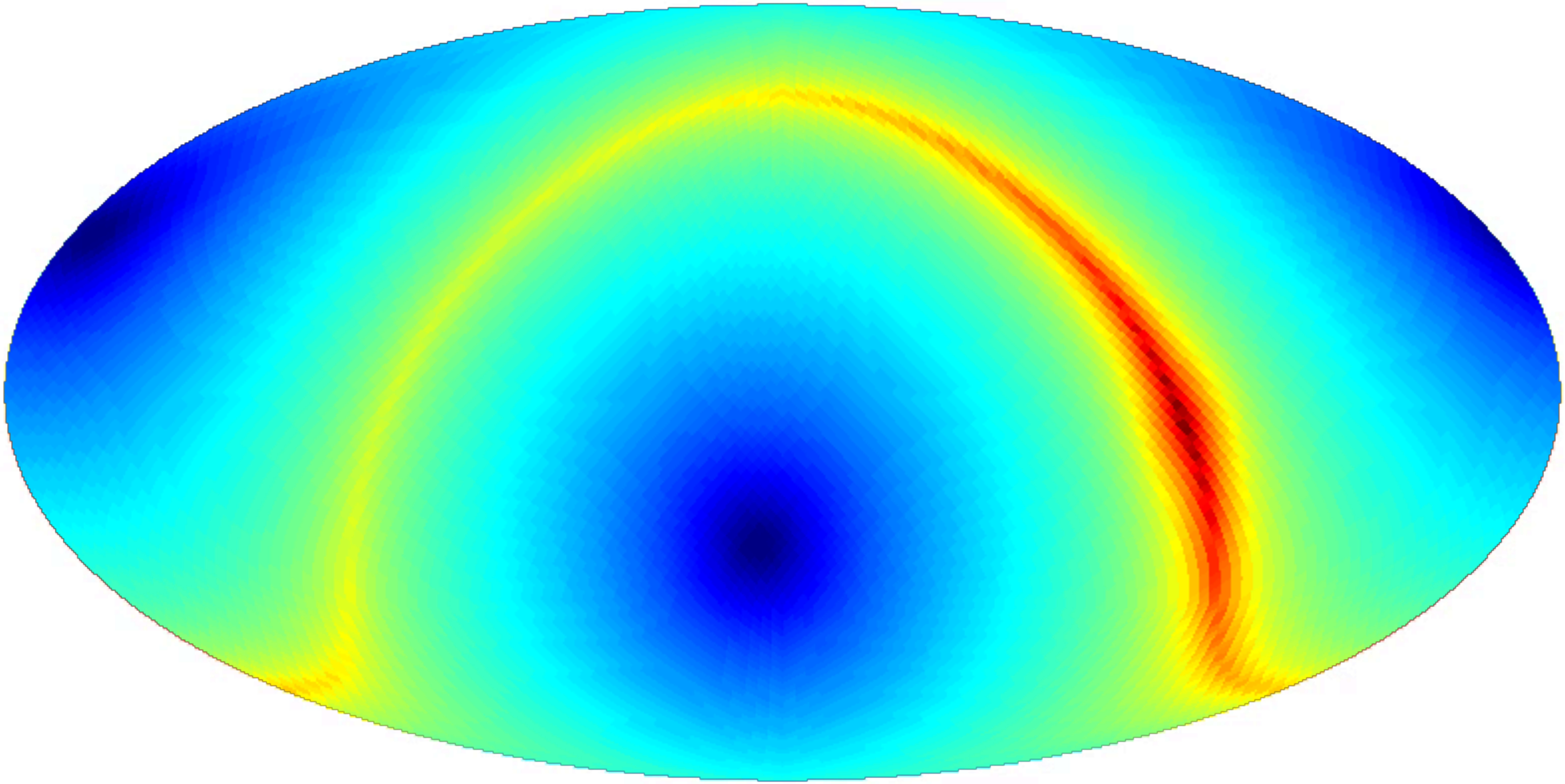

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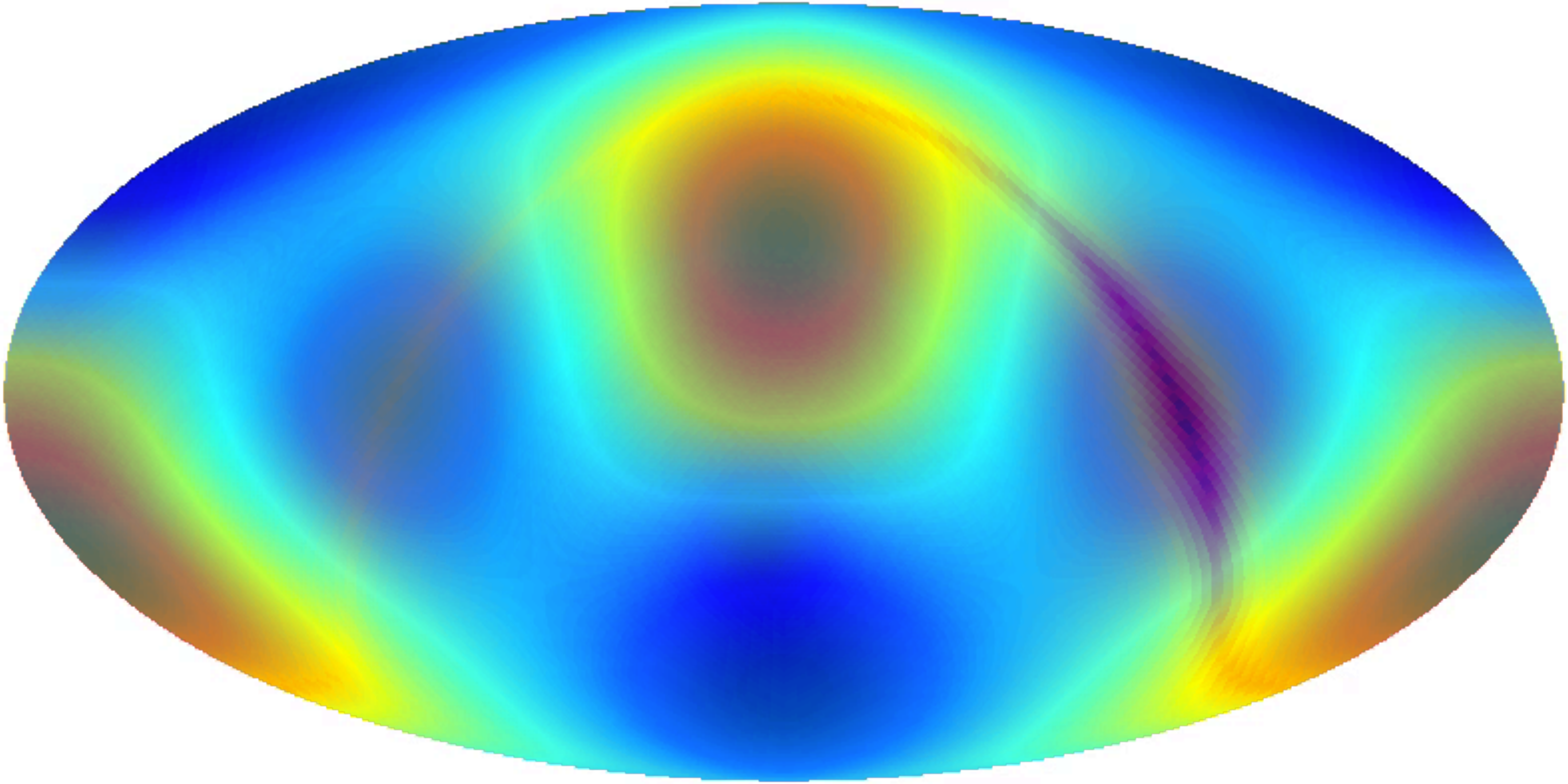
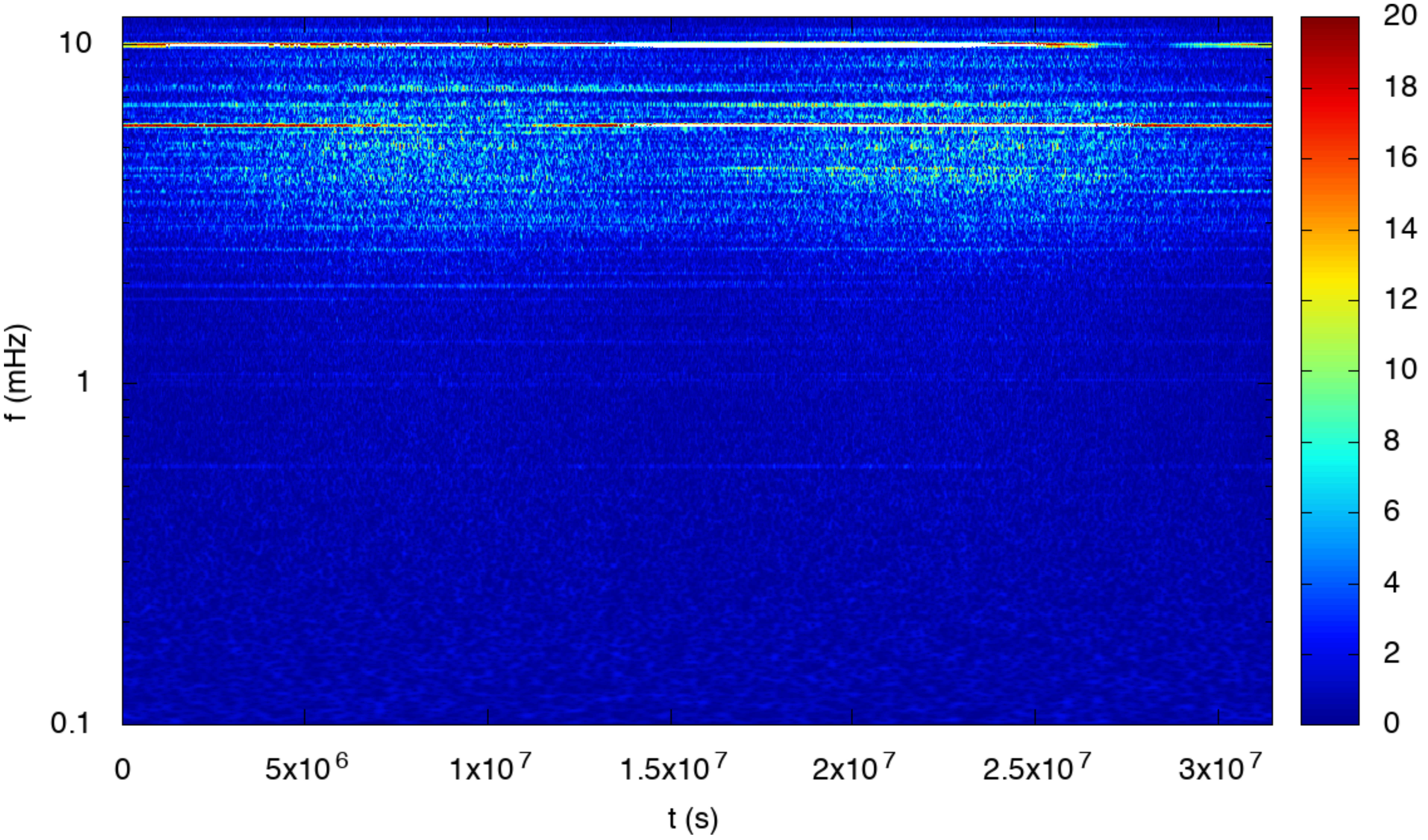
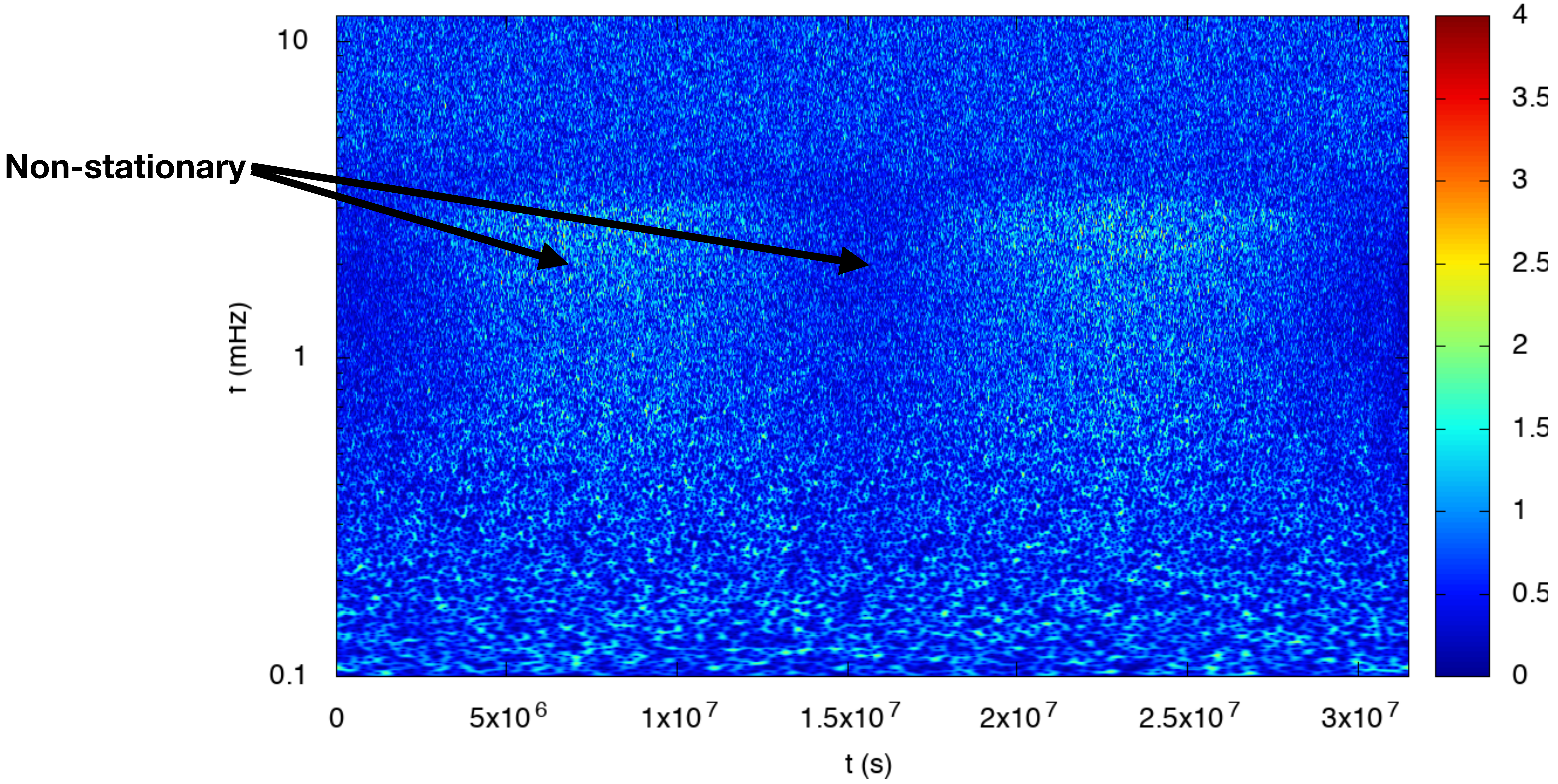


# Whitened Galaxy + Instrument noise spectrogram, 1 year



# Whitened Galactic Confusion + Instrument noise spectrogram, 1 year



# Galaxy + BH + Instrument noise spectrogram, 1 year