Waveform modeling for LIGO parameter estimation: status & challenges for LISA

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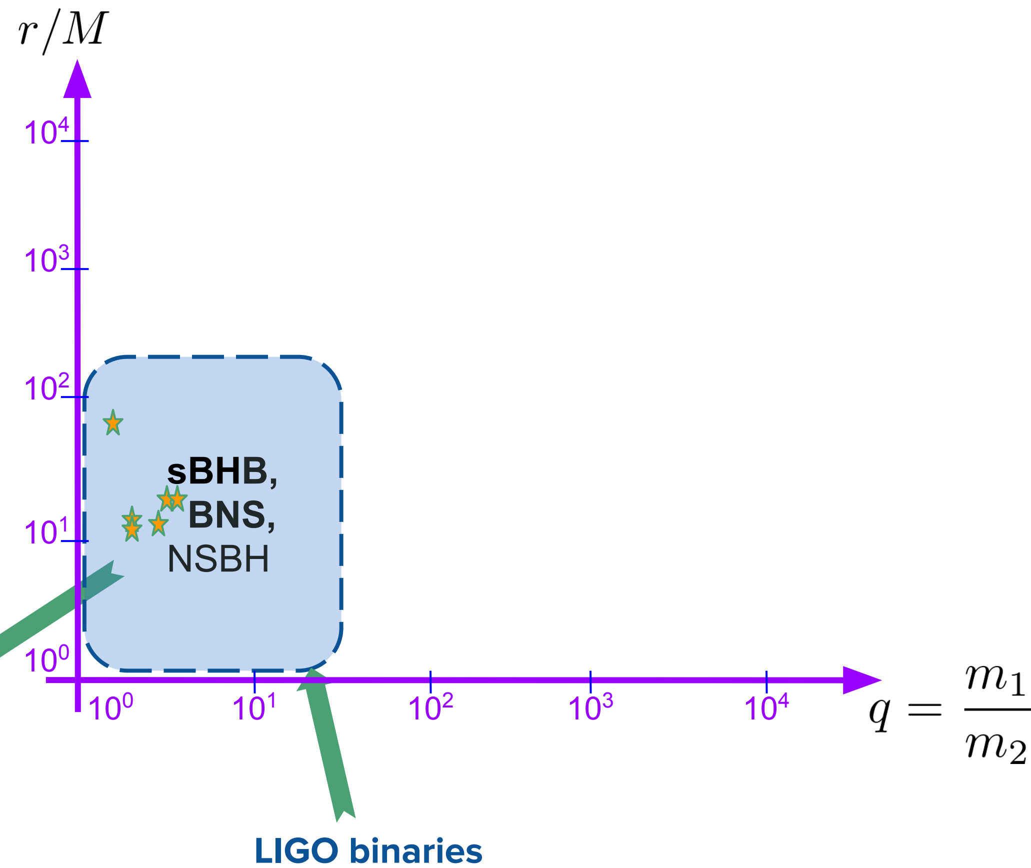
The Architecture of LISA Science Analysis: Imagining the Future
January 16 - 19, 2018

Outline

1. LIGO sources
2. Source modeling:
   a. PN theory
   b. NR (brief)
   c. EOB formalism
   d. Phenomenological models
   e. Simplification of spin-precession
3. Application to LIGO parameter estimation:
   a. PE requirements
   b. Model-order reduction
   c. Reduced-order Quadrature
4. LISA sources
   a. MBH
   b. EMRI & IMRIs
5. Summary

1. LIGO Sources

- LIGO is sensitive to 10–10^3 Hz
- Binary mergers of stellar-collapse black holes and / or neutron stars
- More massive sources visible further away (up to cosmological distances)



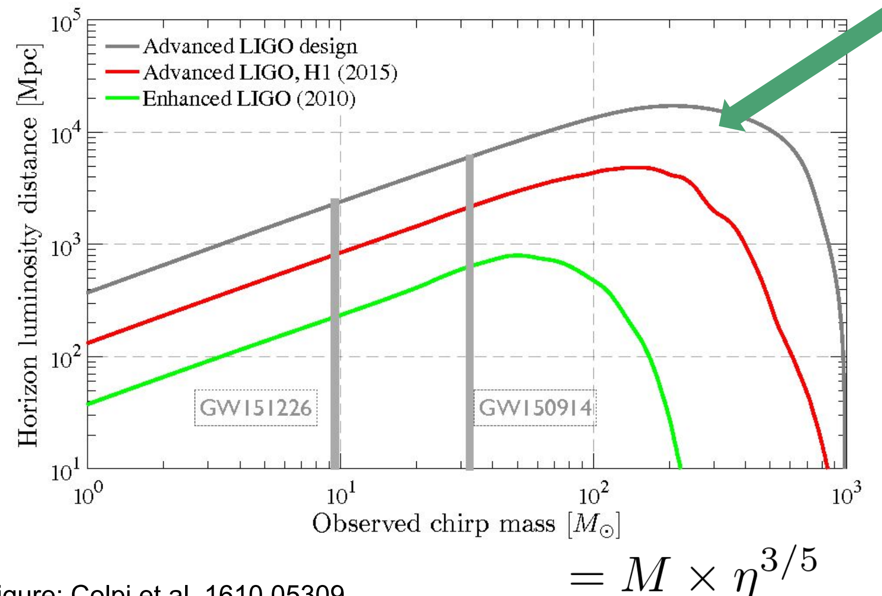
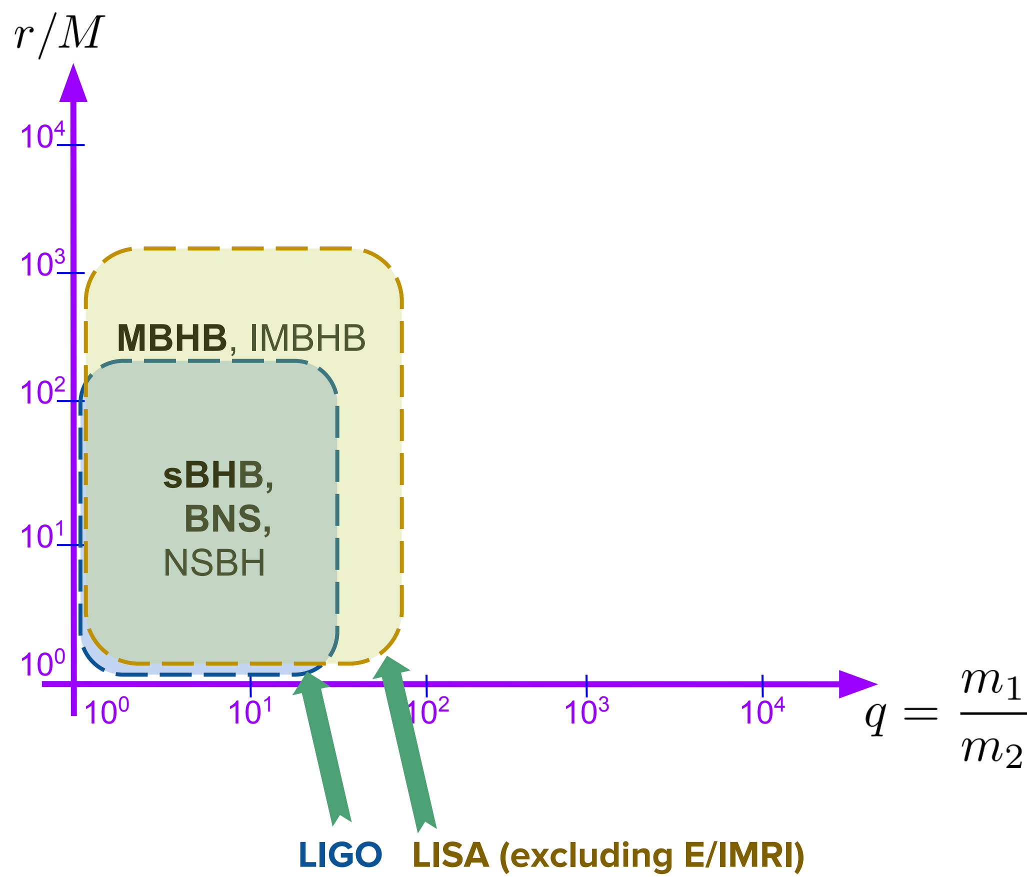


Figure: Colpi et al, 1610.05309

1. LIGO Sources

- LIGO is sensitive to 10–10^3 Hz
- Binary mergers of stellar-collapse black holes and / or neutron stars
- More massive sources visible further away (up to cosmological distances)
- Waveform modeling research into stellar BHB for LIGO will carry over to MBHB / IMBHB sources for LISA

  [Including ranges of parameters not shown, e.g. BH spins, orbital eccentricity]



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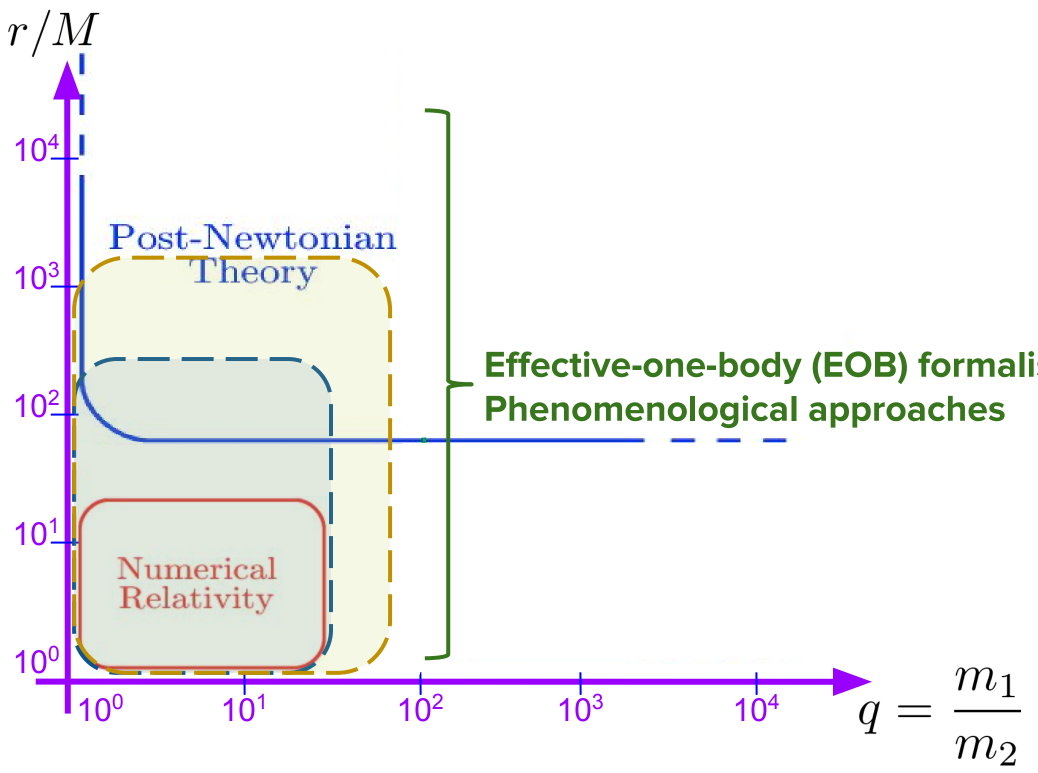
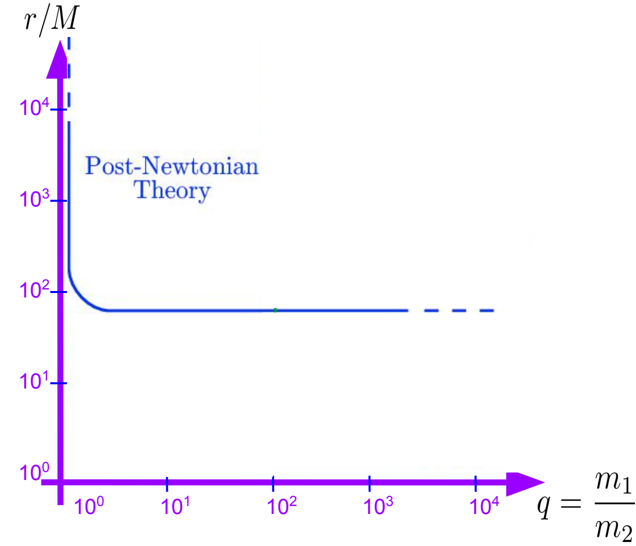
2. Source modeling



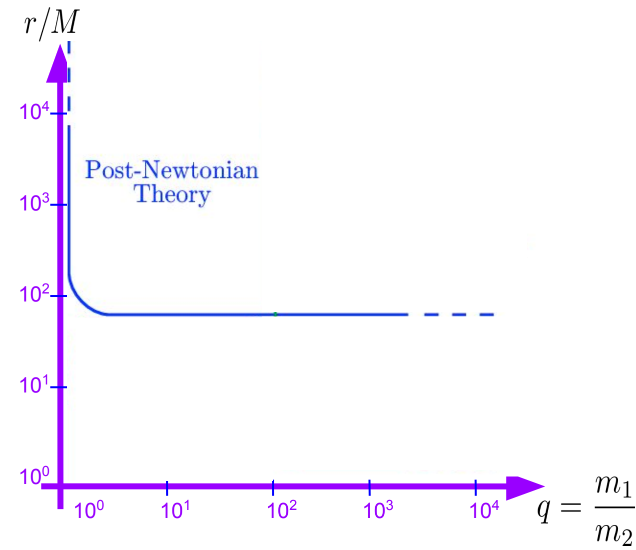
2a. Source modeling: Post Newtonian theory

- Slow-motion weak-field approximation
- Perturbative expansions in orbital velocity (v/c)



2a. Source modeling: Post Newtonian theory

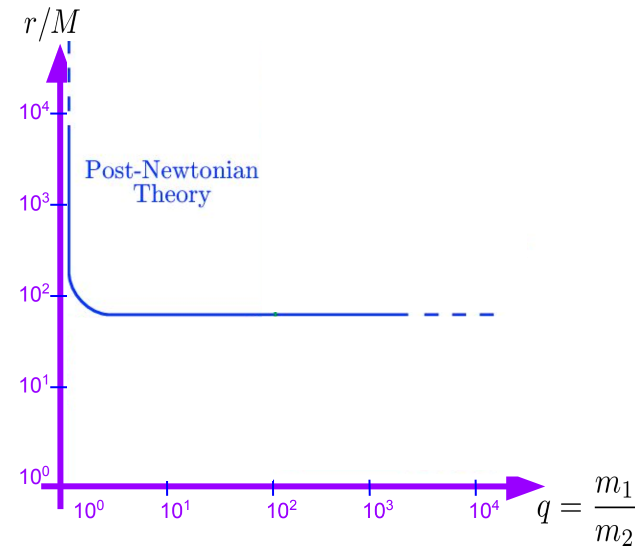
- Slow-motion weak-field approximation
- Perturbative expansions in orbital velocity (v/c)
- Equation of Motion:
  2PN EoM - [Ohta et al, '73],
  3.5PN EoM - [Iyer & Will, '93]
  4PN EoM - [Damour et al, '14; Bernard et al '15]
  1.5PN SO - [Barker et al '75]
  2PN SS - [Kidder et al, '93]
  4PN NNLO SS - [Hartung et al '11, Levi et al '11]
  3.5PN NNLO SO - [Hartung et al '11, Marsat et al '13]



2a. Source modeling: Post Newtonian theory

- Slow-motion weak-field approximation
- Perturbative expansions in orbital velocity (v/c)
- State of the art (circa '14):

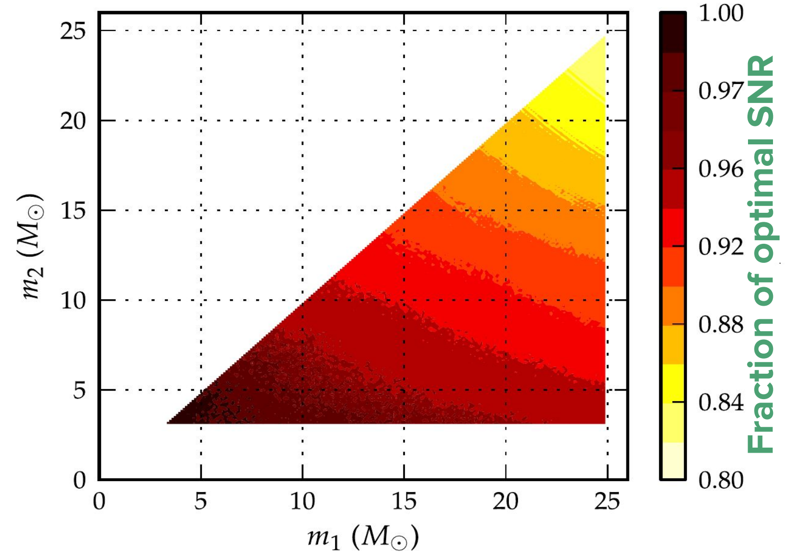
	No Spin	Spin-Linear	Spin-Squared
Conservative Dynamics	4PN[a] [121, 122, 133] [126, 158–164]	3.5PN [52, 54, 141] [140, 165–169]	3PN [52, 54, 138] [137, 170–172]
Energy Flux at Infinity	3.5PN [95, 173, 174]	4PN [175–178]	2PN [53, 54, 179–181]
RR Force	4.5PN [37, 93, 183–185]	4PN [186–188]	4.5PN [189]
Waveform Phase[c]	3.5PN [190]	4PN [175, 177, 178]	2PN [54, 179–181, 191]
Waveform Amplitude[e]	3PN[d] [194–197]	2PN [191, 198]	2PN [53, 54, 191, 198]

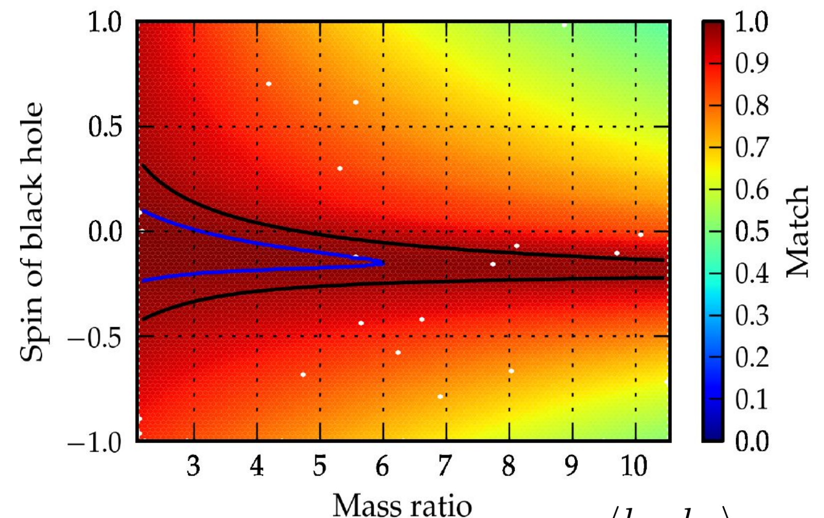


Buonanno & Sathyaprakash(2014), 1410.7832;

2a. Source modeling: Post Newtonian theory

- Accurate enough for detecting NS-NS binaries & low-mass BBHs with LIGO
- However, PN is not nearly sufficient for binary masses $\gtrsim 12 M_\odot$
- PN's performance gets worse with spins in the picture
- If heavy BHs (10 - 60 $M_\odot$) *dominate LIGO detection rates*, PN alone not enough for LIGO detection & PE



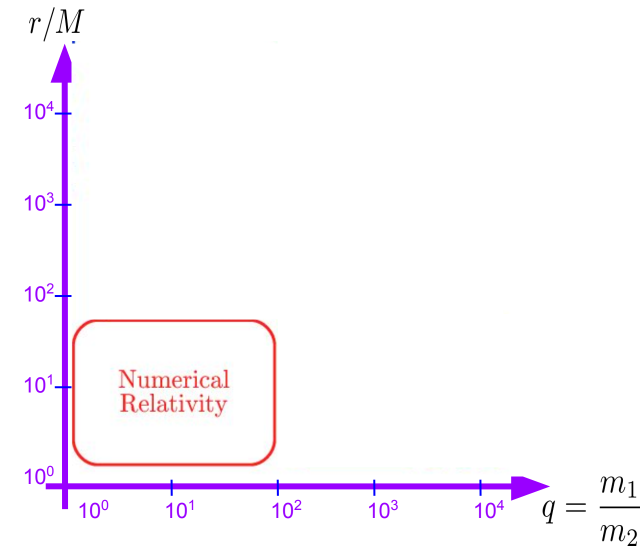


$$\frac{\langle h_1, h_2 \rangle}{|h_1||h_2|}$$

PK et al (2013), 1310.7949; PK et al (2015), 1507.00103;

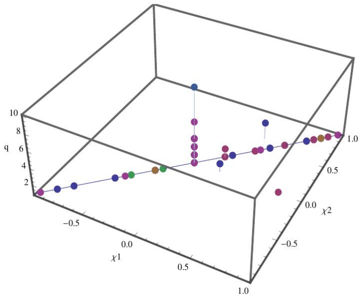
2b. Source modeling: Numerical Relativity

- Direct numerical evolutions of fully-nonlinear Einstein's equations

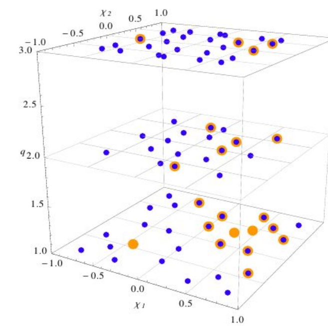


2b. Source modeling: Numerical Relativity

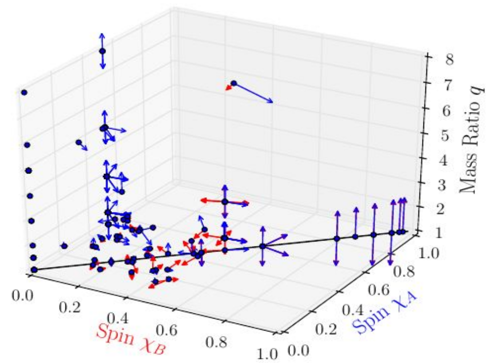
- Direct numerical evolutions of fully-nonlinear Einstein's equations
- Current catalogs



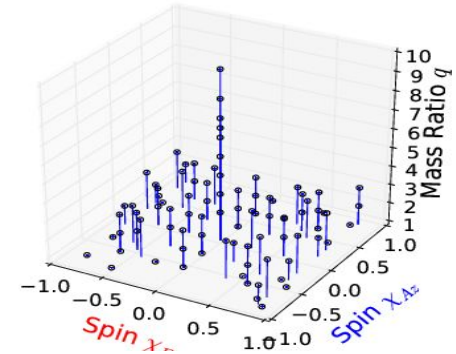
NINJA-2 Catalog (2012)



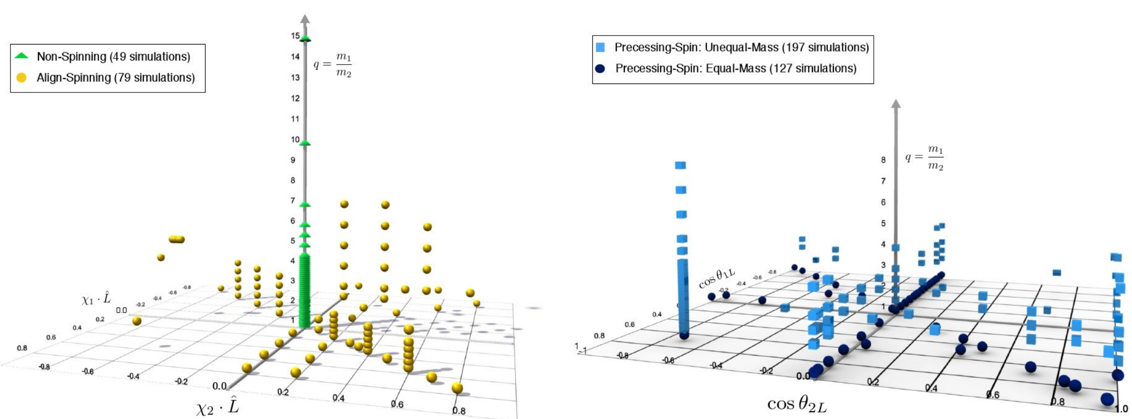
NRAR Catalog (2014)



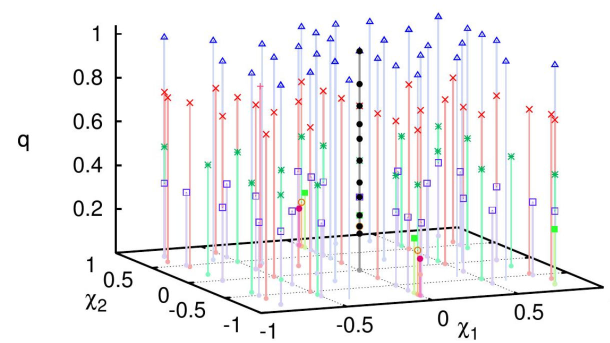
SXS Public Catalog (2013)



Chu, Fong, PK et al (SXS) (2016)



Jani et al (GATech) (2016)



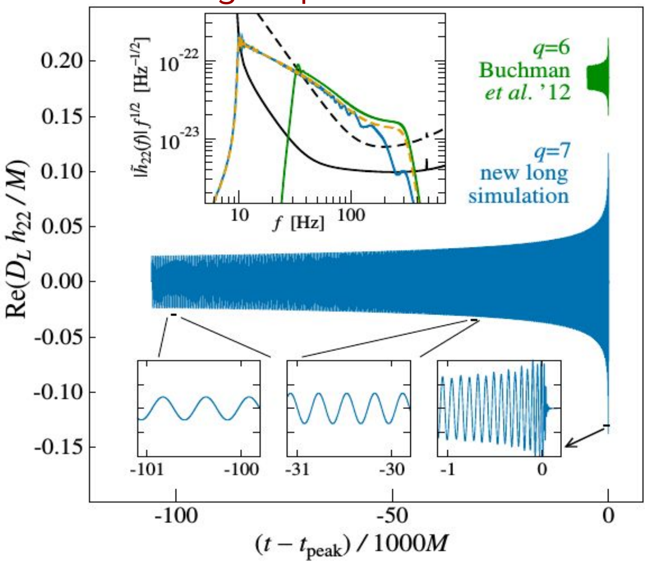
Healy et al (RIT) (2017)

2b. Source modeling: Numerical Relativity

- Direct numerical evolutions of fully-nonlinear Einstein's equations
- Current catalogs
- New frontiers

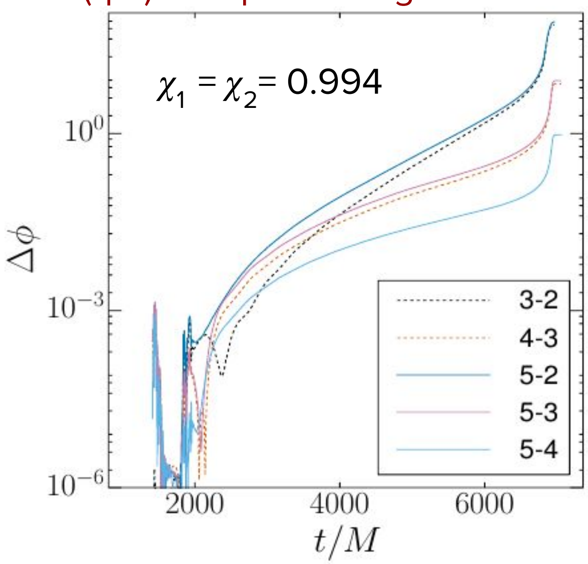
➢ *Critical for GW models of binary mergers*

Length up to ∽170 orbits



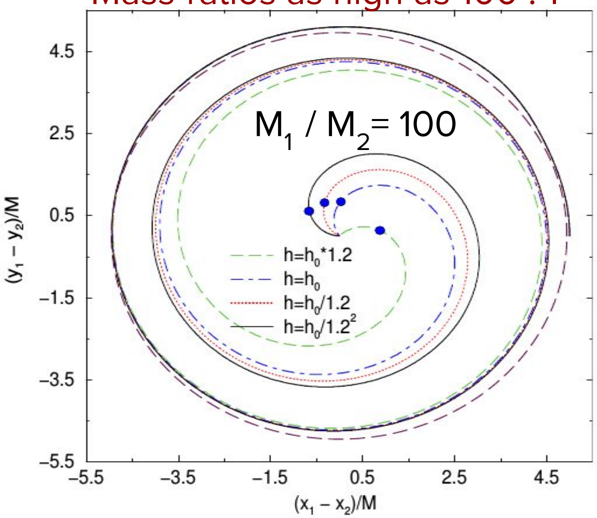
Szilagyi et al (2015)

(q=1) BH spins as high as 0.994



Scheel et al (2014)

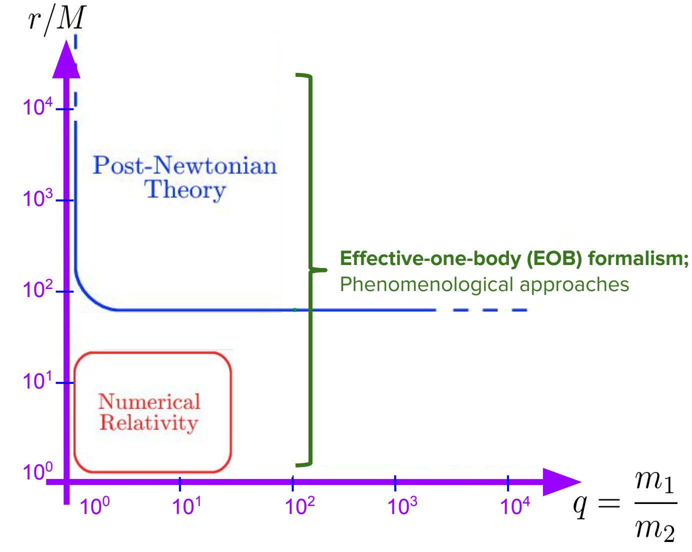
Mass-ratios as high as 100 : 1



Lousto & Zlochower (2011)

2c. Source modeling: Effective one body (EOB)

- General relativistic extension of 2-body to 1-body mapping of Newtonian problem



Buonanno et al (1998), gr-qc/9811091;

2c. Source modeling: Effective one body (EOB)

- General relativistic extension of 2-body to 1-body mapping of Newtonian problem

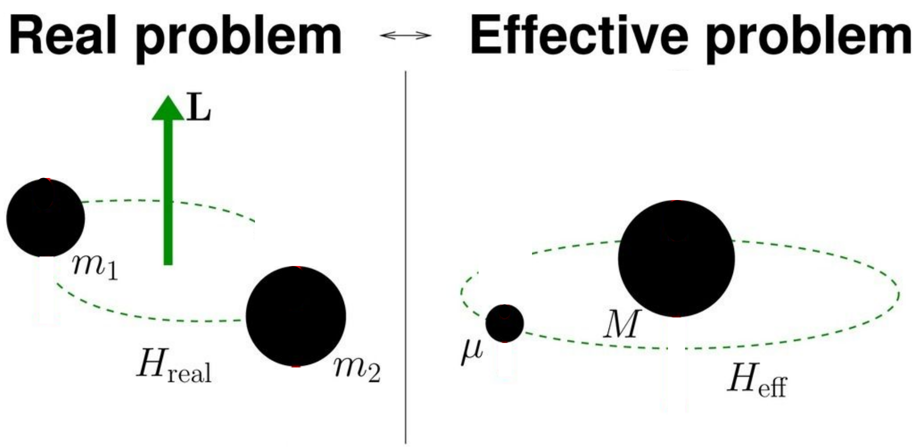
- Conservative dynamics:

$$ds^2_{\rm eff} = -A(R_{\rm eff})dt^2_{\rm eff} + \frac{D(R_{\rm eff})}{A(R_{\rm eff})}dR^2_{\rm eff} + C(R_{\rm eff})R^2_{\rm eff}\, d\Omega^2_{\rm eff}$$

- Identify:
  - $m_1 + m_2 \rightarrow M$;
  - $m_1 m_2 / M \rightarrow \mu$;

- Require the effective spacetime reduce to Schwarzschild at first order leads to (2PN)

- Mapping energy levels between 2-body and EOB description gives



$$H^{2\,\rm body}_{\rm PN} \rightarrow H^{EOB}$$

$$A(R) = 1 - \frac{2M}{R} + 2\eta\left(\frac{M}{R}\right)^3; \quad D(R) = 1 - 6\eta\left(\frac{M}{R}\right)^2$$

Pade re-summed

Buonanno et al (1998), gr-qc/9811091; Image credit: A. Taracchini/AEI

2c. Source modeling: Effective one body (EOB)

- Radiative dynamics:

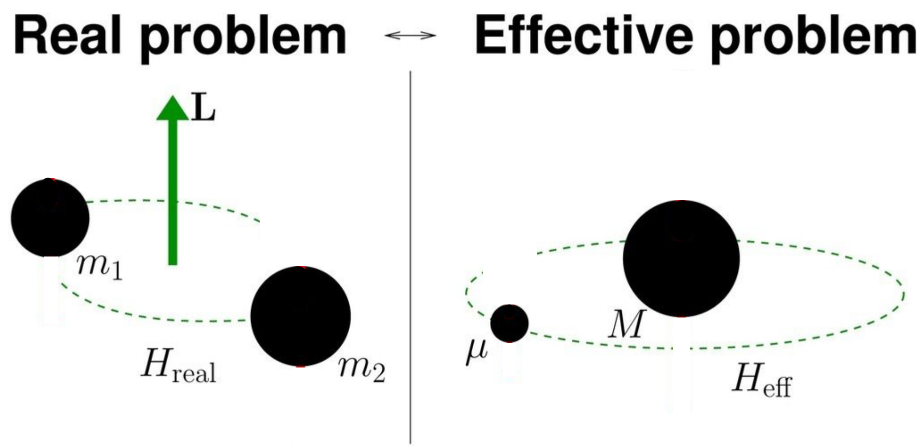
$$\frac{dE}{dt} \propto -\frac{\omega^2}{8\pi} \sum_{l,m} m^2 \left| \frac{\mathcal{R}}{M} h_{lm}(t) \right|^2$$

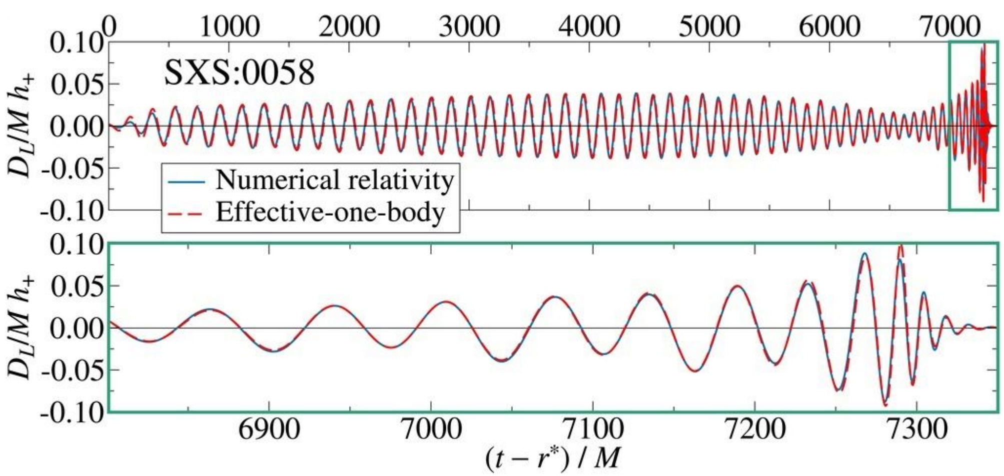
- Waveform multipoles are factorized:

$$h_{lm} = h^A_{lm}\, h^B_{lm}\, T_{lm} e^{i\delta_{lm}}\, c_{lm}\, N_{lm}$$

where, all but the last factor are **re-summed**

- Several free parameters that are calibrated to NR

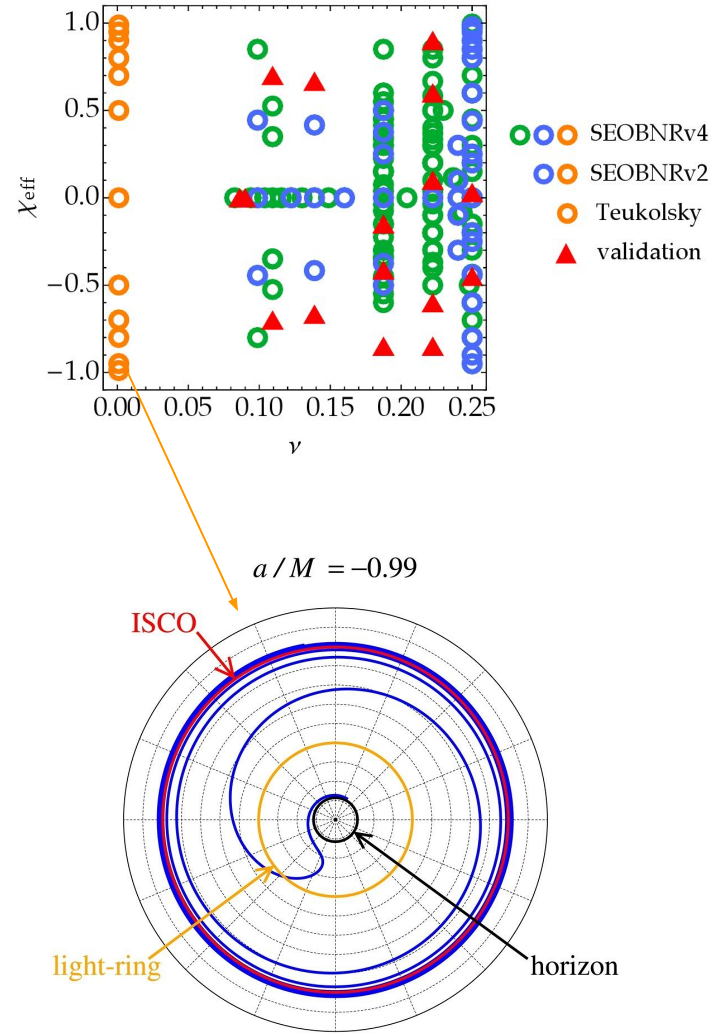




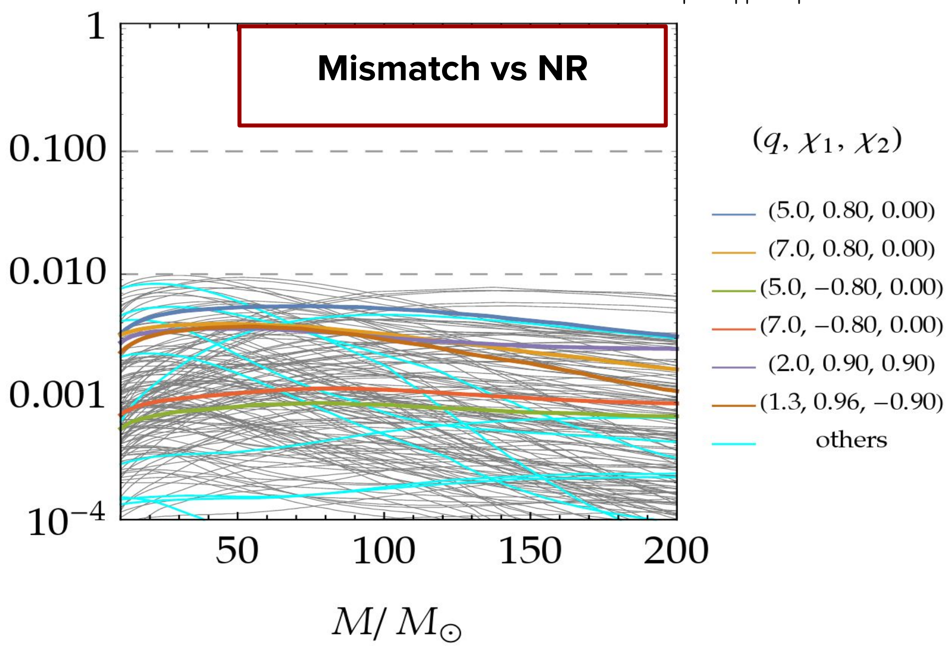
Buonanno et al (1998), gr-qc/9811091; Pan et al (2011), 1106.1021; Image credit: A. Taracchini/AEI

2c. Source modeling: Effective one body (EOB)

- Cutting-edge: EOBv4
- Calibrated to 141 NR + 10 numerical Teukolsky waveforms
- Agreement with NR to *better than 1%*

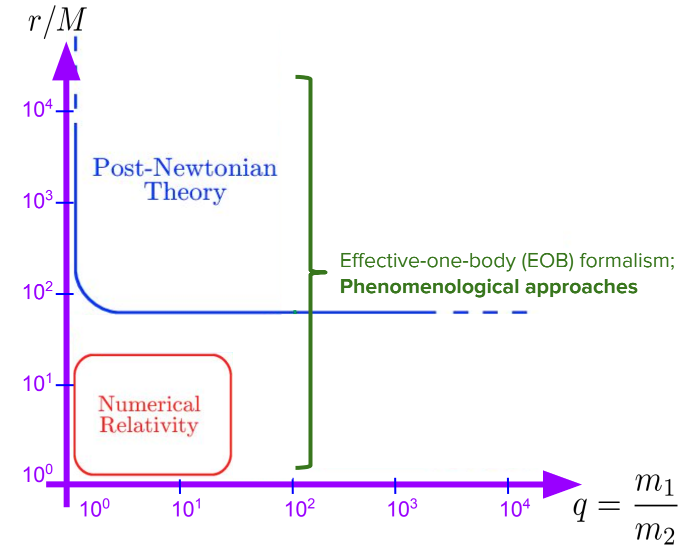


$$\text{Mismatch}(h_1, h_2) = 1 - \frac{\langle h_1, h_2 \rangle}{|h_1||h_2|}$$



Bohe, ..,PK et al (2016), 1611.03703; Image credit: A. Taracchini/AEI

2d. Source modeling: Phenomenological approaches



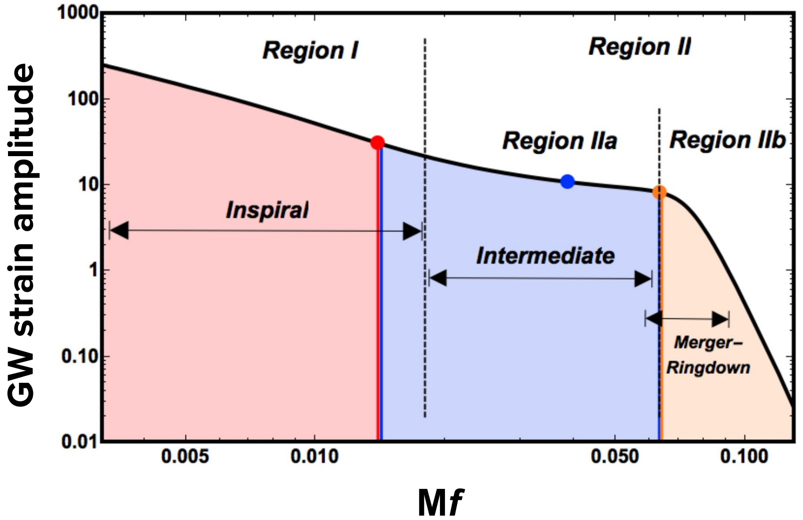
- Guided by need to reduce computational cost of waveform generation, closed-form GW strain models in frequency-domain were developed
- PN-inspired ansatz is taken for amplitude/phase, and PN/EOB+NR hybrid waveforms are used to calibrate the ansatz

$$A_{\rm Int} = A_0 \left(\delta_0 + \delta_1 f + \delta_2 f^2 + \delta_3 f^3 + \delta_4 f^4\right)$$

$$\phi_{\rm Int} = \frac{1}{\eta}\left(\beta_0 + \beta_1 f + \beta_2 \,{\rm Log}(f) - \frac{\beta_3}{3} f^{-3}\right)$$

$$\frac{A_{\rm MR}}{A_0} = \gamma_1 \frac{\gamma_3 f_{\rm damp}}{(f - f_{\rm RD})^2 + (\gamma_3 f_{\rm damp})^2} e^{-\frac{\gamma_2 (f - f_{\rm RD})}{\gamma_3 f_{\rm damp}}}$$

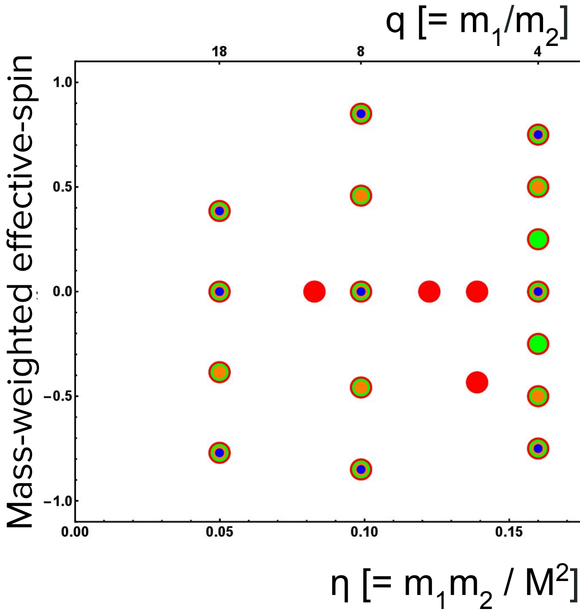
$$\phi_{\rm MR} = \frac{1}{\eta}\left\{\alpha_0 + \alpha_1 f - \alpha_2 f^{-1} + \frac{4}{3}\alpha_3 f^{3/4} + \alpha_4 \tan^{-1}\left(\frac{f - \alpha_5 f_{\rm RD}}{f_{\rm damp}}\right)\right\}.$$

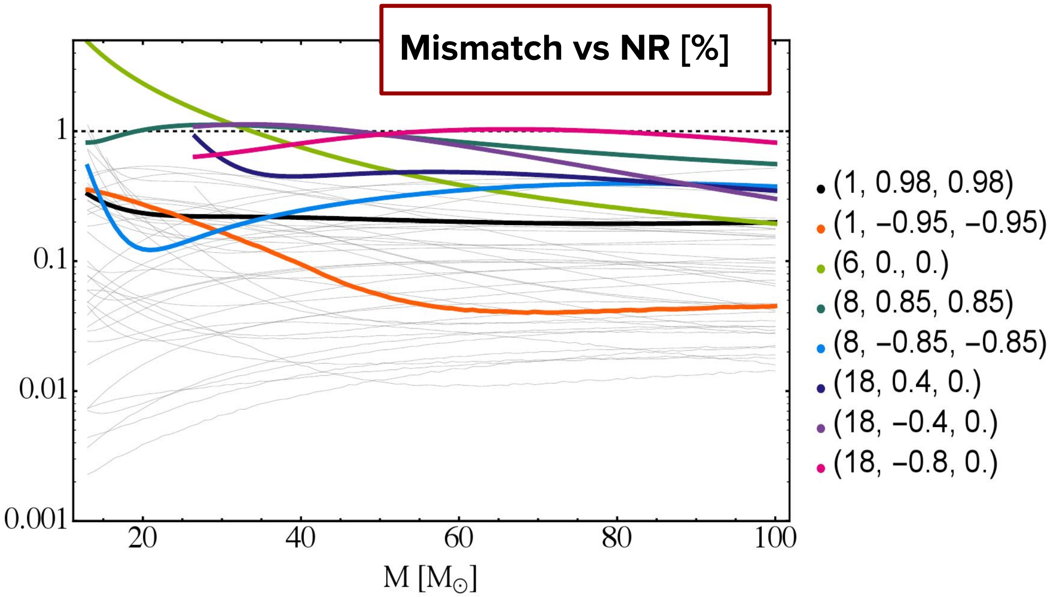


Khan et al (2015), 1508.07253; Santamaria et al (2010), 1005.3306; Ajith et al (2007), 0704.3764/0710.2335; Ajith et al (2009), 0909.2867;

2d. Source modeling: Phenomenological approaches

- Guided by need to reduce computational cost of waveform generation, closed-form GW strain models in frequency-domain were developed
- PN-inspired ansatz is taken for amplitude/phase, and PN/EOB+NR hybrid waveforms are used to calibrate the ansatz
- State of art - PhenomD (version 4)

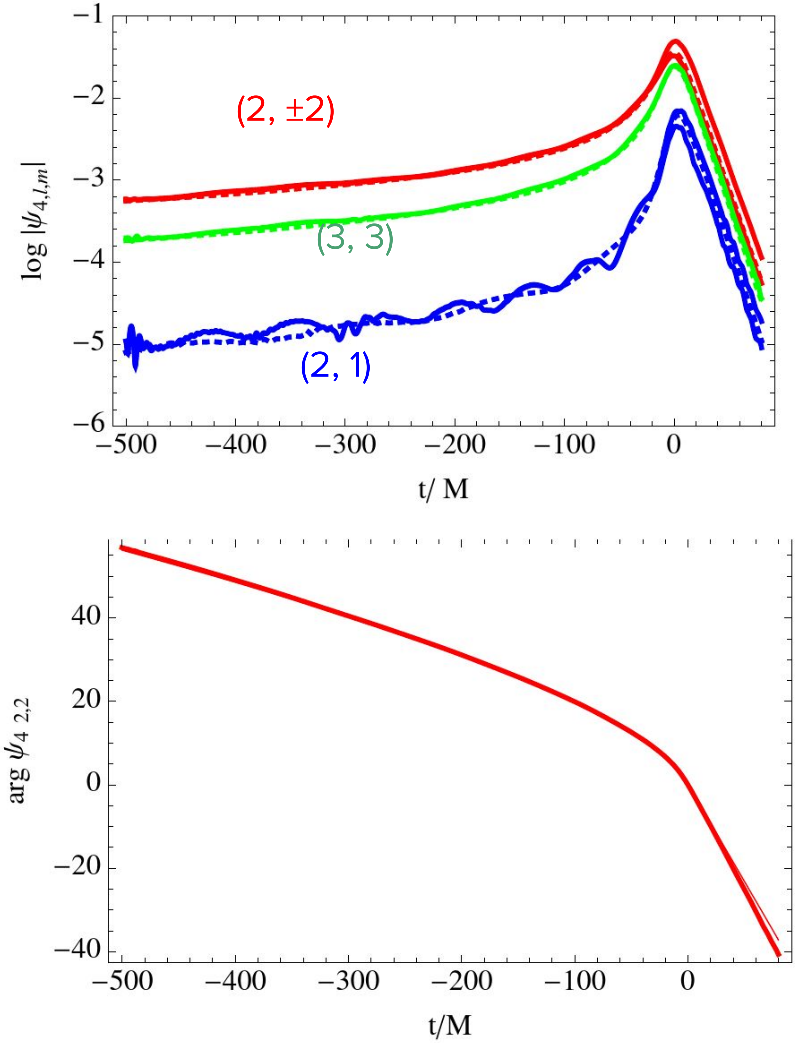




Khan et al (2015), 1508.07253; Santamaria et al (2010), 1005.3306; Ajith et al (2007), 0704.3764/0710.2335; Ajith et al (2009), 0909.2867;

2e. Source modeling: Including spin-induced orbital precession

- So far source models included restricted spin description, with both BH spins (anti)parallel to orbital ang. momentum
- Dominant GW emission directions are ⊥ to the plane of the binary. In a coordinate system aligned with that direction (**QA**), most of the signal power resides in the (l = 2, |m| = 2) spin-weighted spherical harmonics
- ***Schmidt et al <u>identified</u> both GW mode amplitude (in 2010) and phasing (in 2012) of QA-frame waveforms for precessing binaries <u>with</u> equivalent non-precessing-binary waveforms!***
- Application - generic Phenom / EOB models developed by applying a time-dependent rotation to non-precessing binary waveforms.



Schmidt et al (2010), 1012.2879; Boyle et al (2011), 1110.2965; Schmidt et al (2012), 1207.3088; Pekowsky et al (2013), 1304.3176;

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3a. LIGO PE: Model requirements

A. Model accuracy up to reqd. SNR:

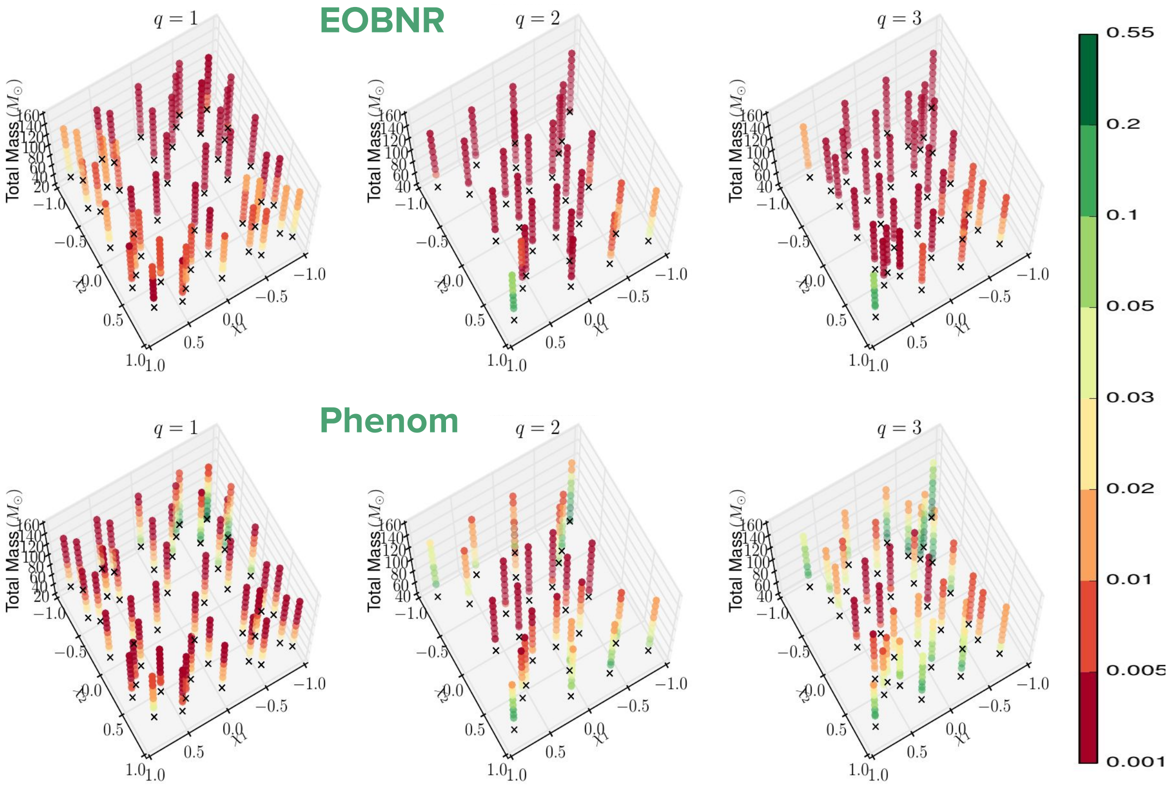
SNR	10	100	1000
Mismatch (vs NR)	< 2 %	< 0.02%	< 2x10^{-4} %

B. Low cost of generation:

a. EOB is expensive - O(10-10^2 s)

$\Rightarrow$ time for PE with 10^7 evaluations ~ *3 - 30 years*!

b. Phenom is inexpensive, but can be less reliable

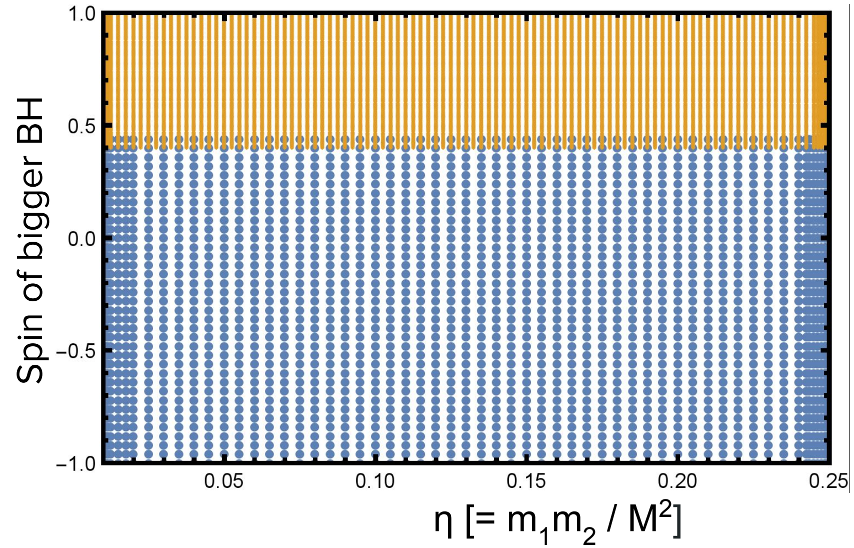


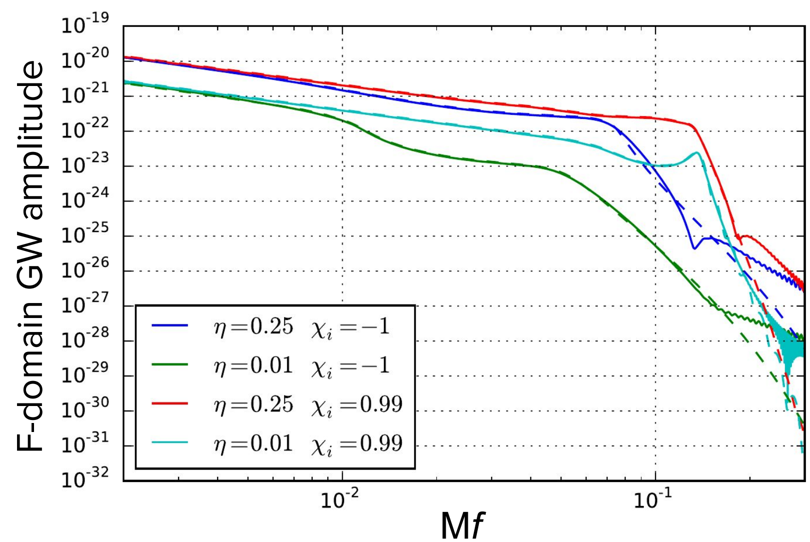
*Lindblom et al (2008), 0809.3844; PK et al (2016), 1601.05396;

3a. LIGO PE: Reduced-order modeling

- ROM is a technique to create surrogate models for computationally expensive waveform models:
  - Define a region of parameter space
  - Compute a basis for GW amplitude & phase
  - Compute projection coefficients for a dense set of training waveforms & interpolate them
  - Store spline-interpolation coefficients on disk

- Evaluation of ROM is straightforward:
  - Read in spline coefficients
  - Evaluate splines at required parameter values
  - Combine with basis vectors to generate GW templates





Purrer et al (2014), 1402.4146; Purrer et al (2015), 1512.02248;

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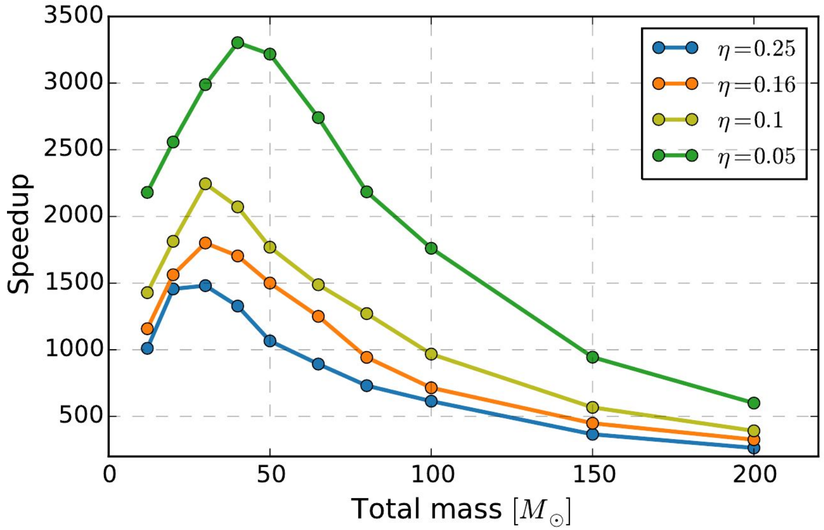
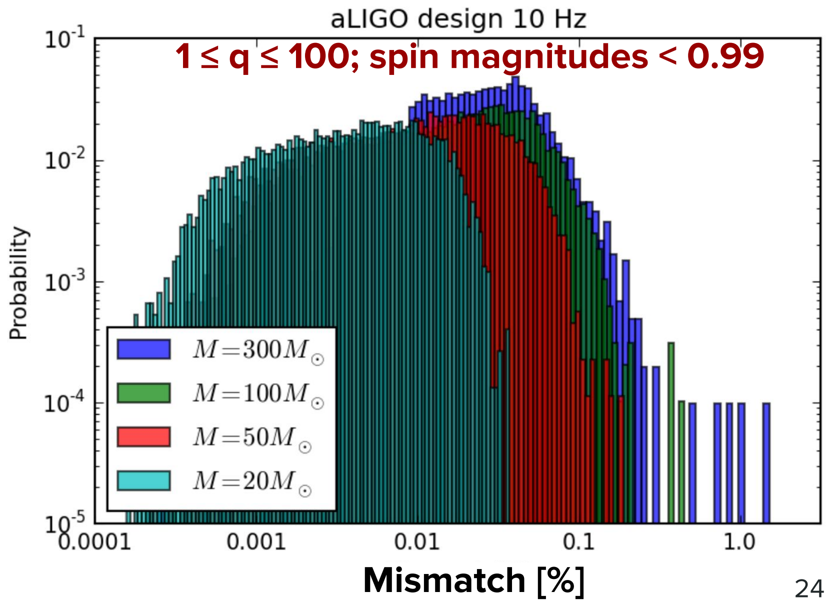
- **Evaluation in polynomial time (esp. EOB)**
- **Marginal loss in accuracy**



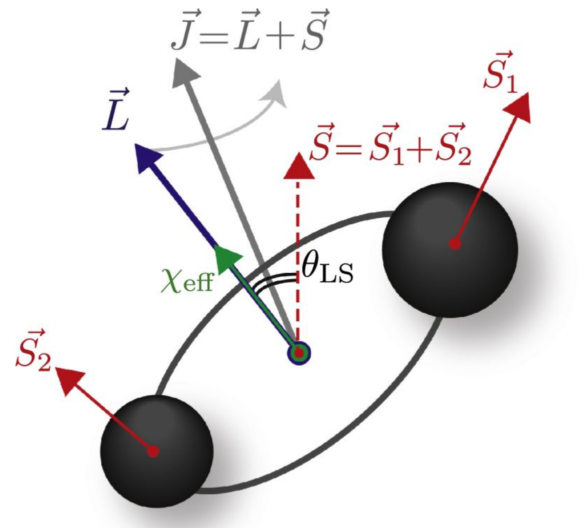


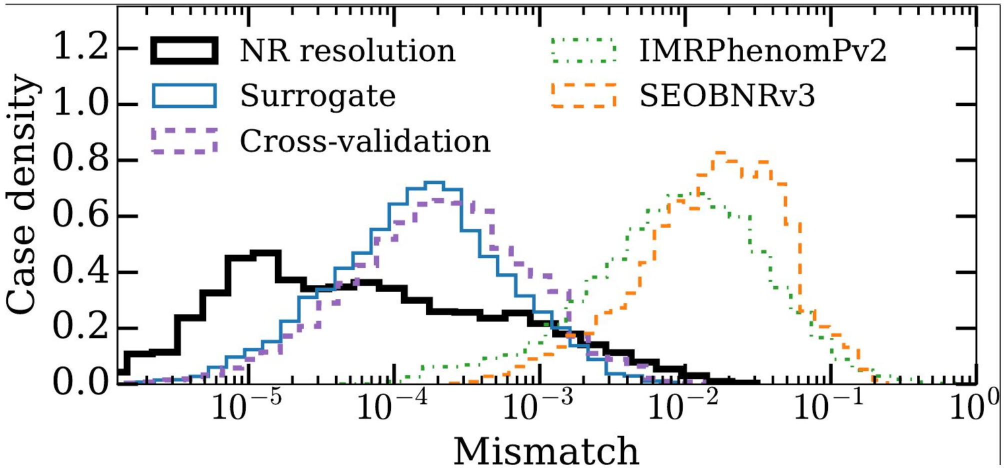
Purrer et al (2014), 1402.4146; Purrer et al (2015), 1512.02248;

3a. LIGO PE: ROM of NR waveforms

- Time-domain surrogates have been built for NR waveforms directly. Used 744 NR simulations:
    - Full-precession; l <= 4 modes
    - **1 ≤ q ≤ 2; spin magnitudes < 0.8**
    - **Length ≈ 4500M**

➢ **Direct application of NR to PE**
➢ **Valuable tool for waveform modeling**





Blackman et al (2017), 1705.07089;
Blackman et al (2017), 1701.00550;
Figure credit: Rodriguez et al 2016 ApJL 832 L2

3a. LIGO PE: Reduced-order Quadrature

- Interpolate portions of Bayesian likelihood directly

$$p(\theta|d) = p(d|\theta)\frac{p(\theta)}{p(d)}$$

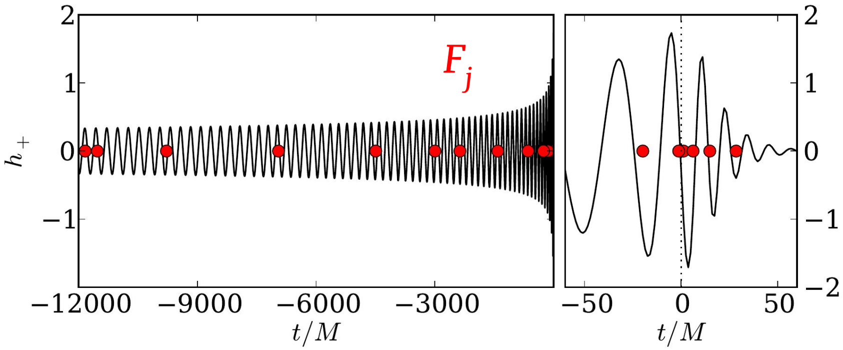
$$\log\mathcal{L} \simeq -2\mathcal{R}\left[\Delta f \sum_{i=1}^{L} \frac{d^*(f_i)h(f_i)}{S_n(f_i)}\right]$$

Canizares et al (2014), 1404.6284;
Field et al (2013), 1308.3565;

3a. LIGO PE: Reduced-order Quadrature

- Interpolate in frequency along-with parameters λ

$$h(f;\lambda) = \sum_{i=1}^{m} c_i(\lambda) e_i(f)$$



- Choose m points in frequency, such that waveform specified there alone can be used to interpolate the whole

$$h(f;\lambda) = \sum_{i=1}^{m} e_i(f) \left(\sum_{j=1}^{m} [e_l(F_k)]^{-1}_{ij} h(F_j;\lambda) \right)$$

$$h(f;\lambda) = \sum_{j=1}^{m} B_j(f)\, h(F_j;\lambda)$$

pre-computed

Canizares et al (2014), 1404.6284;
Field et al (2013), 1308.3565;

3a. LIGO PE: Reduced-order Quadrature

- Simplify computation of Bayesian likelihood:

$$\log \mathcal{L} \simeq -2\mathcal{R}\left[\Delta f \sum_{i=1}^{L} \frac{d^*(f_i)h(f_i)}{S_n(f_i)}\right]$$

using:

$$h(f;\lambda) = \sum_{j=1}^{m} B_j(f)\, h(F_j;\lambda)$$

$$\log \mathcal{L} \simeq \sum_{k=1}^{m} \omega_k\, h(F_k;\lambda)$$

Only λ dependence

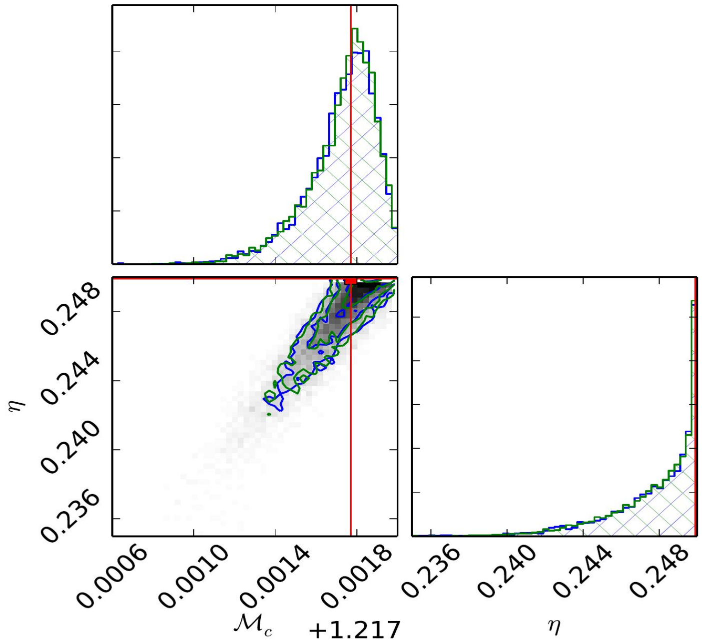
pre-computed

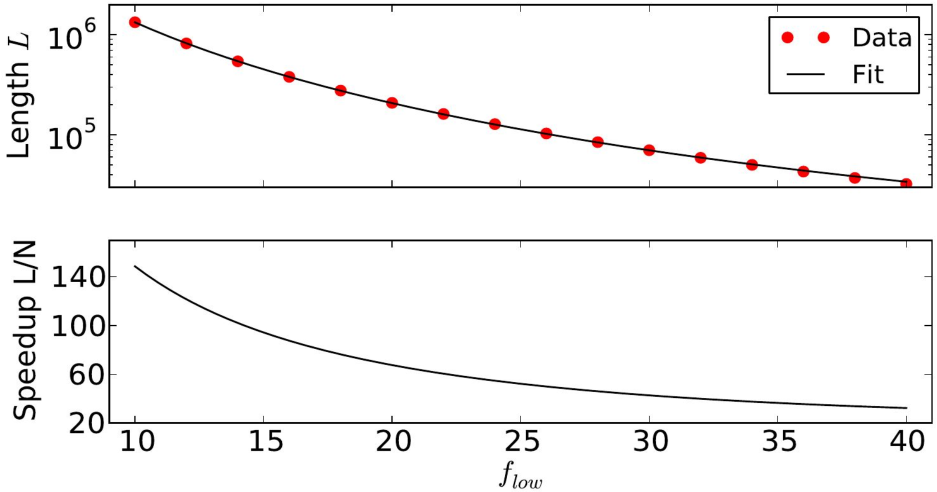
⇒ Instead of L evaluations of h(f), we get away with a much smaller number m

Canizares et al (2014), 1404.6284;
Field et al (2013), 1308.3565;

3a. LIGO PE: Reduced-order Quadrature

- Benefits:
  - Increases speed of PE x 10^2
    [*net speed up for time-domain models, is* 10^4 *–* 10^5 x]
  - **Time for PE reduced from O(weeks) → O(hours)!**
- Limitations:
  - Need h(f) in closed form
  - Sensitive to detector PSD
  - *Extension to LISA PE not straightforward:- will need inclusion of inclination angle, sky angles & detector location in orbit within λ*





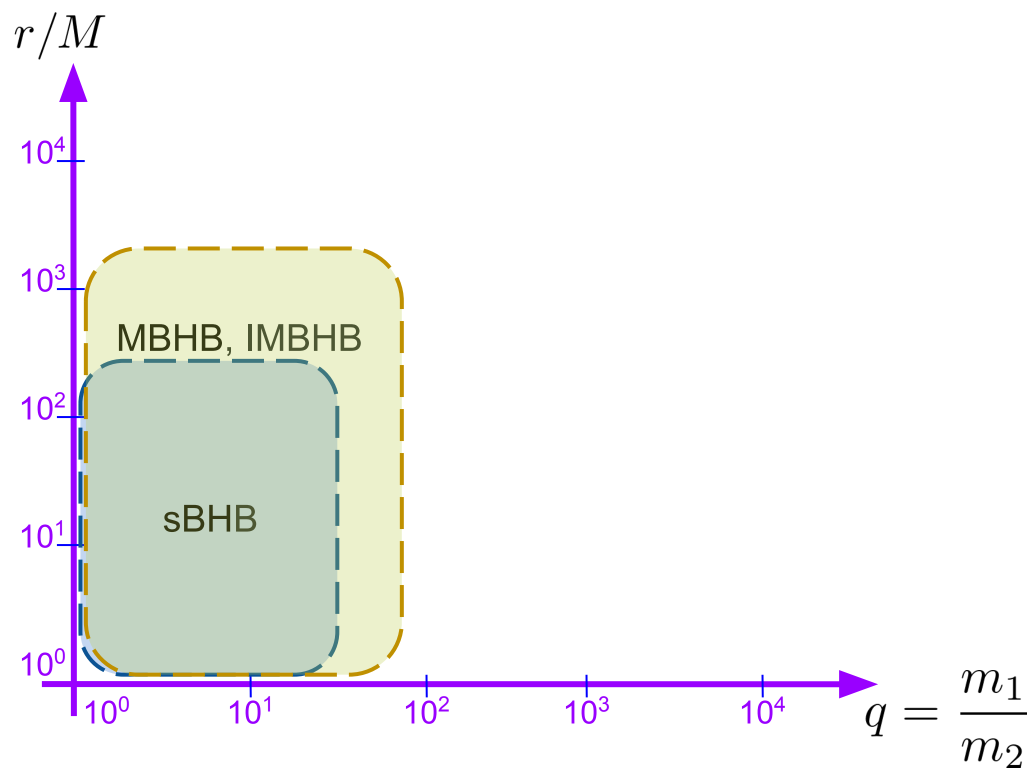
Canizares et al (2014), 1404.6284;
Field et al (2013), 1308.3565;

Outline

1. LIGO sources
2. Source modeling:
    a. PN theory
    b. NR (very brief)
    c. EOB formalism
    d. Phenomenological models
    e. Simplification of spin-precession
3. Application to LIGO parameter estimation:
    a. PE requirements
    b. Model-order reduction
    c. Reduced-order Quadrature
4. LISA sources
    a. MBH
    b. EMRI & IMRIs
5. Summary

4. LISA Sources: MBHB, IMBHB

- Binaries of 10^4 - 10^7 $M_\odot$ BHs
- Cleanest sources, SNRs 10^2 - 10^3 & higher



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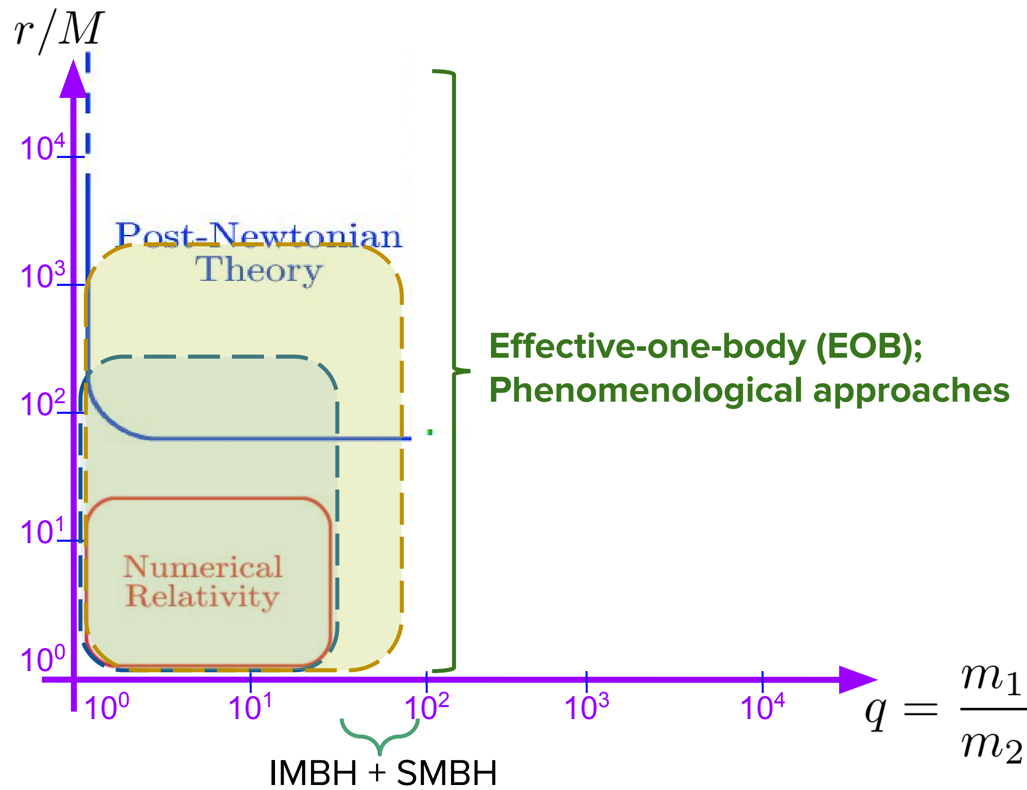
- Two primary challenges:
  - Better accuracy!
  - Include eccentricity alongwith spins for q → 100



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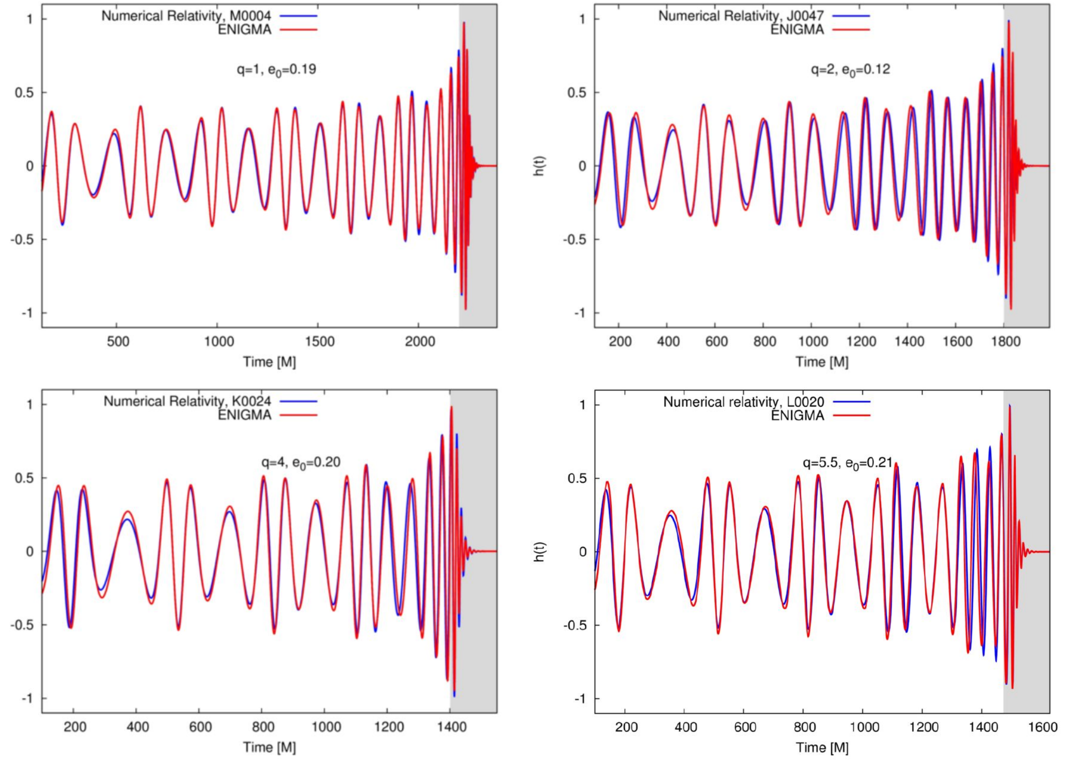
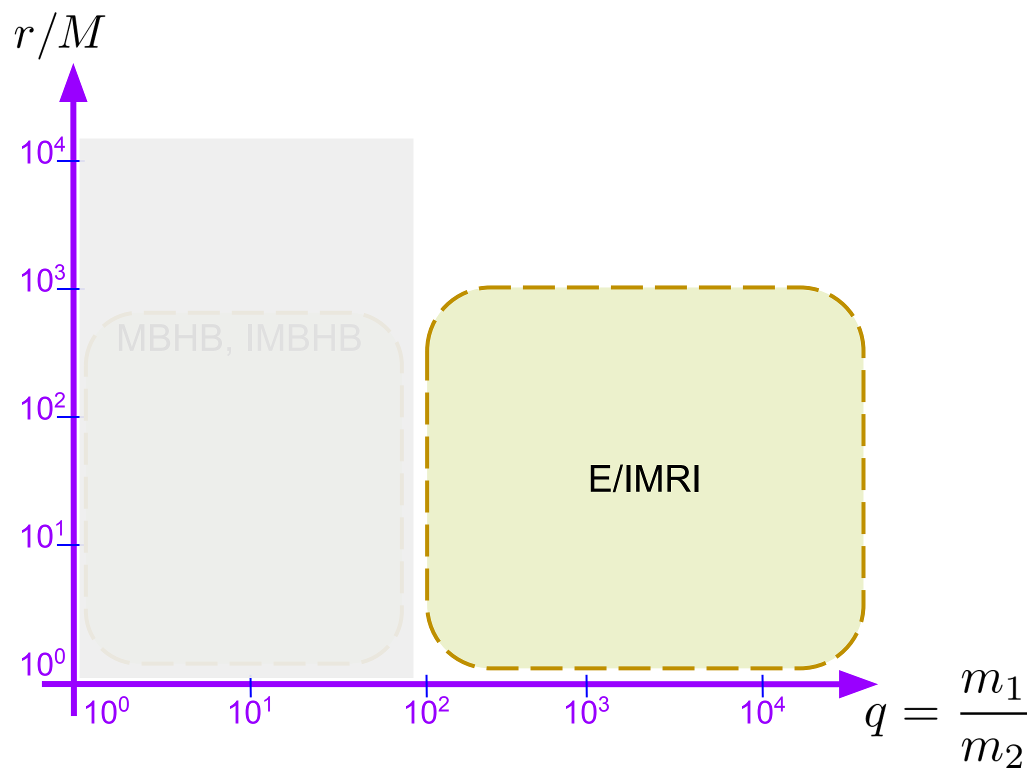
- Cleanest sources, SNRs 10^2 - 10^3 & higher
- Much of LIGO's source modeling carries forward
- Two primary challenges:
    - Better accuracy!
    - Include eccentricity alongwith spins for q → 100
        - Some progress in IMR eccentric modeling (non-spin)
        - Need to extend to high-q & combine with spin effects



Huerta, Moore, PK et al (2017), 1711.06276;

4. LISA Sources: EMRI, IMRI

- EMRIs will be seen by LISA at SNRs 20 - 100s, few every year!



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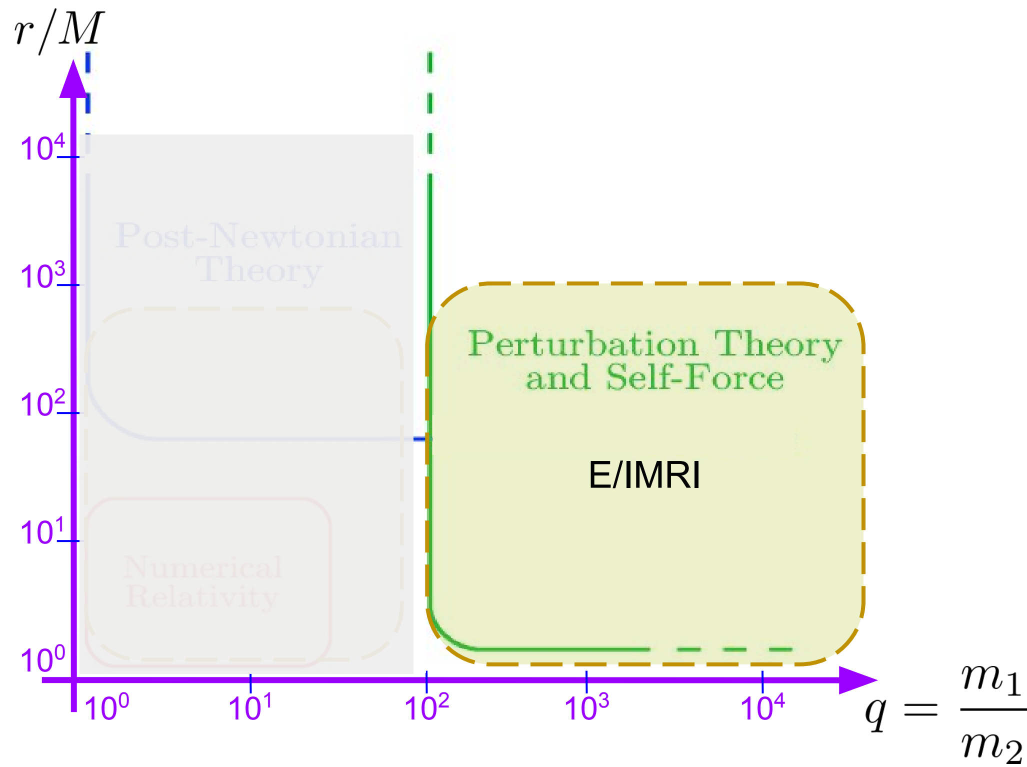

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- Kludge waveforms are available for EMRIs:
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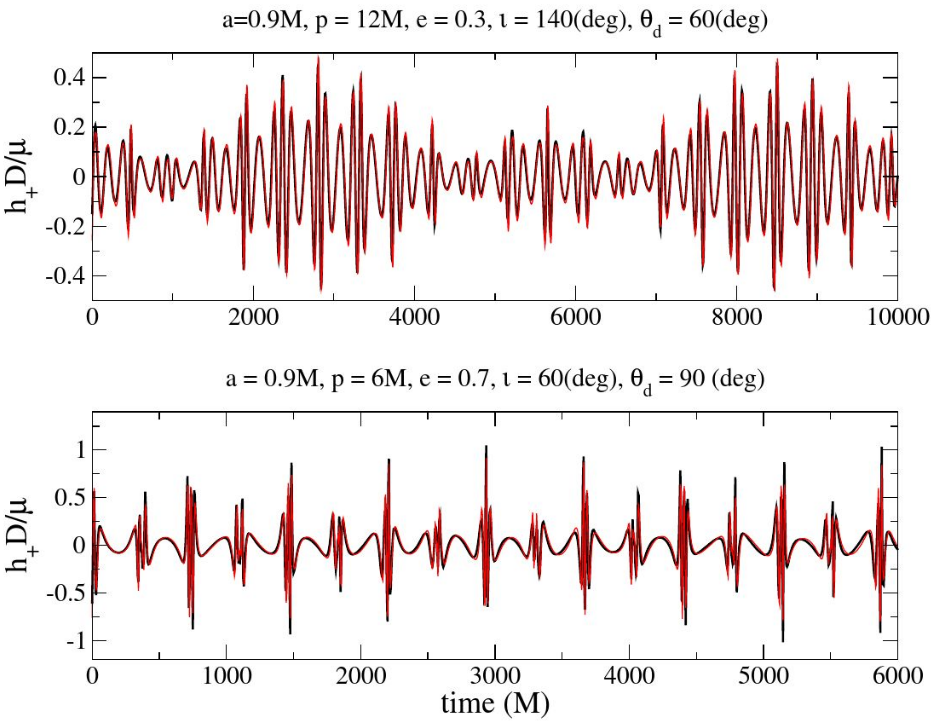
  - Agree with numerical Teukolsky codes



Babak et al (2006), gr-qc/0607007; Gair et al (2005), gr-qc/0510129; Barack et al (2004), gr-qc/0310125;

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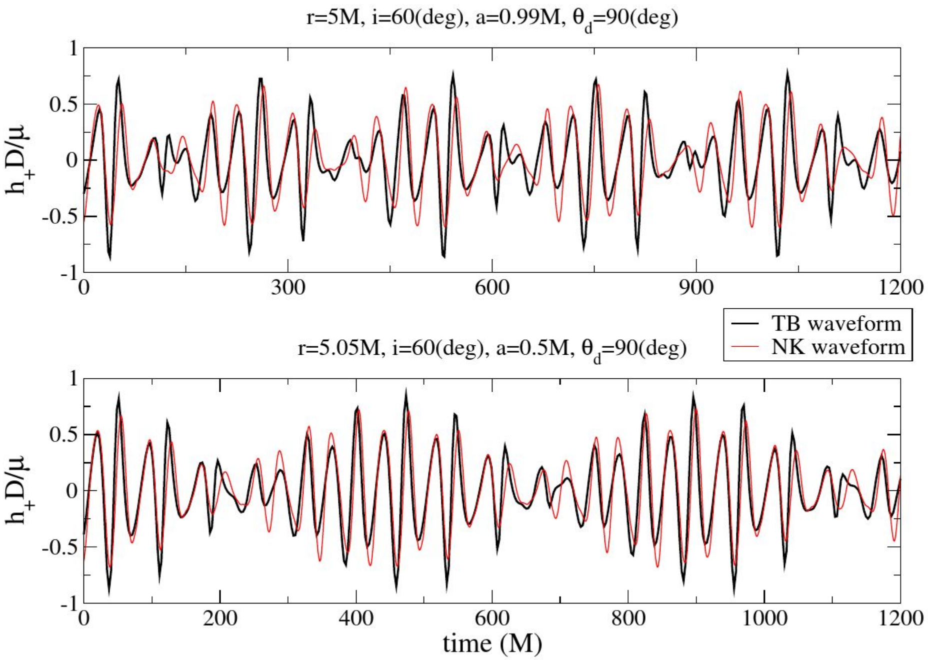
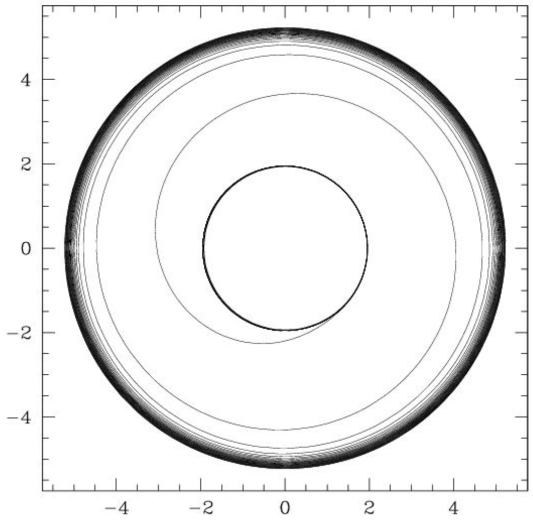
- Kludge waveforms are available for EMRIs:
    - Inexpensive and tested for PE (under idealized conditions)
    - Agree with numerical Teukolsky codes ... up until r ~ 5M
    - Sufficient for detection and maybe even PE - e.g. O(10^{-2}) accuracy if not O(10^{-4})

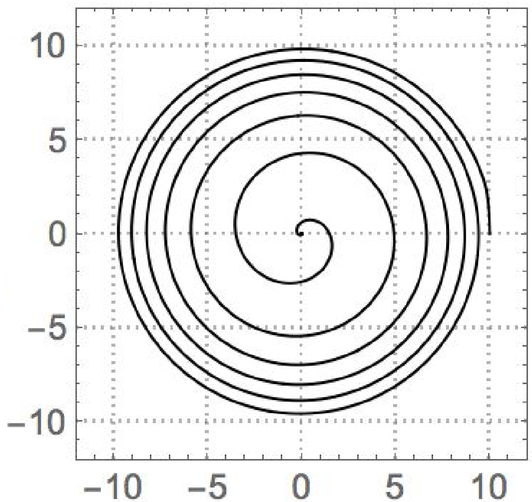


Babak et al (2006), gr-qc/0607007; Gair et al (2005), gr-qc/0510129; Barack et al (2004), gr-qc/0310125;

4. LISA Sources: EMRI, IMRI

- EMRIs will be seen by LISA at SNRs 20 - 100s, few every year!
- Kludge waveforms are available for EMRIs
- Need waveforms from self-force program to:
    - Calibrate *kludge* models, extend to IMRIs
    - Validate models and compare to observed signals
    - Test GR / no-hair theorem: for precision tests, models need to track GW emission to better than O(1) cycle over thousands





Babak et al (2006), gr-qc/0607007; Gair et al (2005), gr-qc/0510129; Barack et al (2004), gr-qc/0310125; Image credit: arXiv:1003.0485 & einsteintoolkit.org;

5. Summary

- For LISA observations of MBHBs with q ~ O(10):
  - EOB / Phenom are state of art for circular IMR with high-order spin effects. Work to SNRs of 40-50.

- For MBHBs with q ~ O(100):
  - EOB incorporates information from test-particle limit. Best model at present for binaries in quasi-circular orbits. Will need further calibration/validation against NR.

- IMBH + MBH will have (A) higher mass-ratios, & (B) non-negligible residual eccentricity, simultaneously. Need further development.

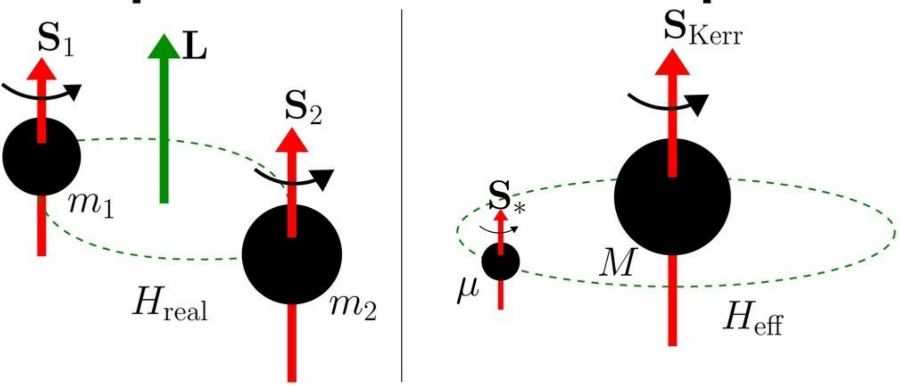
- LISA can record SNRs ~ O(10-10^2). *Need overall better accuracy in all above approaches.*

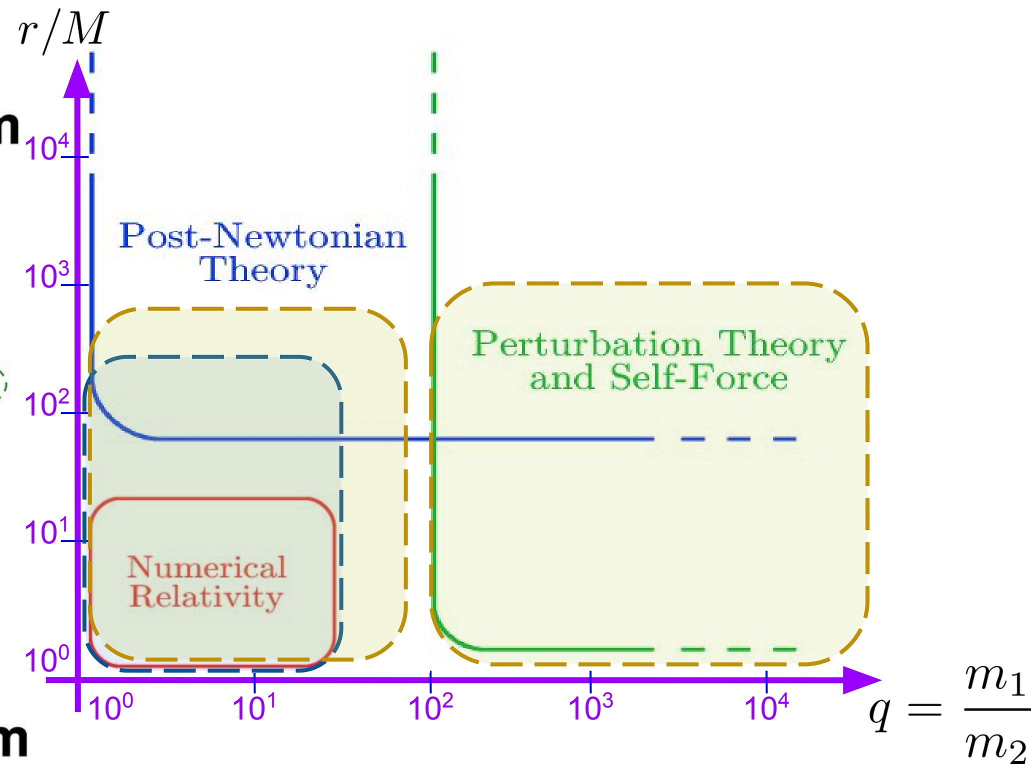
- EMRI and IMRIs need waveforms from self-force program, for calibration and validation of models, especially for precision tests of General Relativity, no-hair theorem, etc. Kludge models may suffice for detection.

- PE computational challenges can be *aided* with reduced-order modeling of expensive source models / reduced-order quadrature rules for Bayesian inferencing. Need development.

Extras

Real problem ↔ Effective problem





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