Jet Propulsion Laboratory
California Institute of Technology

Reduced-order modeling and gravitational waveforms

Chad Galley (JPL/Caltech)

The Architecture of LISA Science Analysis: Imaging the Future, Caltech, January 18, 2018

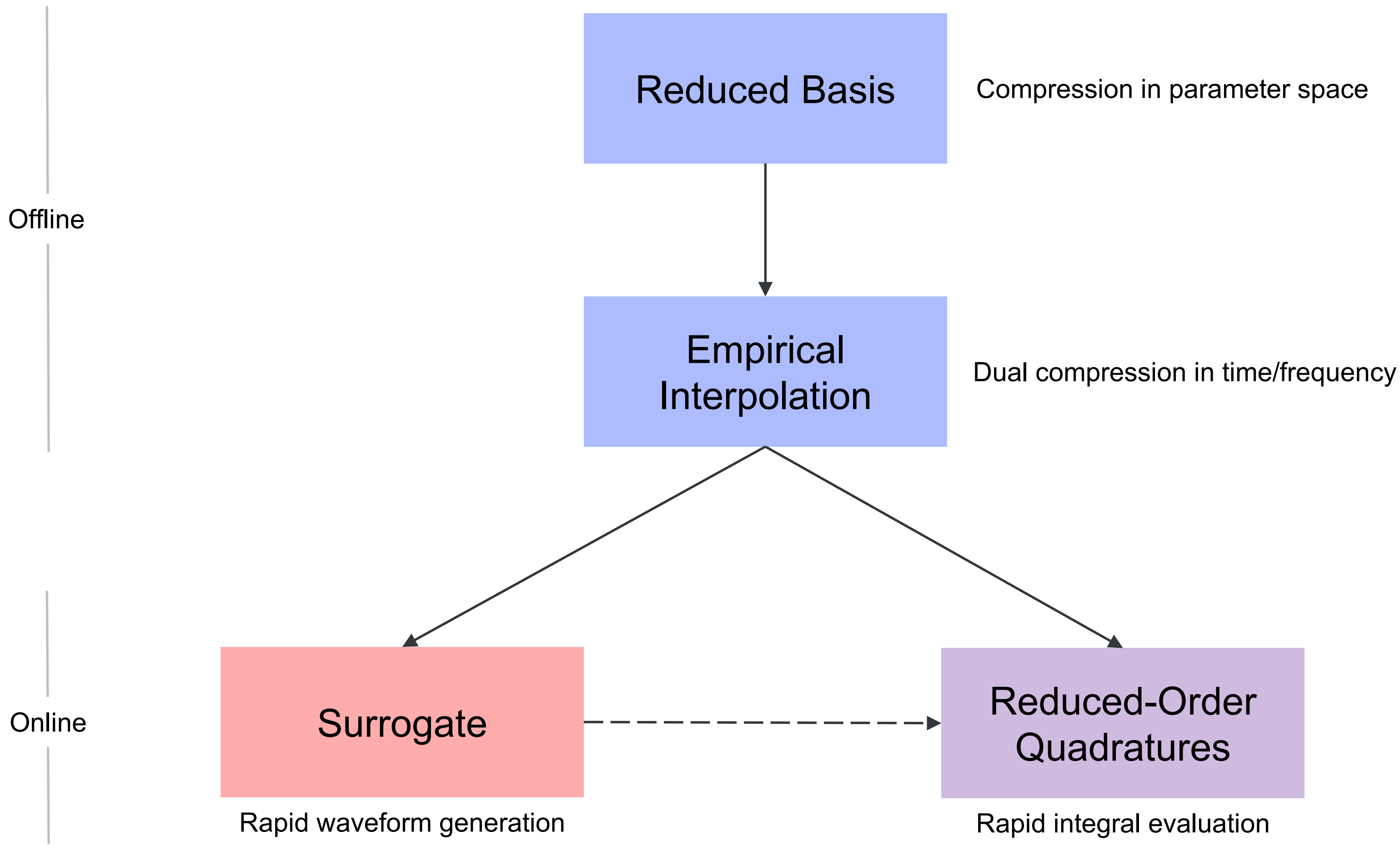
Motivation

- Generating waveforms can be expensive, time-consuming, and a bottleneck for practical data analysis applications
  - Template bank generation for gravitational wave searches ("curse of dimensionality")
  - Multiple waveform queries for parameter estimation (e.g., with stochastic methods)
  - Parameter space mapping, exploration, and discovery (i.e., science!)
  - Accessibility to broader scientific communities and the public

- Goals: To cheaply and quickly predict gravitational waveforms that are otherwise prohibitively expensive to mass-produce.
  - Numerical relativity waveforms of compact binary coalescences
    - Can take weeks to months to complete one simulation and corresponding waveform
    - 3 points only in each of the 7 parameter dimensions requires 2187 simulations!
  - Extreme Mass Ratio Inspirals (EMRIs)
  - Continuous gravitational waves

Reduced-Order Modeling (ROM)

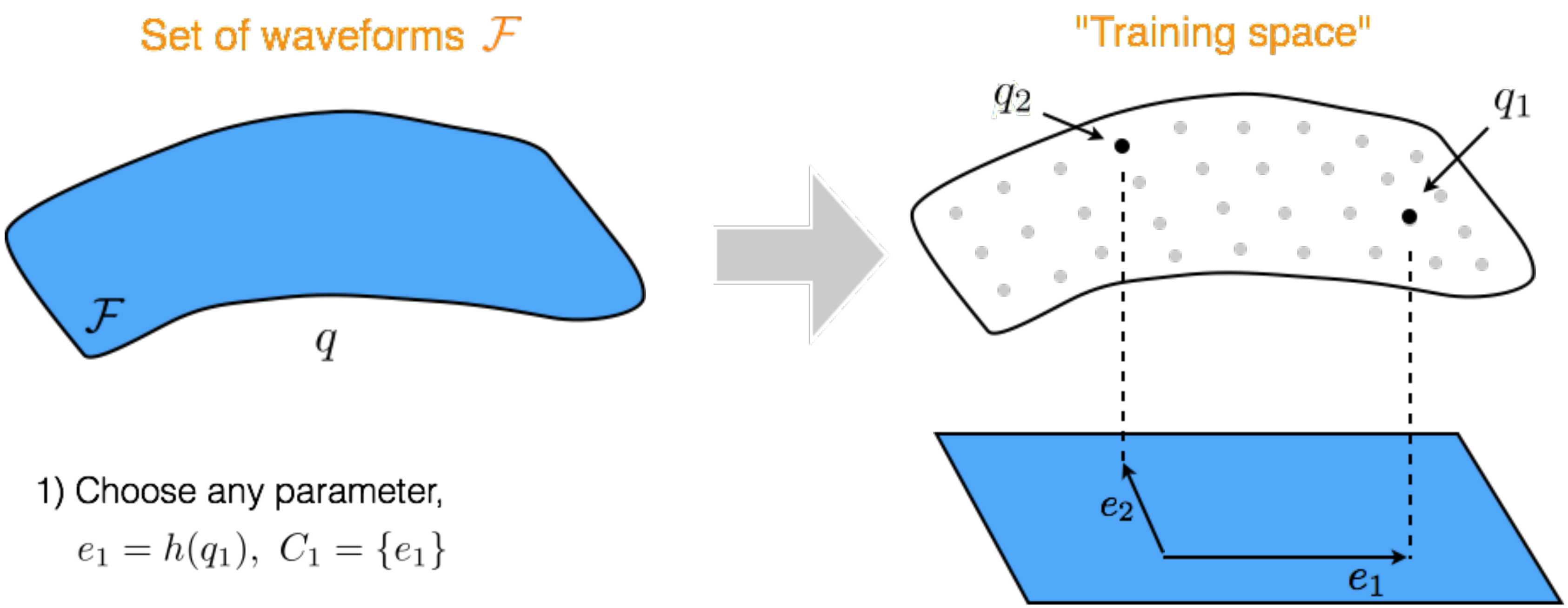
Overview



Reduced Basis

Basic Idea

Can find a linear approximation space that is nearly optimal



1) Choose any parameter,

$$e_1 = h(q_1),\ C_1 = \{e_1\}$$

2) <u>Greedy search</u> - Find the parameter that maximizes:

$$||h_q - P_1(h_q)||,\ P_1(h_q) = e_1\langle e_1, h_q\rangle$$

3) Orthogonalization to get basis vector e_2

$$C_2 = \{e_1, e_2\},\ C_1 \subset C_2$$

Output:

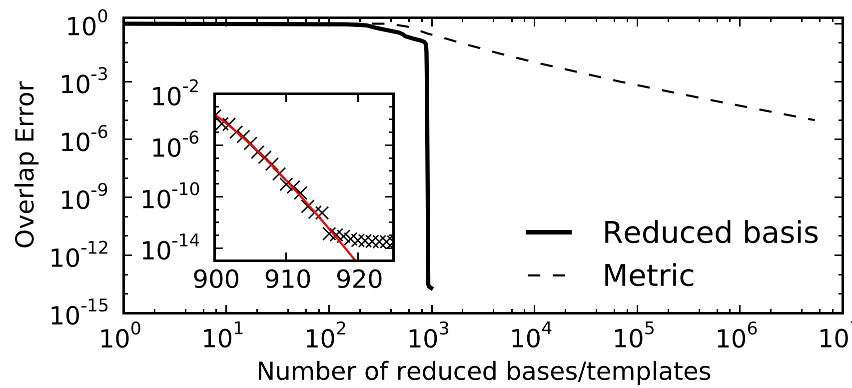
1) "Most relevant" parameters and waveforms
2) A nested/hierarchical basis
3) Maximum projection errors converge (super-)exponentially

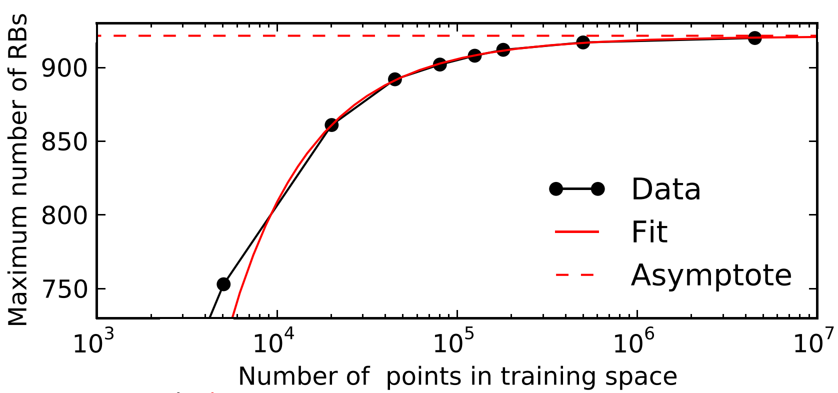
Reduced Basis

Some results and lessons learned

- Nonspinning PN inspirals [Field et al, PRL (2011)]

$$m_1, m_2 \in [1,3]M_\odot$$
$$f_{\min} = 40Hz$$



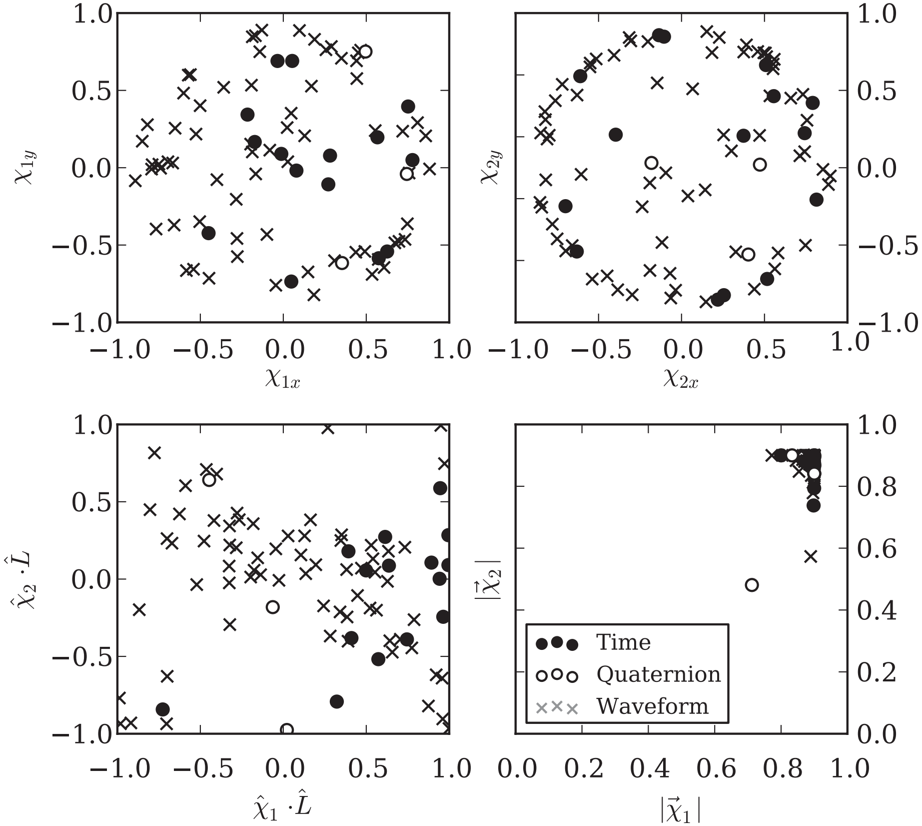


- The reduced basis asymptotes to span the continuum of waveforms, not just the training set.

Reduced Basis

Some results and lessons learned

- Precessing PN inspirals [Blackman et al, PRL (2014)]



Error	Basis size 1D	2D	3D	7D
$\lesssim 10^{-2}$	4	6	7	13
$\lesssim 10^{-4}$	4	7	8	20
$\lesssim 3 \times 10^{-8}$	6	15	23	50

$q \in [1, 10]$

$||\vec{\chi}_{1,2}|| \in [0, 0.9]$

200 cycles

- Arrange and transform the training data into a form that is smooth with parameter variations (i.e., "boring")

- The greedy algorithm is highly flexible and can adapt to many different types of strategies
  - Randomly resample the training set after each iteration [Blackman et al, PRL (2014)]
  - Use an error metric that is suitable to the problem and parameterization
  - Divide and conquer the training space (with random resampling) [Galley (unpublished)]

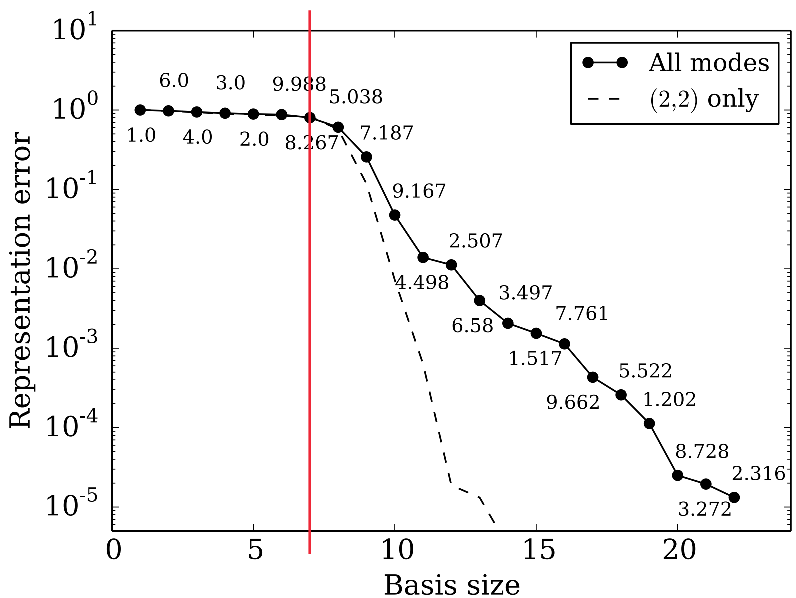
Reduced Basis

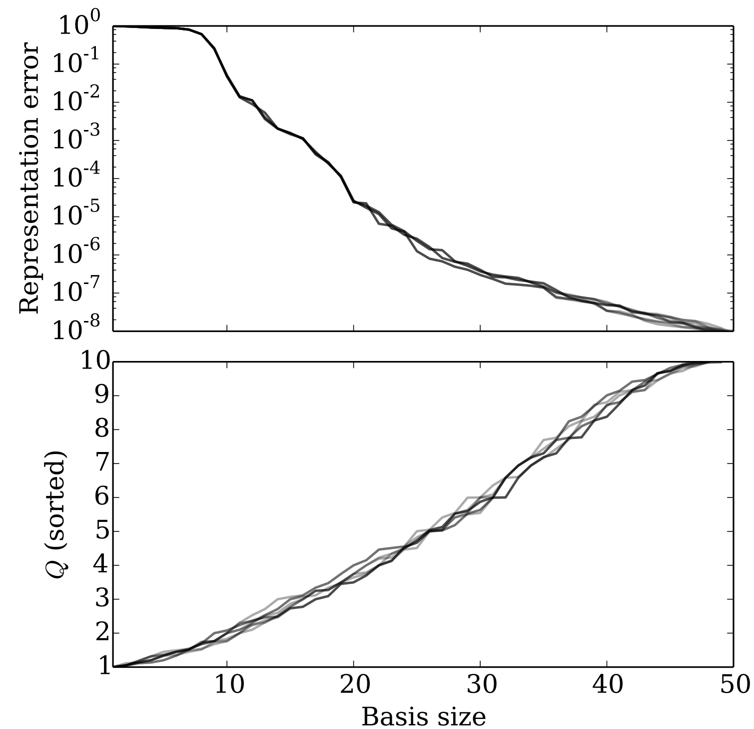
Some results and lessons learned

- Nonspinning EOBNR inspiral-merger-ringdown waveforms [Blackman et al, PRL (2015)]

$q \in [1, 10]$

25-31 cycles before peak amplitude





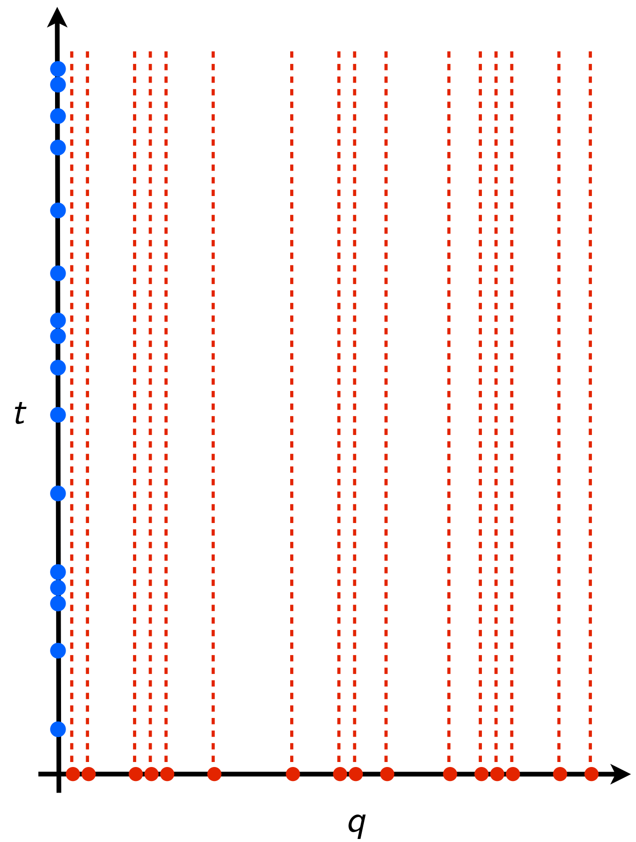
- Lower-accuracy waveform models (e.g., Phenom?, ?EOB?) are helpful to inform for which parameters to run expensive simulations
- Parameters selected by greedy algorithm are robust

Empirical Interpolation

Barrault et al (2004)
Maday et al (2009)

Basic Idea

- Empirical interpolation is similar to the standard interpolation problem but constructed using the application-specific reduced basis instead of a generic basis (e.g., Chebyshev polynomials)
  - Interpolation nodes are selected by another greedy algorithm that minimizes the interpolation error

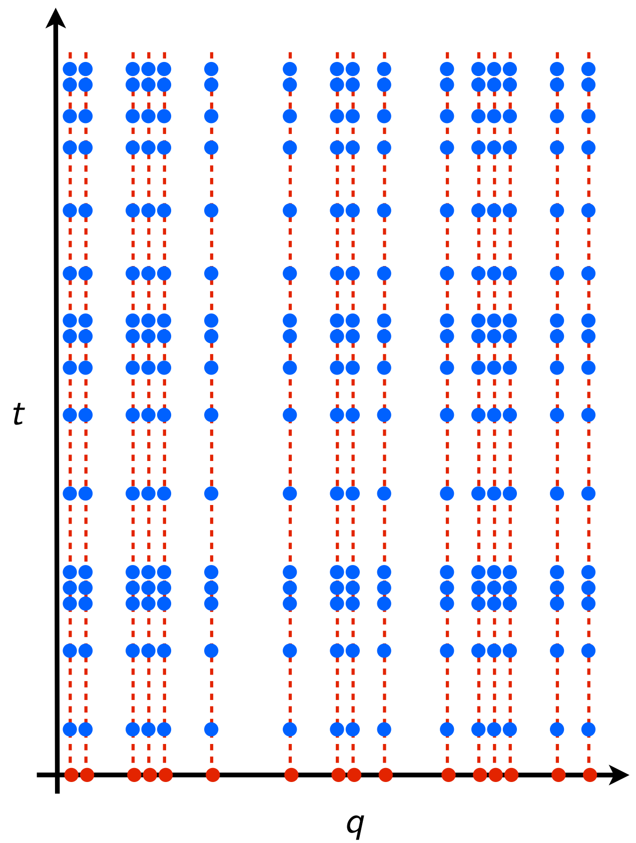


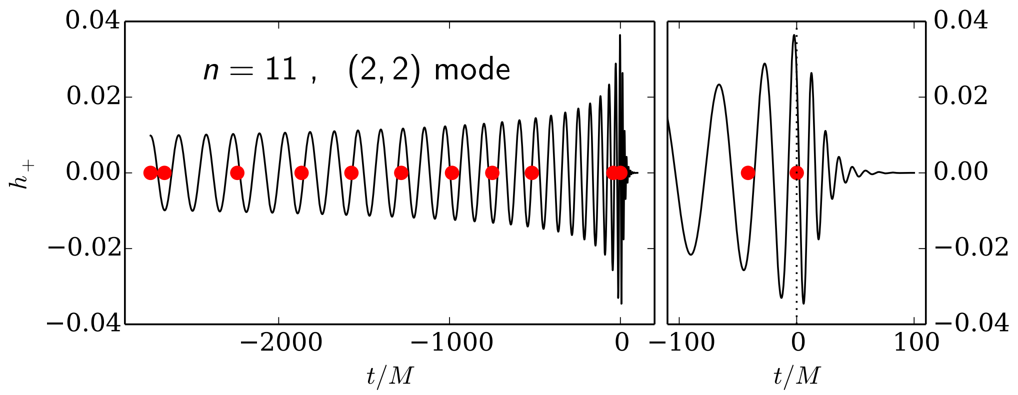
Empirical Interpolation

Basic Idea

Barrault et al (2004)
Maday et al (2009)

- Empirical interpolation is similar to the standard interpolation problem but constructed using the application-specific reduced basis instead of a generic basis (e.g., Chebyshev polynomials)
  - Interpolation nodes are selected by another greedy algorithm that minimizes the interpolation error

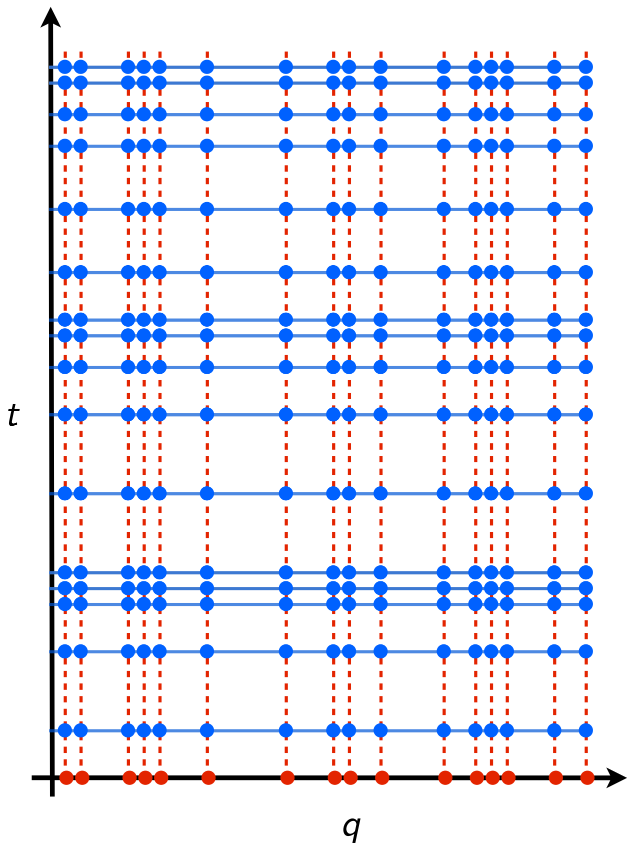




Surrogate

Basic Idea

- Surrogate is constructed by fitting for the parameter variation at each empirical interpolation node (e.g., time)



$$A_S^{\ell m}(t;q) \equiv \sum_{i=1}^{n_A} B_{A,i}^{\ell m}(t)\mathcal{A}_i^{\ell m}(q)$$

$$\varphi_S^{\ell m}(t;q) \equiv \sum_{i=1}^{n_\varphi} B_{\varphi,i}^{\ell m}(t)\Phi_i^{\ell m}(q)$$

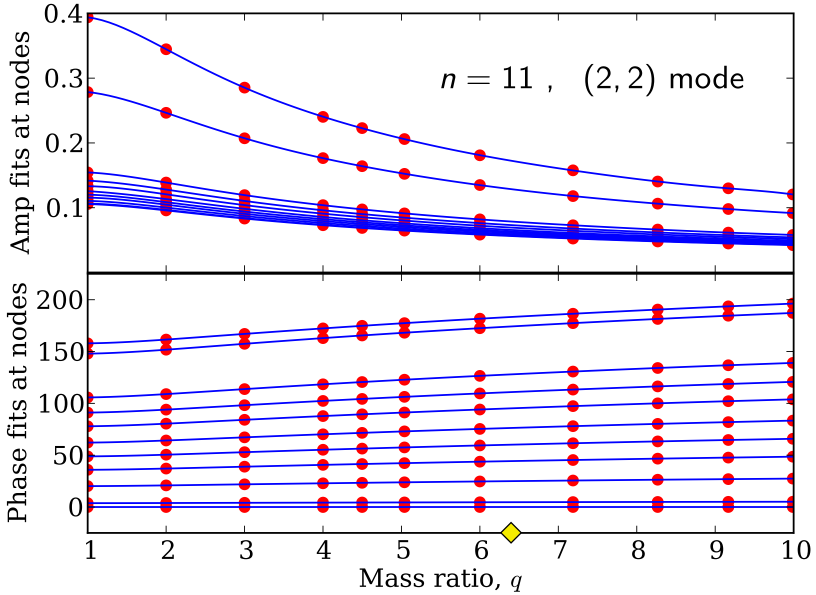
$$h_S^{\ell m}(t;q) \equiv A_S^{\ell m}(t;q)e^{-i\varphi_S^{\ell m}(t;q)}$$

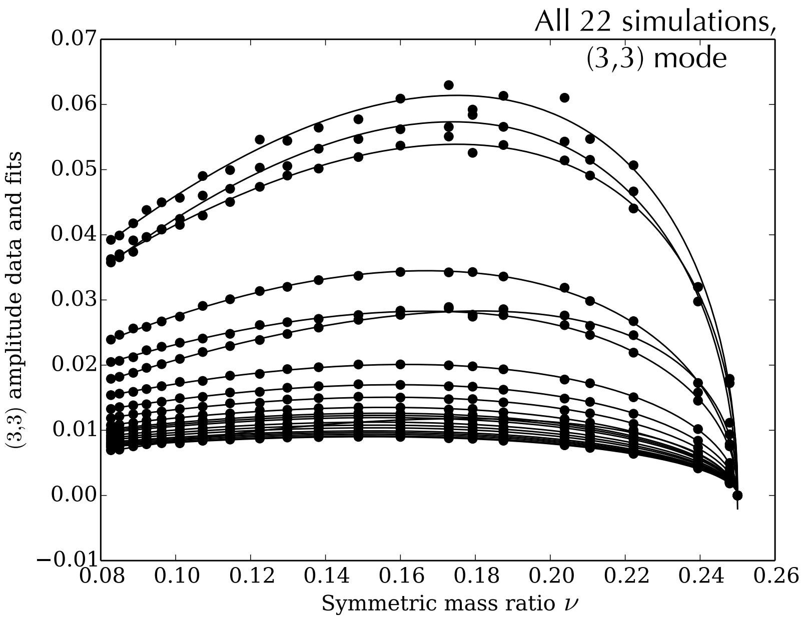
$$h_S(t,\theta,\phi;q) \equiv \sum_{\ell,m} h_S^{\ell m}(t;q)\,{}_{-2}Y_{\ell m}(\theta,\phi)$$

- Speed to evaluate a NR surrogate for BBH multi-modal waveforms is < 1 sec

Surrogate

Basic Idea





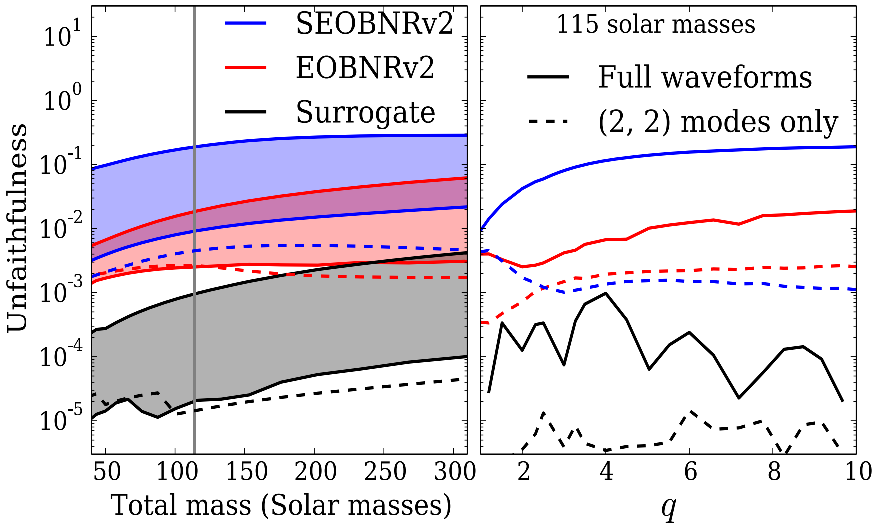
Surrogate

Some results

- NR surrogates are comprehensively the most accurate BBH waveform models to date

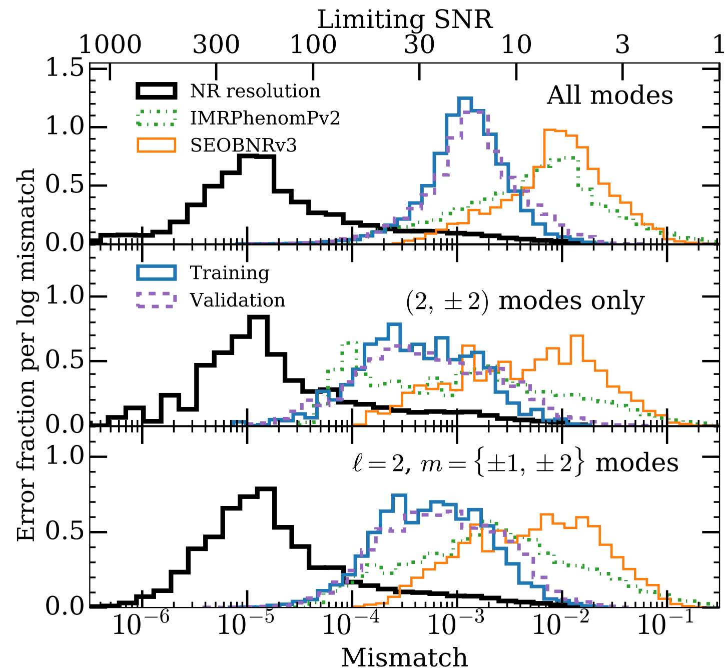
NR nonspinning BBH

Blackman, et al, PRL (2015)



NR precessing BBH

Blackman, et al, PRD (2017)



- Speed-up of surrogate evaluation relative to a SpEC non-spinning BBH simulation is ~ 10^8
  - SpEC: 9.3 days on 48 procs
  - Surrogate: ~ 10 msec on 1 proc

Fast and accurate reduced-order surrogate models

Waveform generation is no longer a bottleneck for data analysis!

Short-comings and mitigations

- ROMs are accurate within the parameter domain of the training space
  - Caution must be taken when extrapolating outside the training space
  - Mitigations:
    - Increase the domain of the training space
    - Find a smoother waveform parameterization so that extrapolation is less severe

- Offline generation of the training set can take a long time
  - Example: Took 2 years to generate 22 NR training waveforms for the non-spinning BBH surrogate
  - Mitigations:
    - Progress with time: Took ~2 years to generate 276 NR training waveforms for the precessing BBH surrogate [Blackman et al, PRD (2017)]
    - Adjust the training space sampling strategy and waveform parameterization for the Reduced Basis greedy algorithm

- ROM works best on a training set of $C^{infinity}$ functions
  - Can still work on C^n functions but yields a less compact reduced basis (e.g., EOBNRv2)
  - Mitigation: Try to transform the to reduce the impact of finite continuities

- Reduced-Order Quadratures rely on the linearity of the waveform itself
  - Smart (i.e., nonlinear) parameterizations for RB construction are not helpful here
  - Mitigations:
    - None but one can still use a surrogate (if needed) for rapid waveform generation in the integrand

Summary and Outlook

- Reduced-order modeling has proven to be a uniquely powerful tool for rapid waveform generation

- Data analysis problems that weren't possible a few years ago are now feasible

- Reduced Basis greedy algorithm outputs a gold mine of information that could be used
  - Detection? Estimate quickly a prior on parameters?
  - What can we use for LISA?

- Surrogate modeling is also effective in other applications relevant for LISA
  - (Scalar) Self-force evaluation, consistent orbital evolution, and waveform generation
  - EMRI waveform surrogates?

Summary and Outlook

- Numerical Relativity surrogate model waveform data are available at

  www.black-holes.org

  - Nonspinning BBHs
  - Nonprecessing BBHs
  - Precessing BBHs

- *RomPy* is a Python code used to build generic reduced-order surrogate models

  bitbucket.org/chadgalley/rompy

  - Python2
  - Example iPython notebooks
  - Stable but more development desired
  - Python3 and C++ back-end (for speed) in progress

Backup

Reduced-Order Quadratures

Basic Idea

- Speed up the computation of match integrals and likelihoods

  Example: Given a surrogate waveform:

$$h(f;\vec{\lambda}) = \sum_{i=1}^{N_{RB}} B_i(f) H_i(F_i;\vec{\lambda})$$

  Example: Overlap integral

$$\int_{f_{\min}}^{f_{\max}} df\, \frac{h(f;\vec{\lambda}) d^*(f)}{S_n(f)} = \sum_{i=1}^{N_{RB}} H_i(F_i;\vec{\lambda}) \left[\int_{f_{\min}}^{f_{\max}} df\, \frac{B_i(f) d^*(f)}{S_n(f)} \right]$$

$$= \sum_{i=1}^{N_{RB}} H_i(F_i;\vec{\lambda}) w_i$$

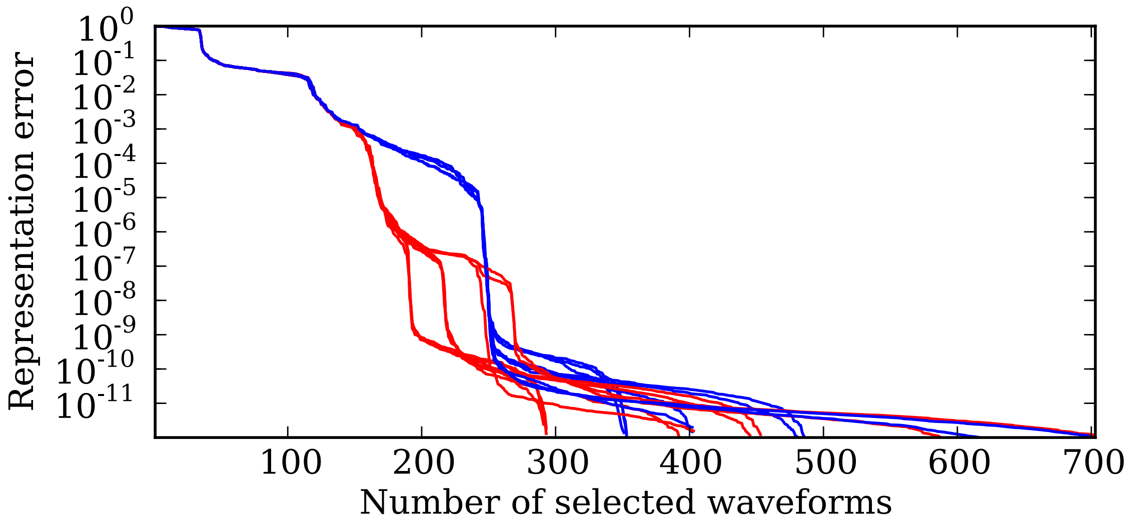
- Quadrature weights w_i can be computed once the data is available: "Start-up" phase

- Once weights are computed, the integral is evaluated as a simple quadrature

Surrogate

Lessons learned

- Not aligning the training set waveforms at peak amplitudes results in a much larger reduced basis size and little robustness, e.g., to different time samplings of the waveforms



Jet Propulsion Laboratory
California Institute of Technology

jpl.nasa.gov