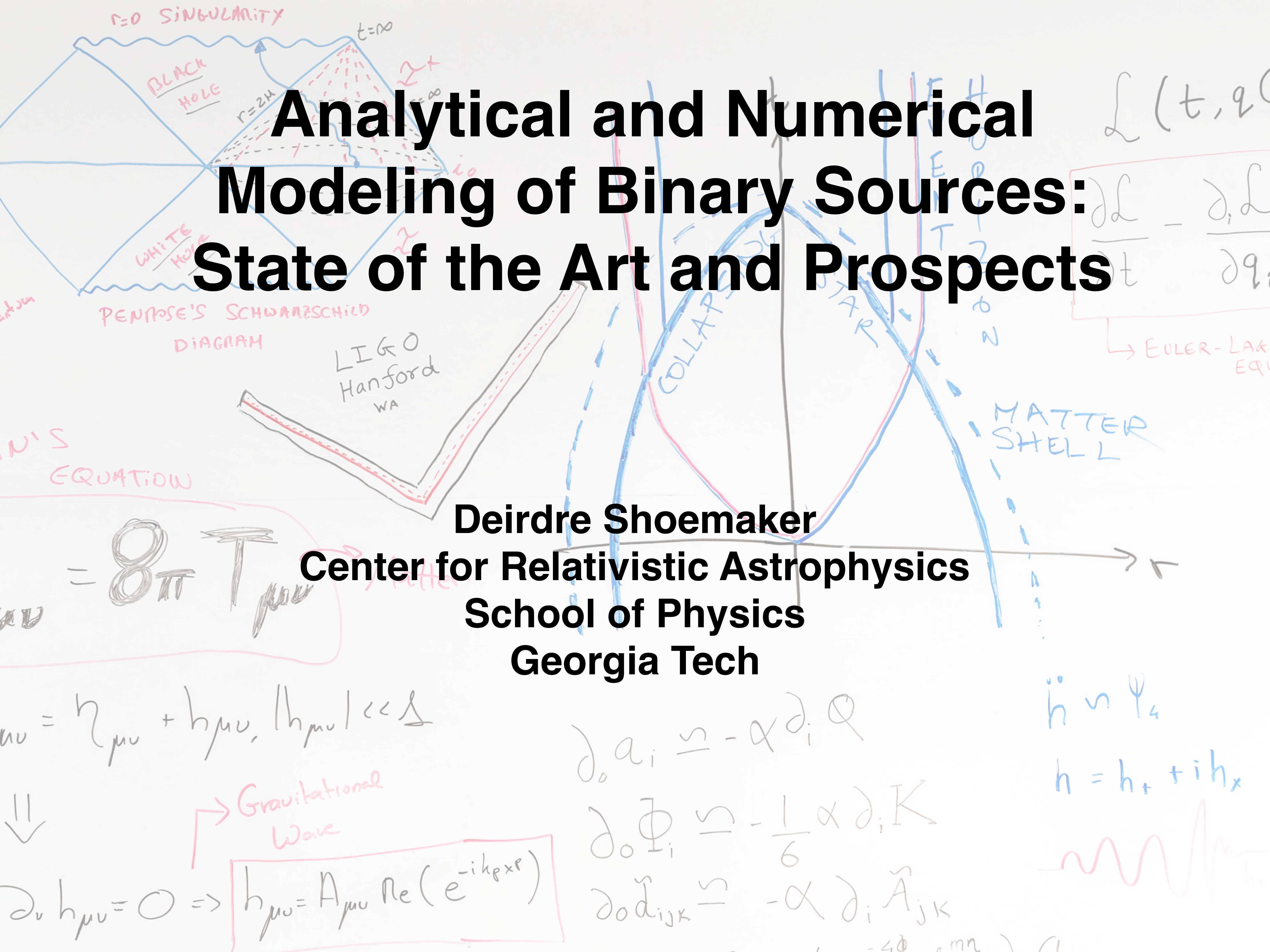


Analytical and Numerical Modeling of Binary Sources: State of the Art and Prospects

Deirdre Shoemaker
Center for Relativistic Astrophysics
School of Physics
Georgia Tech

Binary Sources

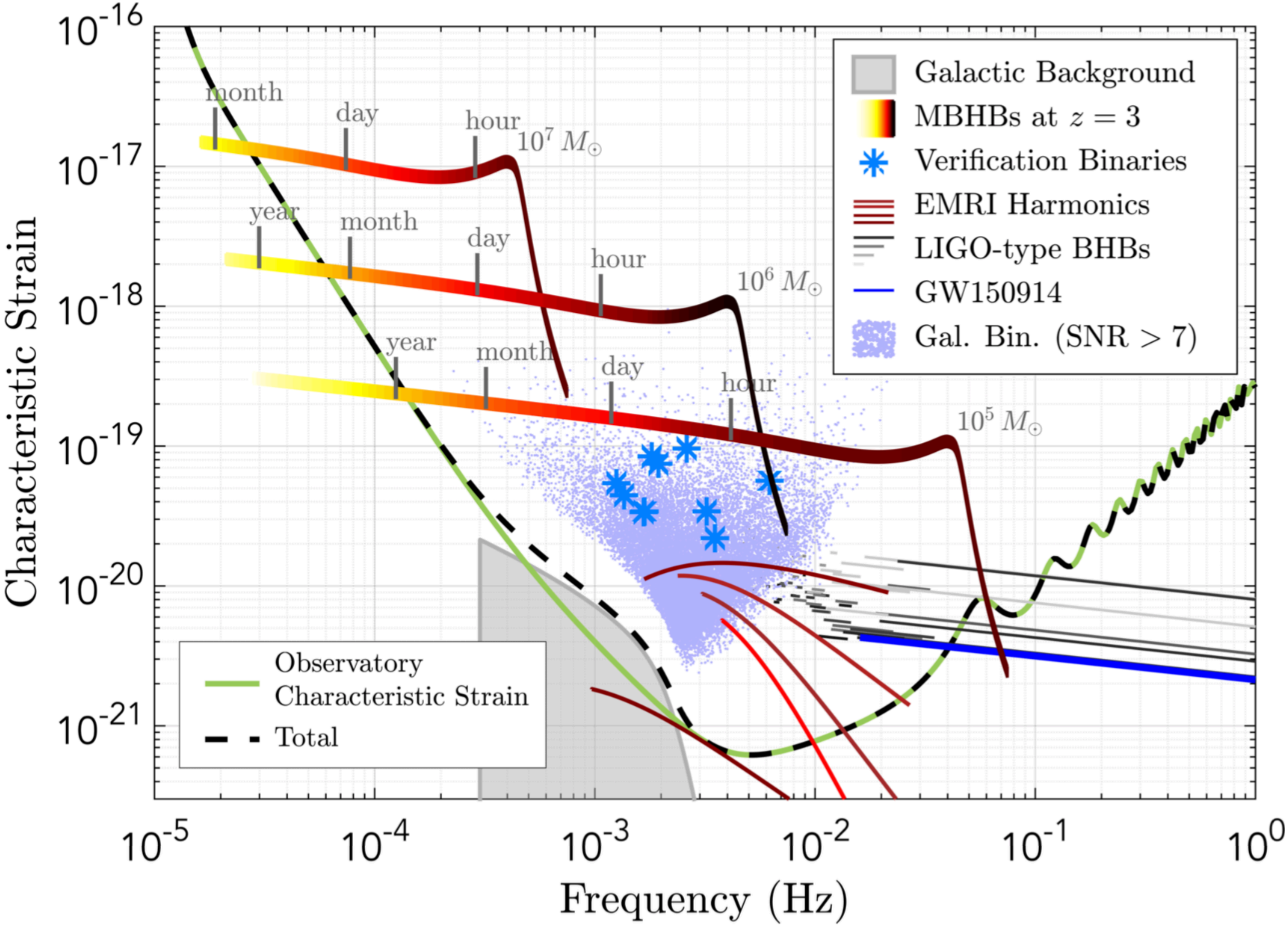


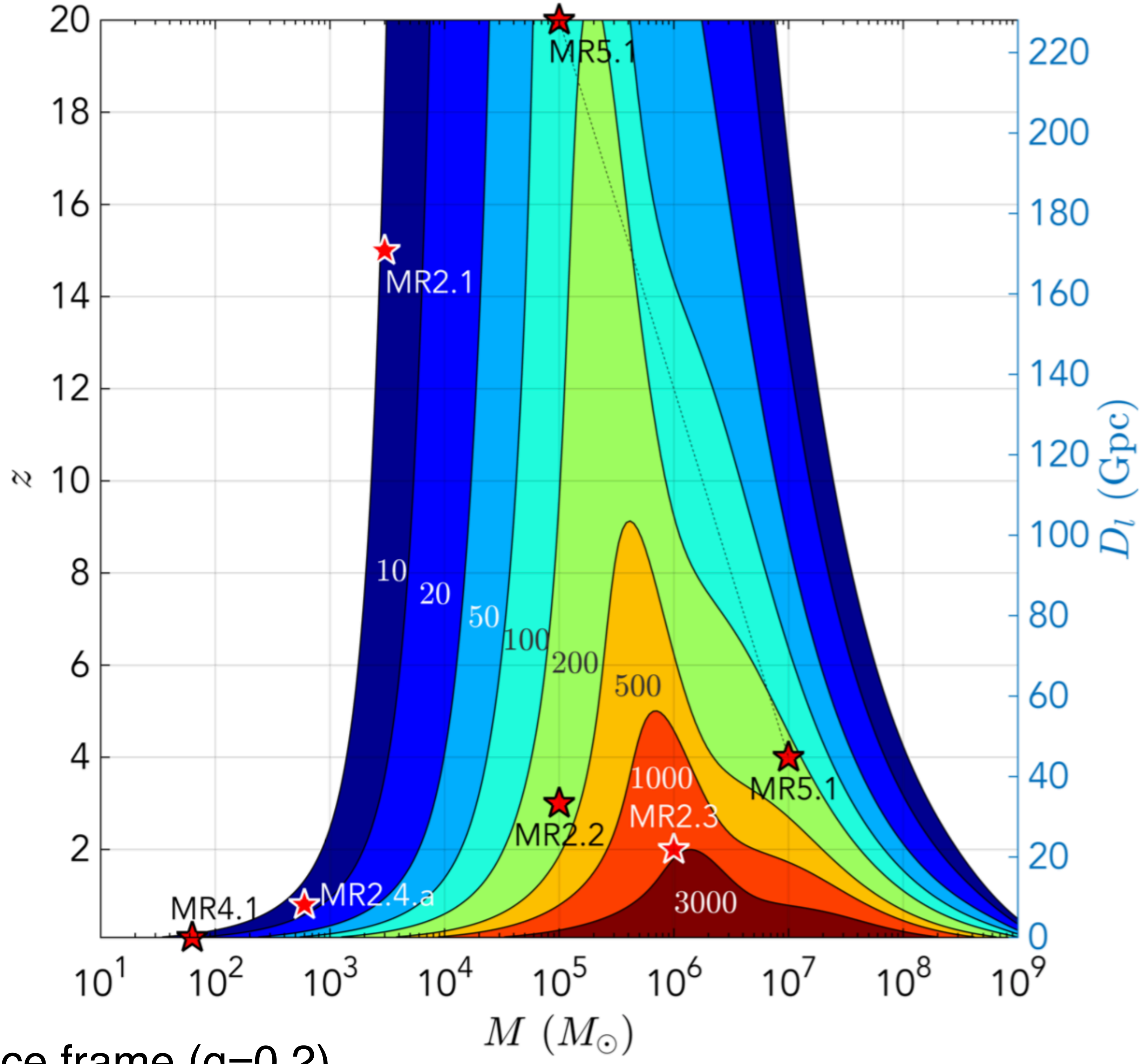
Figure 1 of LISA L3 Document

Binary Source Objectives

10^3 - 10^7 M_{solar}

- SO2 (Science Objective)Trace the origin, growth and merger history of massive black holes across cosmic ages
- SI2.2 (Science Investigation) Study growth mechanisms of MBHs from earliest quasars - includes measuring dimensionless spin parameters to 0.1 and misalignment of spin to 10 degrees
- SI2.3 Observe EM counterparts to merging BBHs (requirement on localization)
- SI2.4 Test existence of IMBH (masses to 30% from inspiral)
- SO.4 Origins of stellar mass BHs (multi-band GW astro)
- SI5.1 use ringdown to from merging BBHs to test if post-merger BHs are GR (more than one mode)

Contours of constant SNR



in source frame (q=0.2)
Stars are threshold binaries

Figure 3 of LISA L3 Document

Some Info on Binary Sources

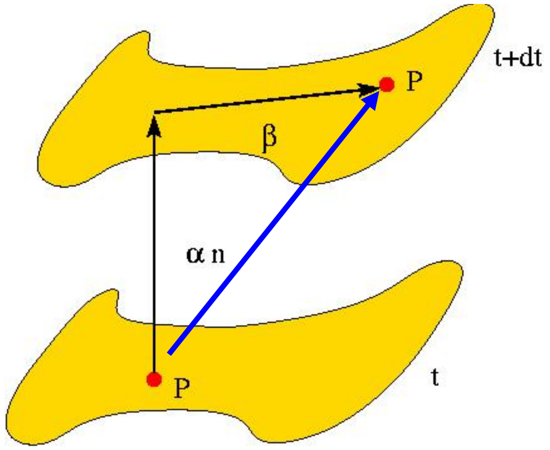
- BBH of 10^3 - 10^7 M_{solar}
- Luminous in GWs: SMBHBs 10^{26} L_{solar} (compared to a SN at 10^{14} L_{solar})
- Event rate of ~100/yr
- Parameters (17 d) measured
  Component masses to <1% error
  luminosity distance 1 - 50%
  time of merger within minutes

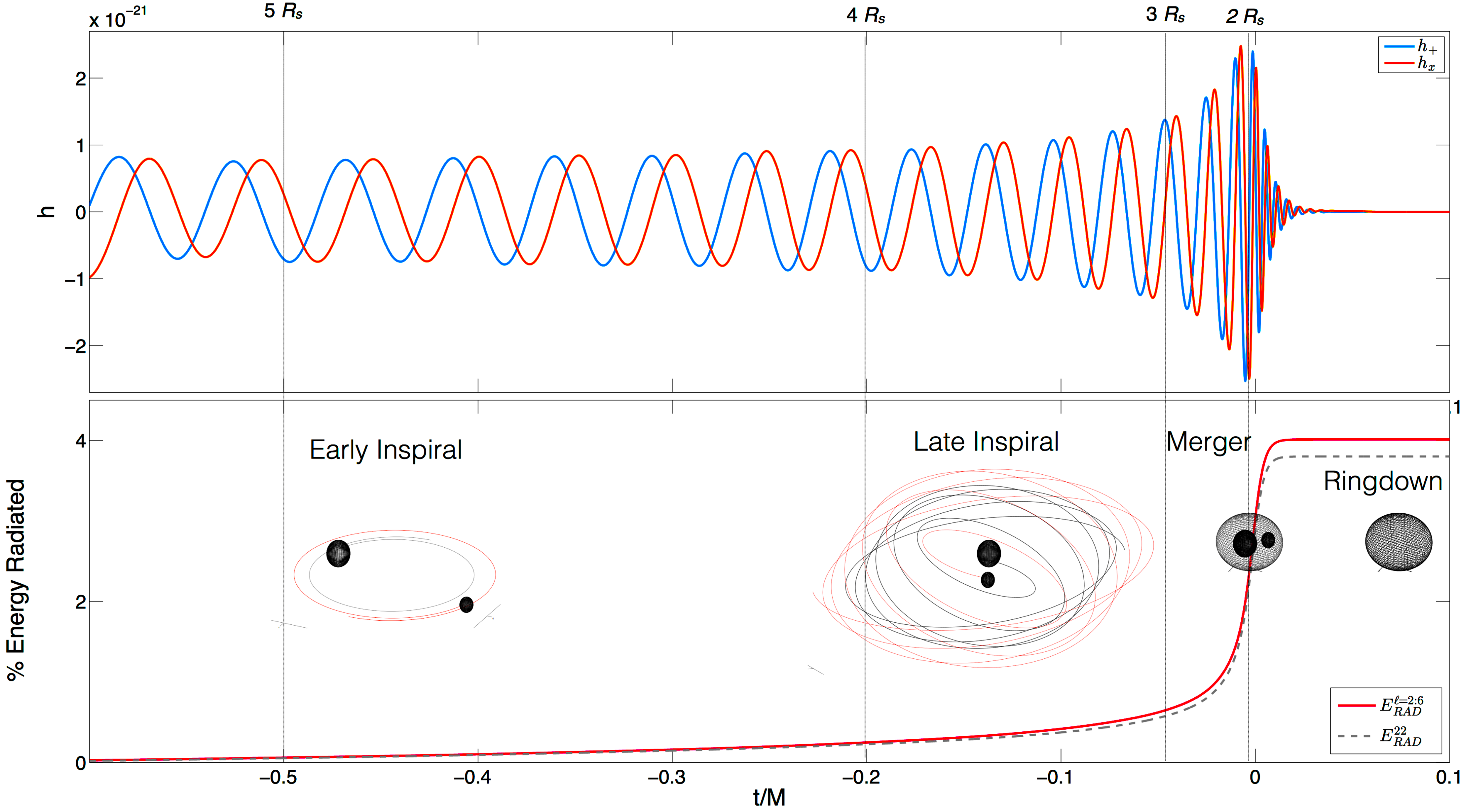
See E. Porter 2014

Einstein's Equations of General Relativity

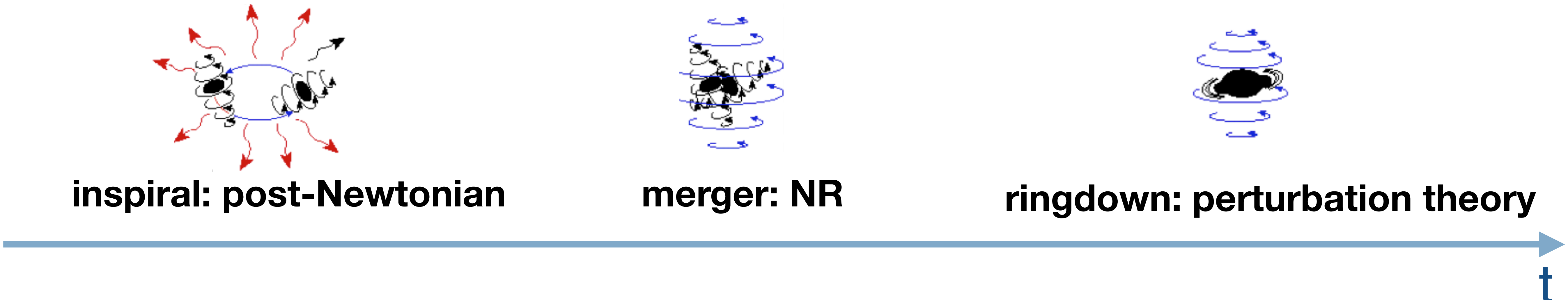
$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$G_{\mu\nu} \propto F(\partial_\alpha \partial_\beta g_{\mu\nu}, \partial_\alpha g_{\mu\nu} \partial_\beta g_{\lambda\nu}, \ldots)$$





Anatomy of a Black Hole Binary

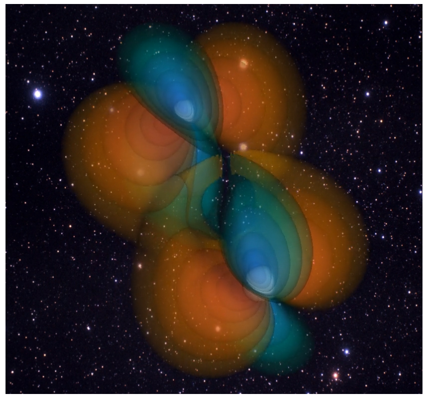


inspiral: post-Newtonian

merger: NR

ringdown: perturbation theory

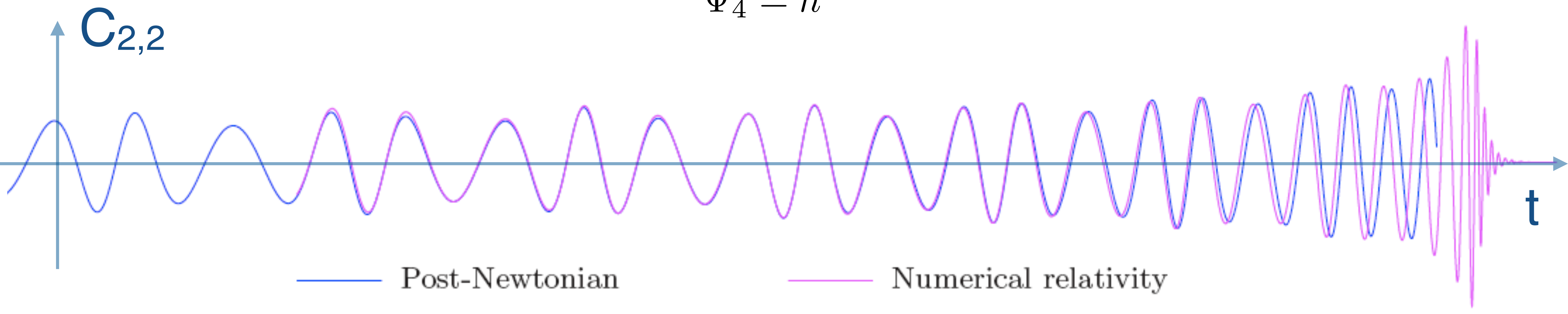
total mass, mass ratio, angular momentum, individual spins, eccentricity...



final mass and spin vector

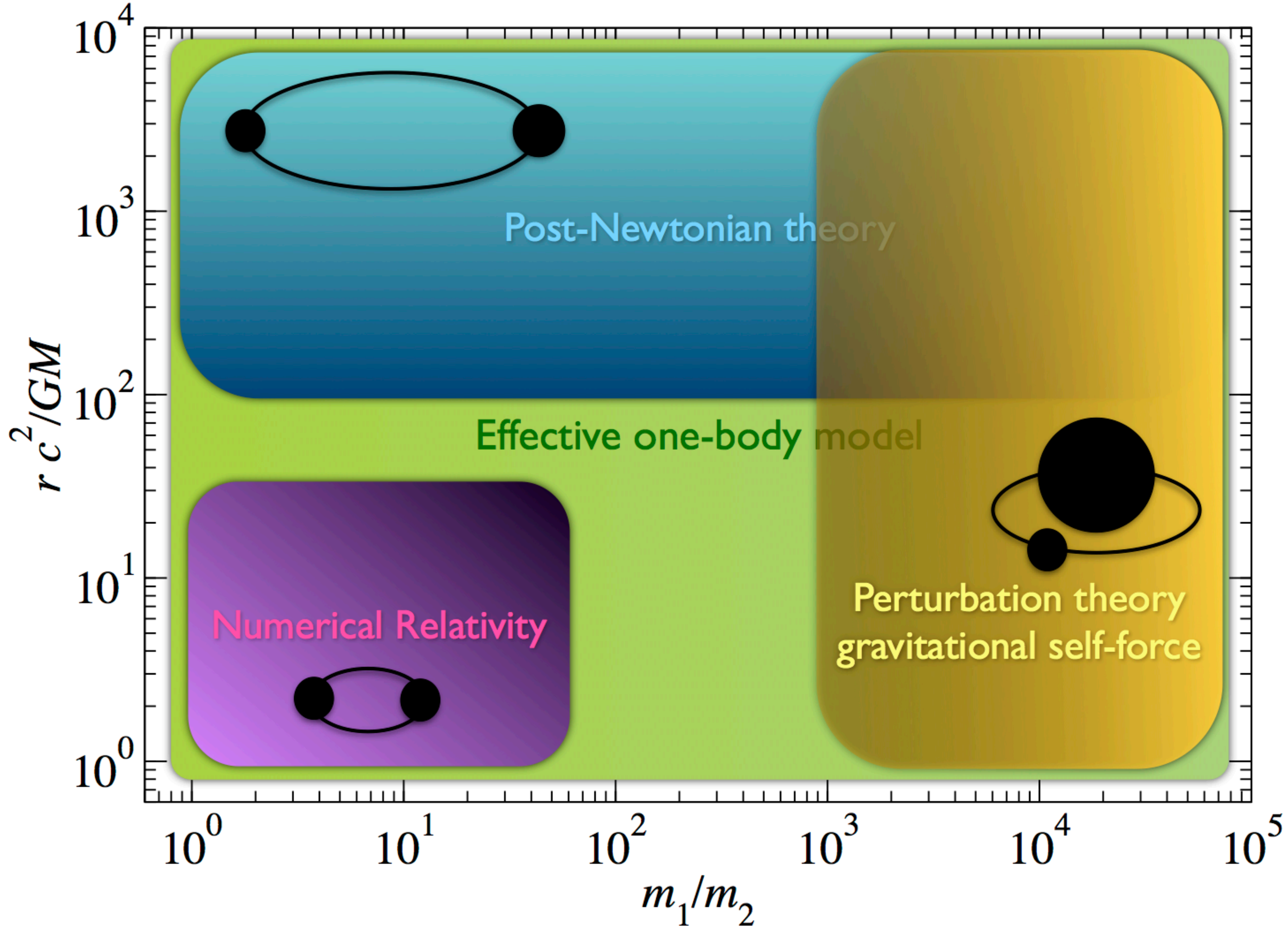
$$rM\Psi_4(\iota,\phi,t) = \sum_{l,m} {}_{-2}Y_{\ell m}(\iota,\phi)C_{\ell m}(t)$$

$$\Psi_4 = \ddot{h}$$



The Landscape

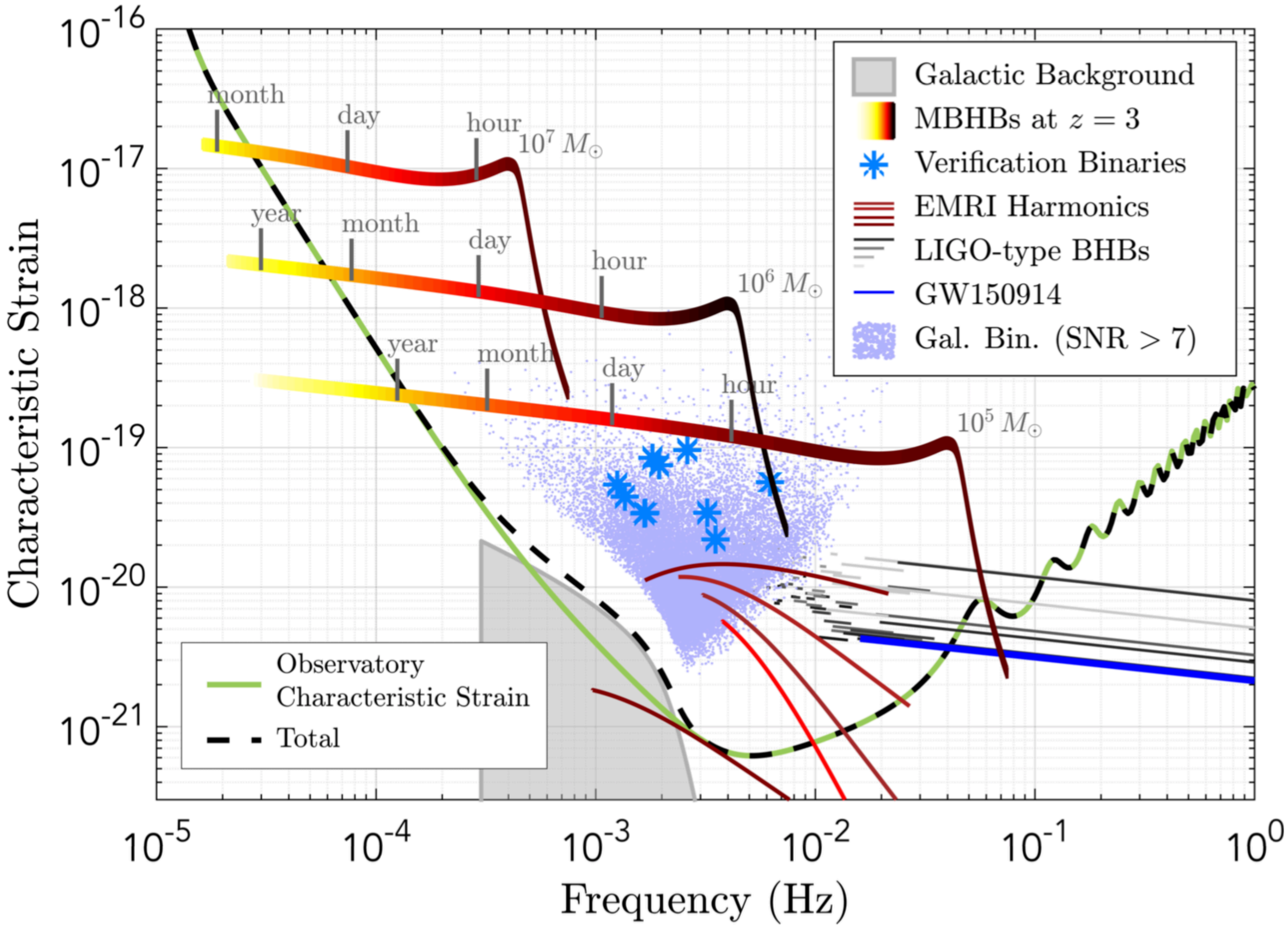
Buonanno and Sathyaprakash 2014



Two parameters determine the **range of validity** of each method:

$$\frac{G\,M}{r\,c^2} \sim \frac{v^2}{c^2} \quad \& \quad \frac{m_2}{m_1}$$

Inspiral Waveform



Post-Newtonian

Assuming a 2-orthogonal detectors

strain: $h(t) = h_+(\xi(t))F_+ + h_\times(\xi(t))F_\times$

Polarizations to 2PN

$h_{+,x}= 2Gm\eta\, x \{ H^{(0)} + x^{1/2}H^{(1/2)} + xH^{(1)} + x^{3/2}H^{(3/2)} + x^2H^{(2)}\}/c^2D_L$

$x=(GM\omega/c^3)^{2/3}$

orbital frequency for a circular orbit @ 2PN

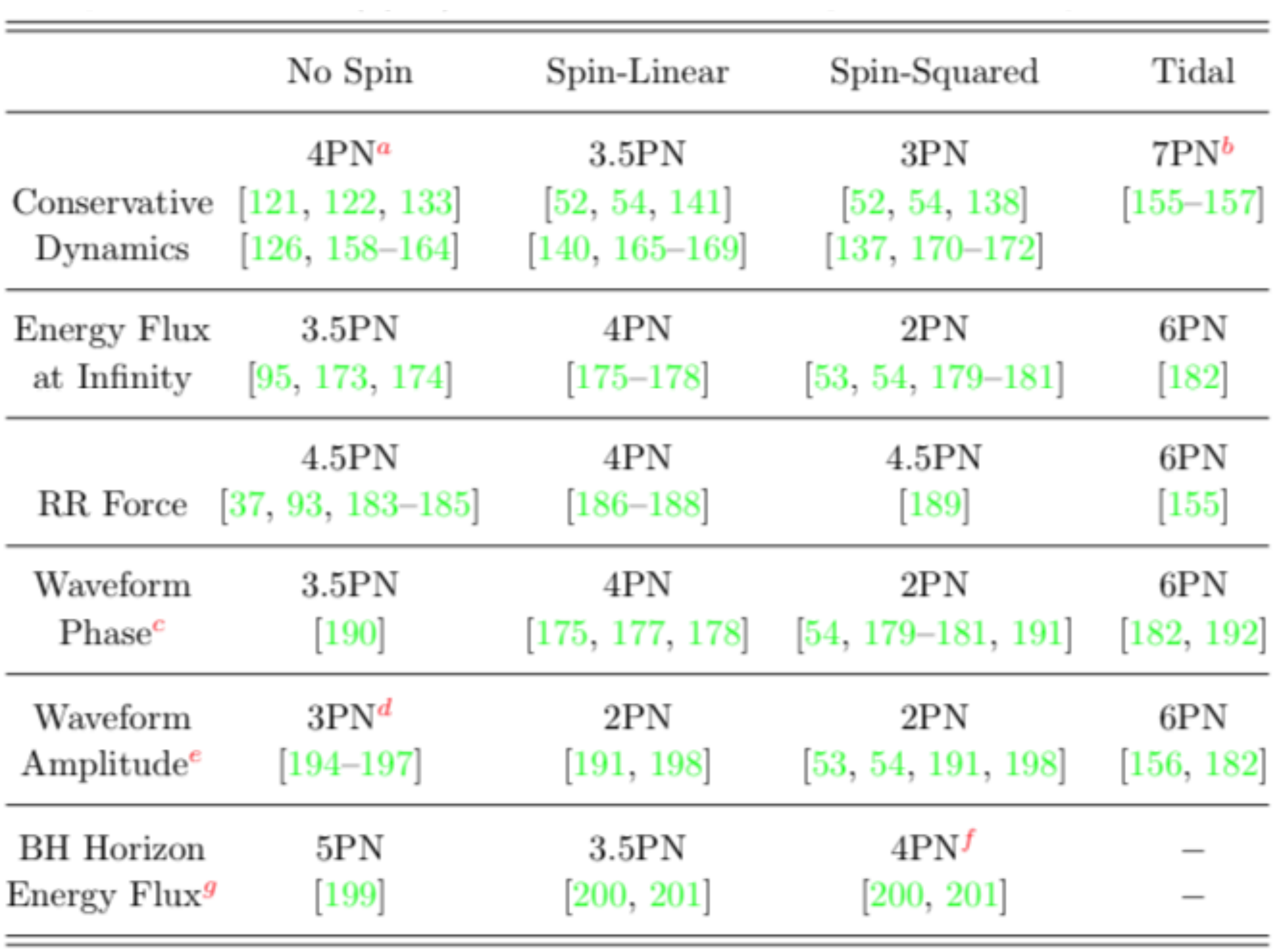
$\omega = d\Phi_{0rb}/dt$ is

orbital phase

$\Phi(t)=\phi_c^{orb}-1 / \eta \{\Theta^{5/8}+ f(\eta)\, \Theta^{3/8}-a\Theta^{1/4}+ f(\eta,\eta^2)\Theta^{1/8}\}$

$\Theta(t)= c^3\eta\, (t_c-t)/5Gm$

Post-Newtonian



	No Spin	Spin-Linear	Spin-Squared	Tidal
Conservative Dynamics	4PN[a] [121, 122, 133] [126, 158–164]	3.5PN [52, 54, 141] [140, 165–169]	3PN [52, 54, 138] [137, 170–172]	7PN[b] [155–157]
Energy Flux at Infinity	3.5PN [95, 173, 174]	4PN [175–178]	2PN [53, 54, 179–181]	6PN [182]
RR Force	4.5PN [37, 93, 183–185]	4PN [186–188]	4.5PN [189]	6PN [155]
Waveform Phase[c]	3.5PN [190]	4PN [175, 177, 178]	2PN [54, 179–181, 191]	6PN [182, 192]
Waveform Amplitude[e]	3PN[d] [194–197]	2PN [191, 198]	2PN [53, 54, 191, 198]	6PN [156, 182]
BH Horizon Energy Flux[g]	5PN [199]	3.5PN [200, 201]	4PN[f] [200, 201]	– –

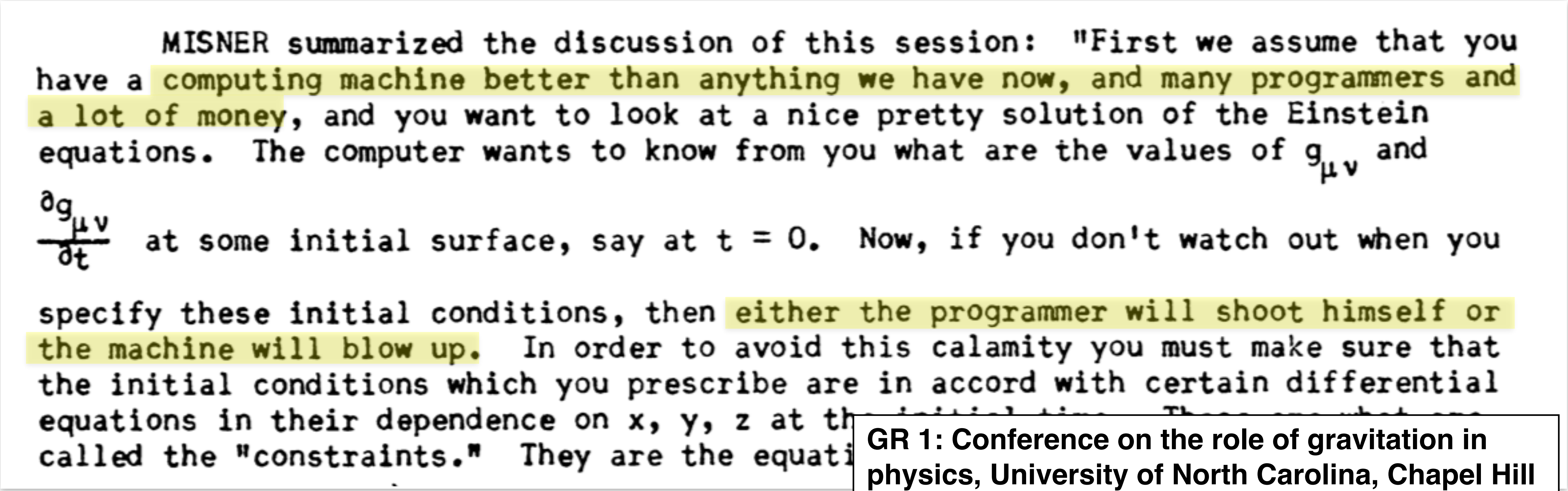
Table 6.1 from Buonanno and Sathyaprakash 2014 and references therein
See also Blanchet 2016 for complete description of PN

Inspiral Waveforms

- Newtonian quadrupole (Cutler 1998)
- PN expansion (Hughes 2002)
- simple pression (Vecchio 2004)
- Spin-induced precession (Lang & Hughes 2006)
- full PN no spin-precession (Arun et al 18 2007 and Porter & Cornish 2008)
- full, spinning no precession 2008 (Trias & Sintes 2007)
- Higher harmonics (Porter & Cornish 2008)
- Eccentricity: depending on formation scenario
  TaylorF2 for LIGO (Huerta et al 2014)
- 2PN spin-orbit precession (Klein, Jetzer & Sereno 2017)

The Building of an Inspiral Merger Ringdowm Waveform

Numerical Relativity



MISNER summarized the discussion of this session: "First we assume that you have a computing machine better than anything we have now, and many programmers and a lot of money, and you want to look at a nice pretty solution of the Einstein equations. The computer wants to know from you what are the values of $g_{\mu\nu}$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ at some initial surface, say at $t = 0$. Now, if you don't watch out when you specify these initial conditions, then either the programmer will shoot himself or the machine will blow up. In order to avoid this calamity you must make sure that the initial conditions which you prescribe are in accord with certain differential equations in their dependence on x, y, z at th

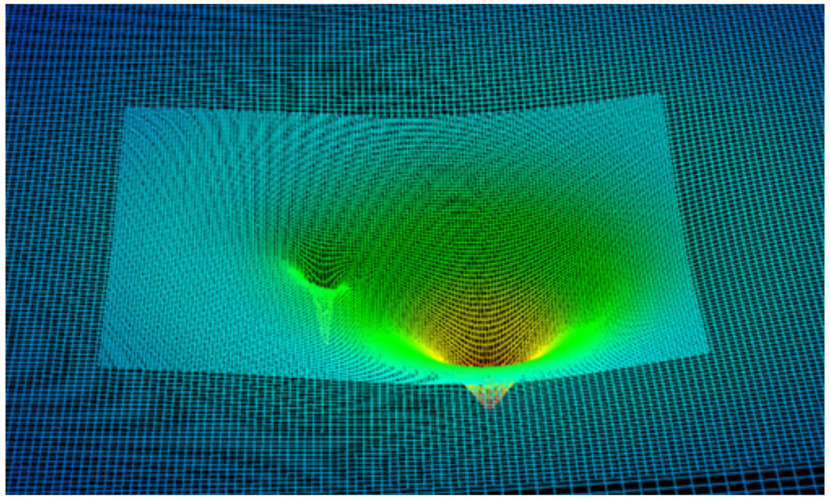
called the "constraints." They are the equati

GR 1: Conference on the role of gravitation in physics, University of North Carolina, Chapel Hill [January 18-23, 1957]

The 2-body problem of binary black hole took decades and supercomputers.

2005 Pretorius
Binary inspiral and merger

Phys.Rev.Lett. 95 (2005) 121101

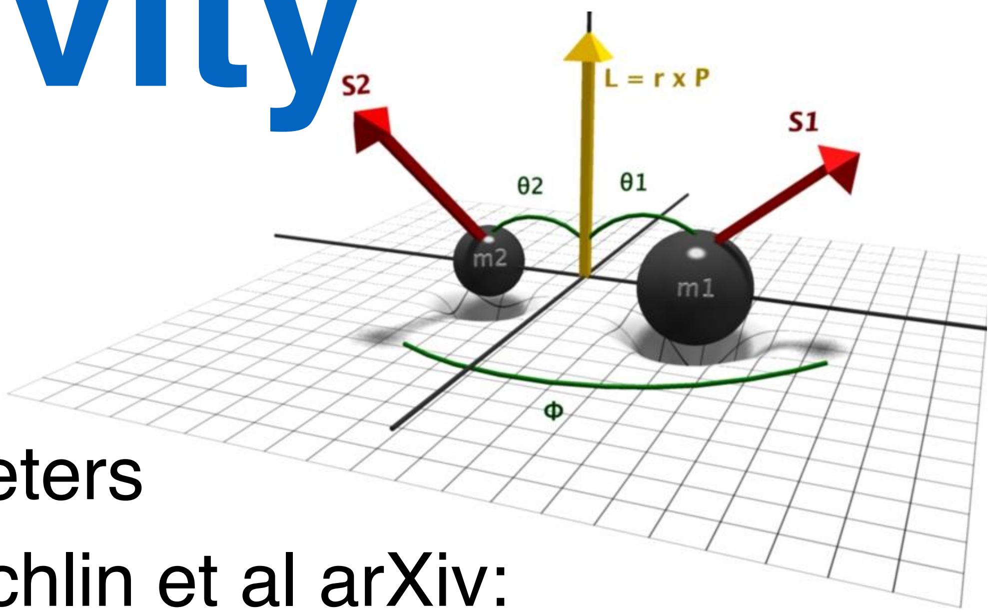


2006 RIT and NASA
Moving Punctures Method

Campanelli, Lousto, Zlochower
Phys.Rev.Lett. 96 (2006) 111101

Baker, Centrella, Choi,
Koppitz, van Meter
Phys.Rev.Lett. 96 (2006)
111102

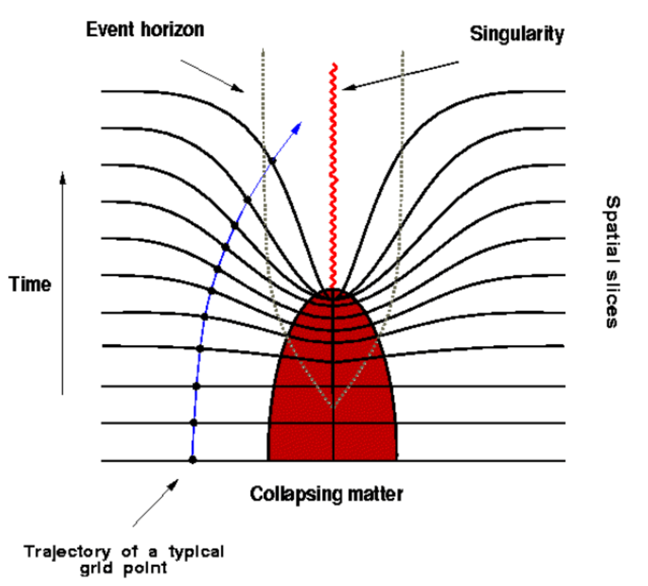
Numerical Relativity



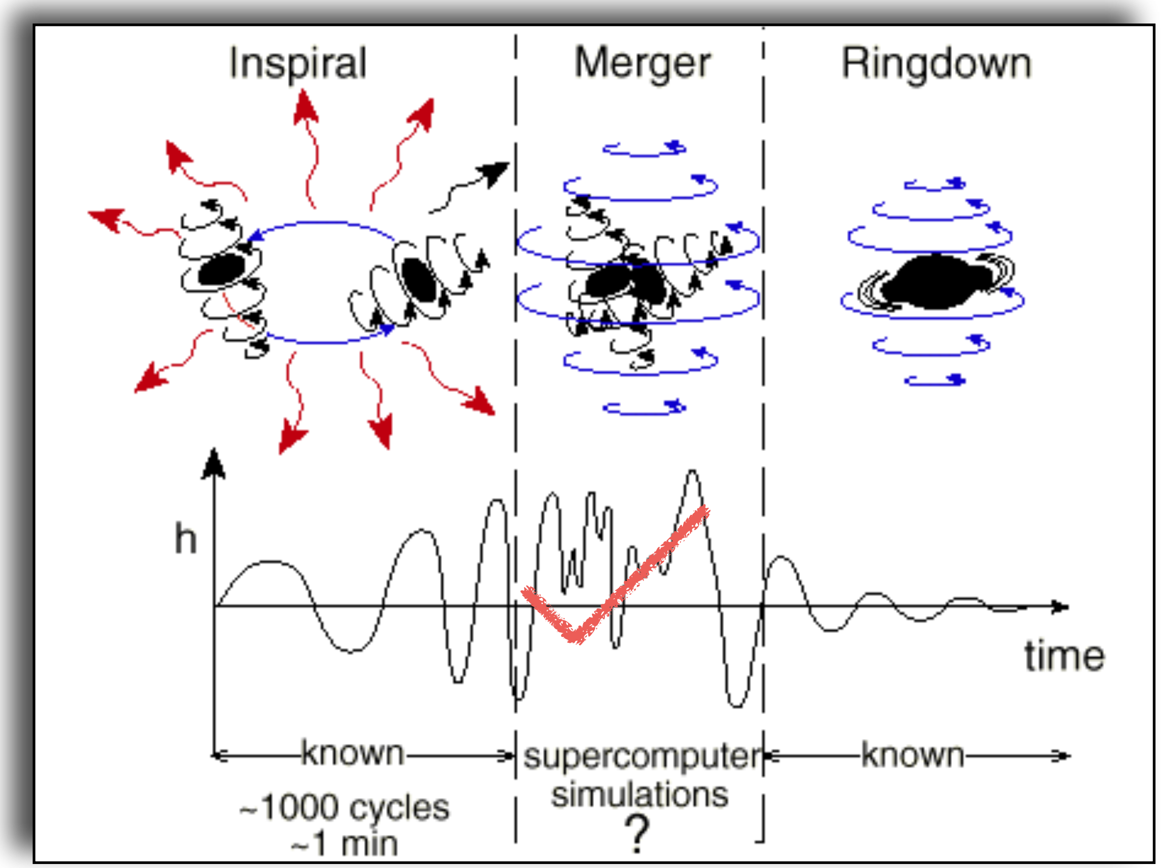
- Initial Data
  - solve initial data equations for a set of parameters
  - extreme spins (Lovelace et al CQG 2012, Ruchlin et al arXiv: 1410.8607)
  - “extreme” mass ratios (Lousto et al PRL 2011, Ossokine 1506.01689)
  - need smart method for choosing parameters

- Evolution Equations (+gauge, boundary ,…)
  - Einstein Toolkit (Loeffer et al CQG 2012)
  - SpEC (Szilagyi et al PRD 2009)
  - weeks to months of compute time (more orbits, longer!)

- Extracting radiation
  - Reisswig & Pollney CQG 2011



What made it possible?



Holy Grail Obtained!

Fundamental Gravitational Physics Explored

Orbital Hang Ups

Gravitational Recoil

Black Hole Remnant

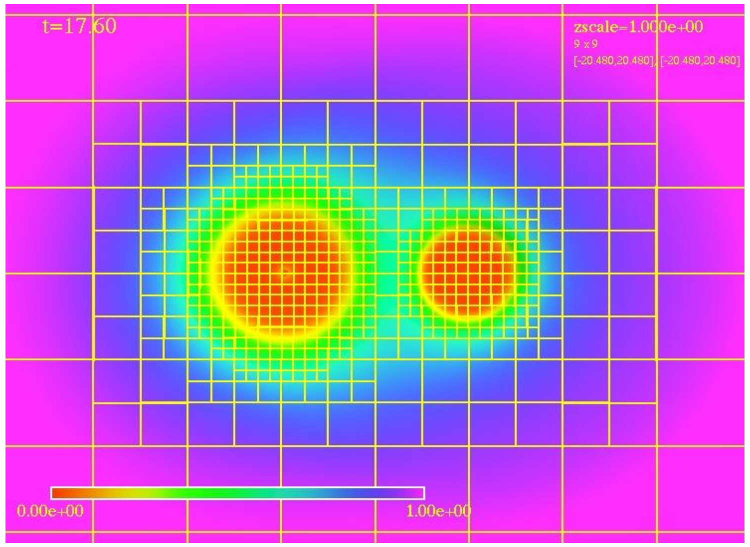
Black Hole Triplets

Extreme Orbits

Gravitational Wave

Why did it take so long?

We did not have the appropriate package of Mathematical Tools (e.g. Gauges, Formulations) and Computational Infrastructure (e.g. Adaptive Mesh Refinements, Hardware, etc.)



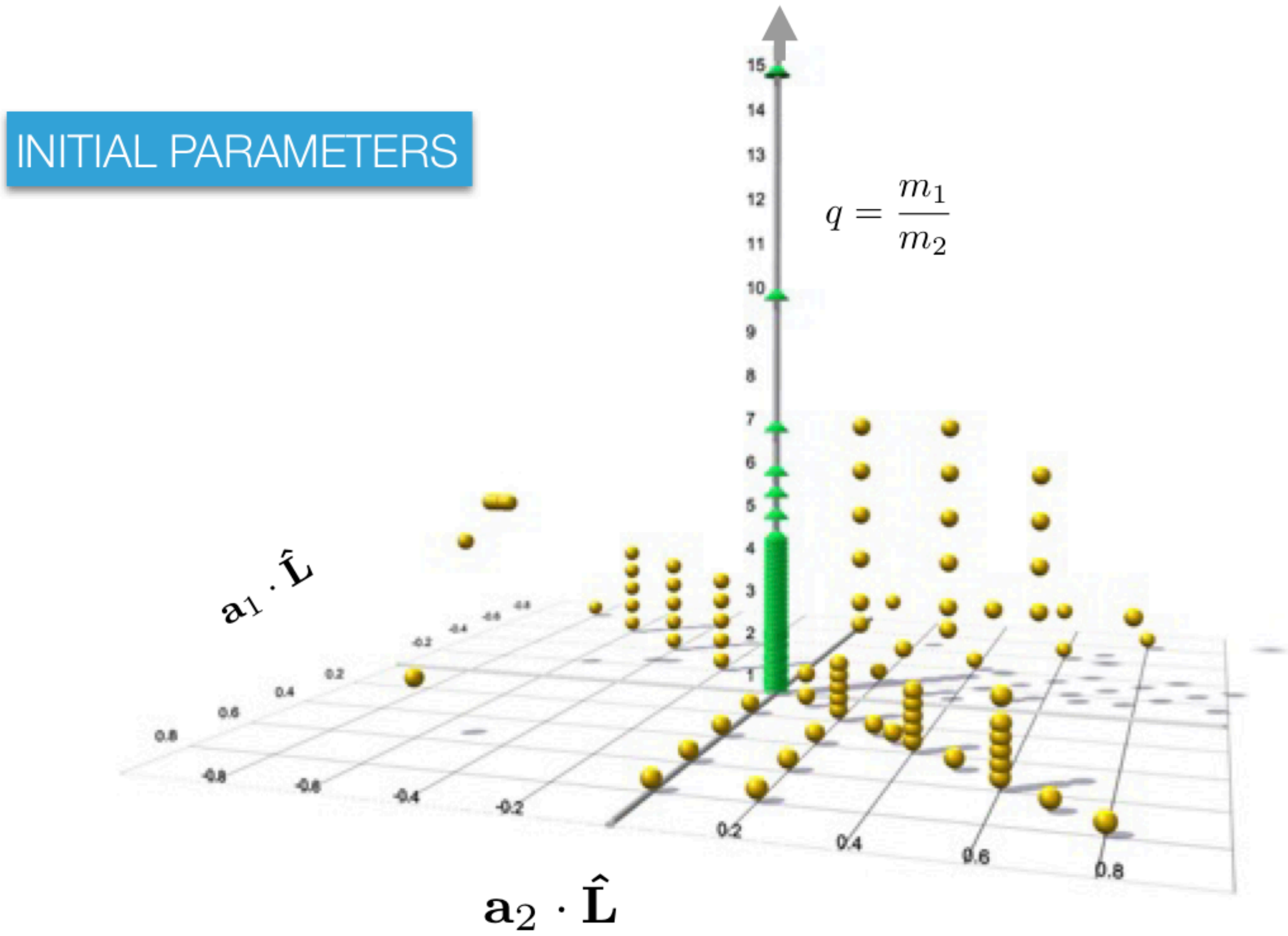
Dale Choi (NASA-GSFC)

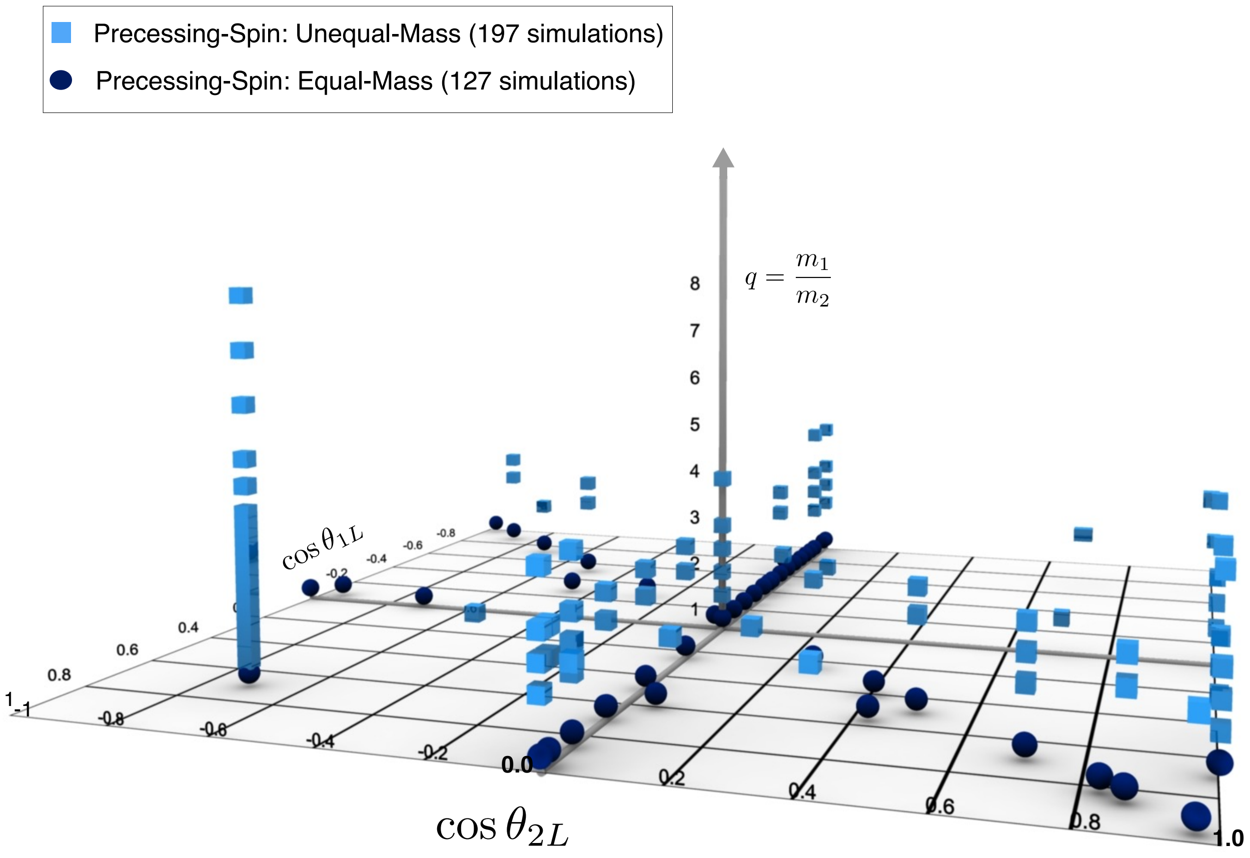
State of Art: NR Waveforms

- **GT public catalog of a few hundred BBH simulations with many processing systems at einstein.gatech.edu/catalog**

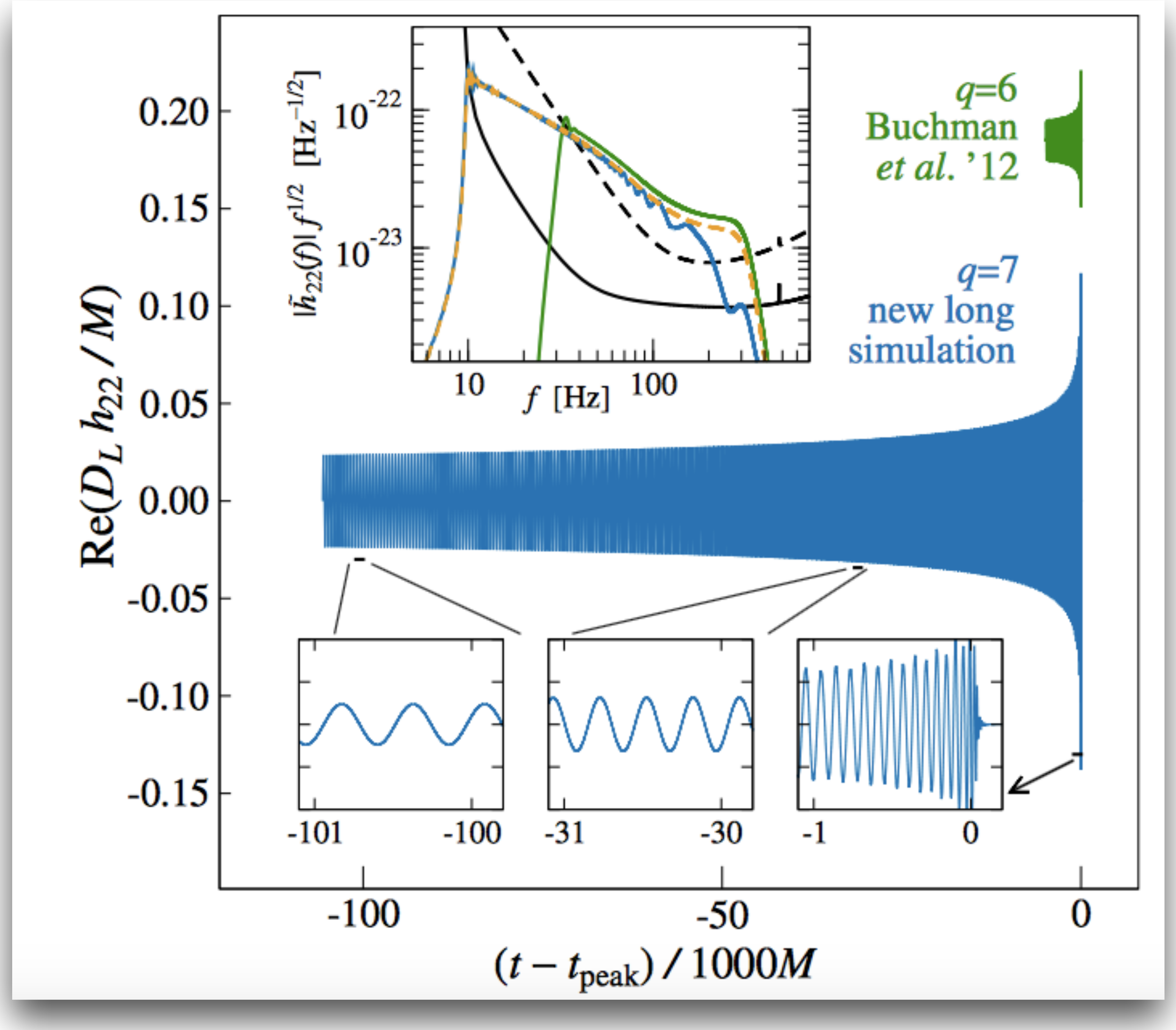
- **SXS public catalog of a few hundred long BBH simulations and some extremely spins at black-hole.org**

- **RIT catalog at ccrg.rit.edu**





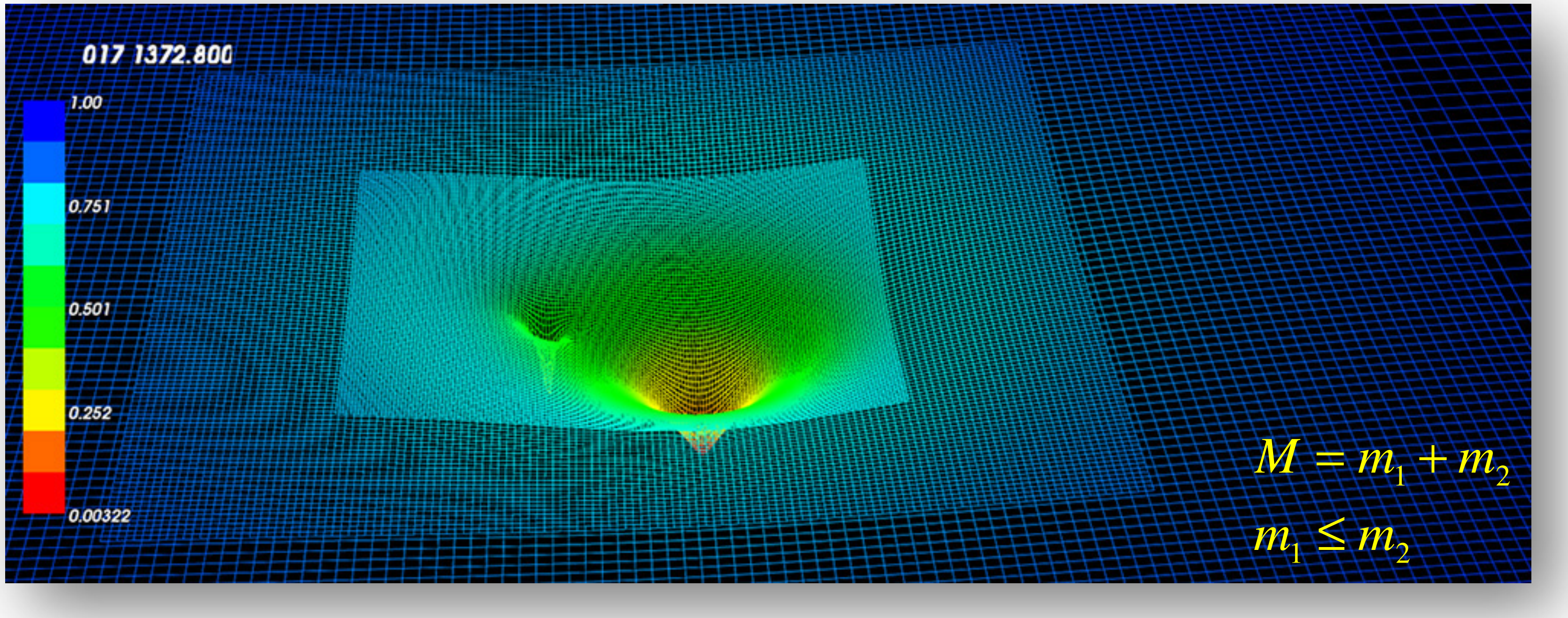
Numbers of cycles



350 NR GW cycles
45.5 M
q=7
arXiv:1502.04953
Szilagyi et al

EOB formalism accurately describes the inspiral dynamics 20 to 176 orbits before merger for this case

Computational Cost (NR)



$M = m_1 + m_2$

$m_1 \leq m_2$

Scales:

- Size of the smallest black hole: m_1
- Wavelength of the waveform in the wave zone: 16M
- Distance between the binary system and the wave-zone: 256M

Resolutions:

- At the black hole: M/256
- For waveform extraction: 2M
- At coarsest mesh: 4M

$$h_{\max} = 4\,M = 2^2\,M$$

$$h_{\min} = \frac{M}{256} = 2^{-8}\,M$$

$$\#\,\text{refinements} = \log_2\left(\frac{h_{\max}}{h_{\min}}\right) = 10$$

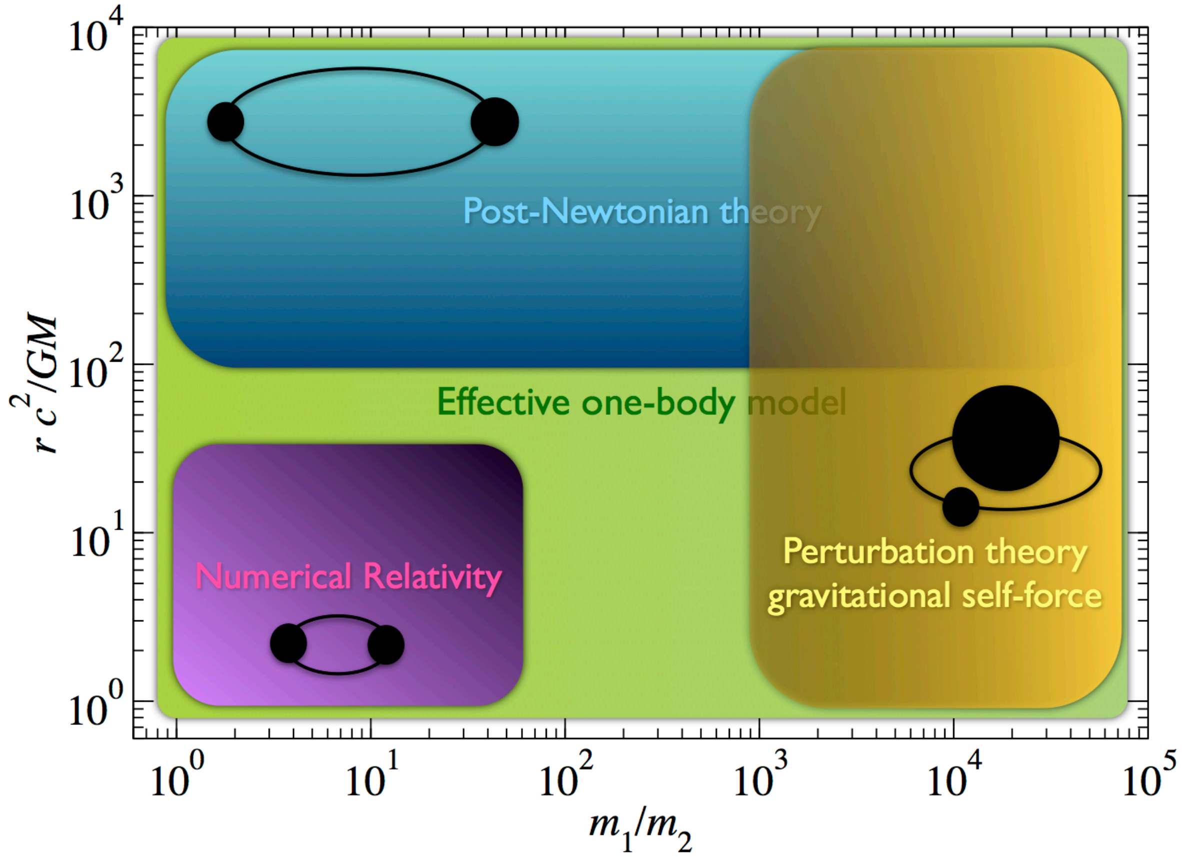
$$L = 512\,M$$

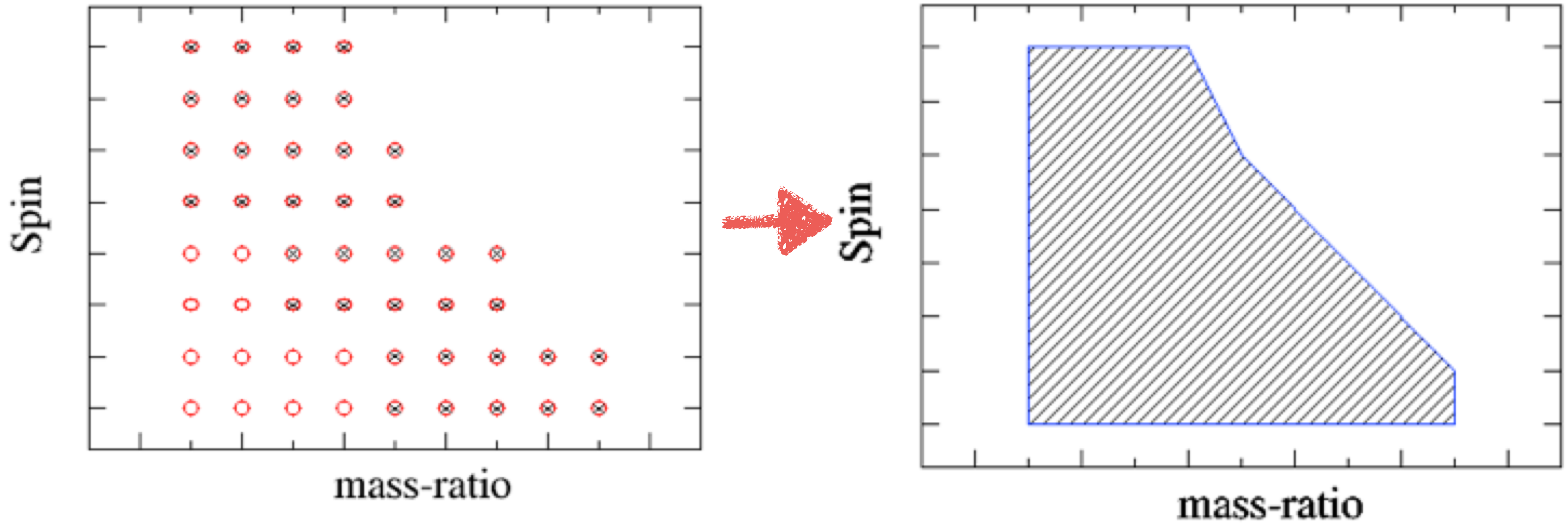
$$\#\,\text{of grid-points per refinement} = 128^3$$

$$\text{Total}\ \#\,\text{of grid-points} = 20 \times 128^3 \approx 4 \times 10^7$$

Model Waveforms

f(Post-Newtonian/EOB + NR, parameters) = waveform

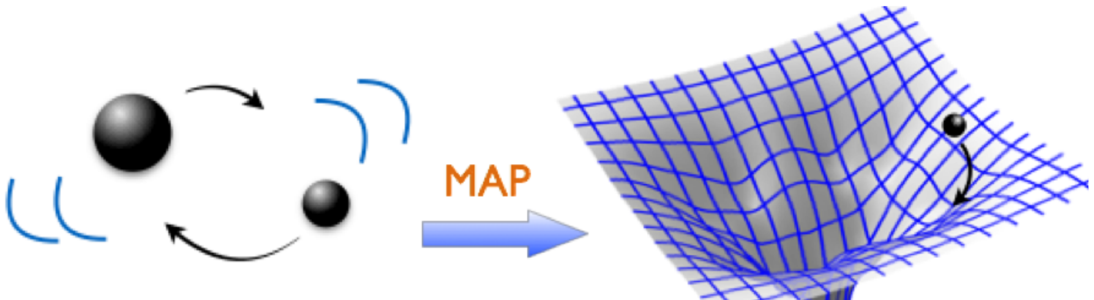




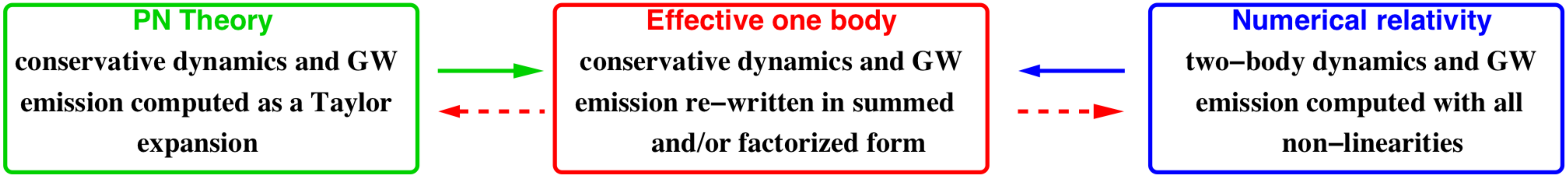
2 Major Models

- EOBNR (Buonanno et al 2007)
- Phenom (Ajith et al 2007)

EOBNR Waveforms



(credit: Hinderer)

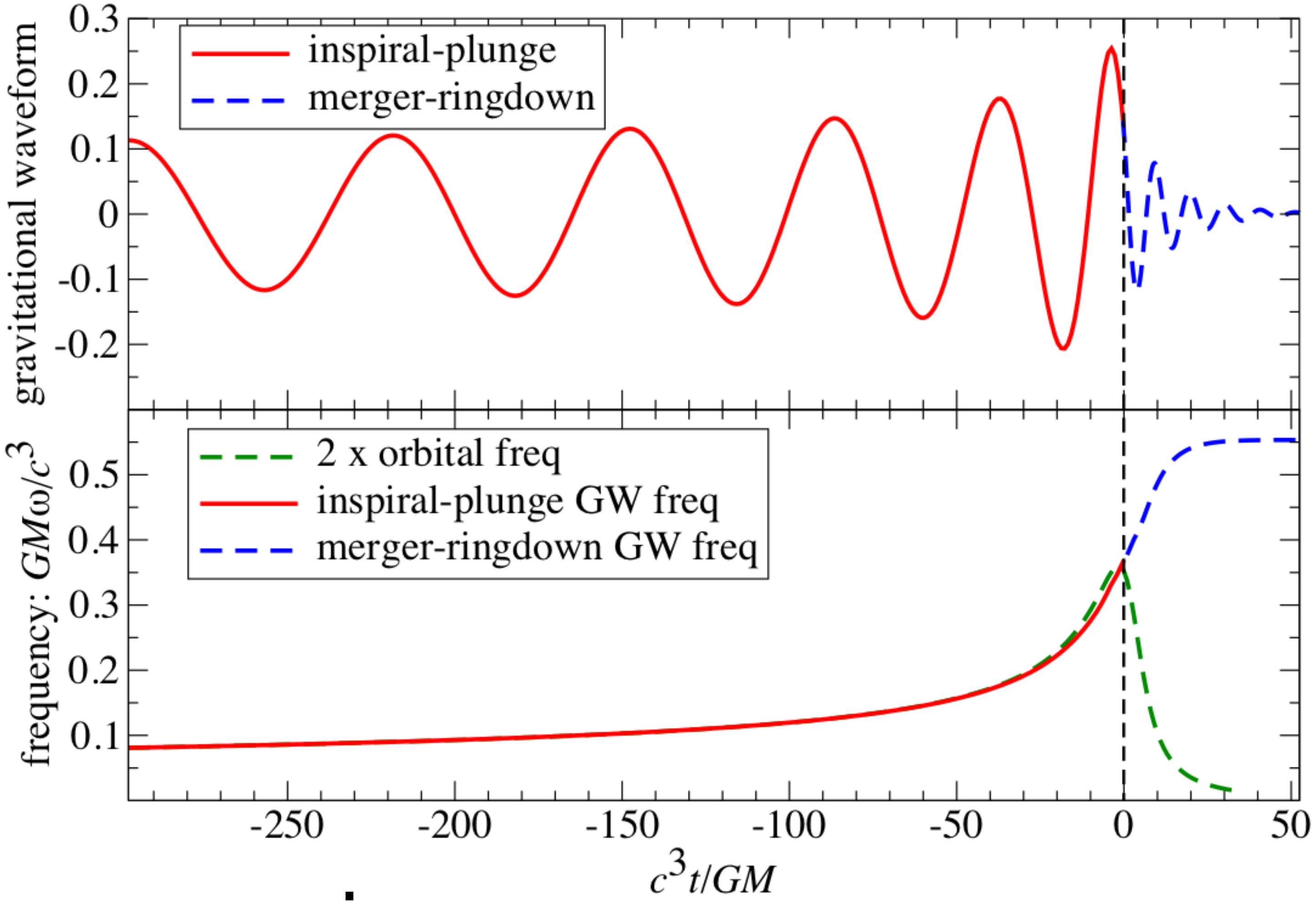


PN Theory
conservative dynamics and GW emission computed as a Taylor expansion

Effective one body
conservative dynamics and GW emission re–written in summed and/or factorized form

Numerical relativity
two–body dynamics and GW emission computed with all non–linearities

EOBNR waveform model combines EOB fitted to NR waveforms (Buonanno et al 2007, Buonanno & Damour 1990, 2000)



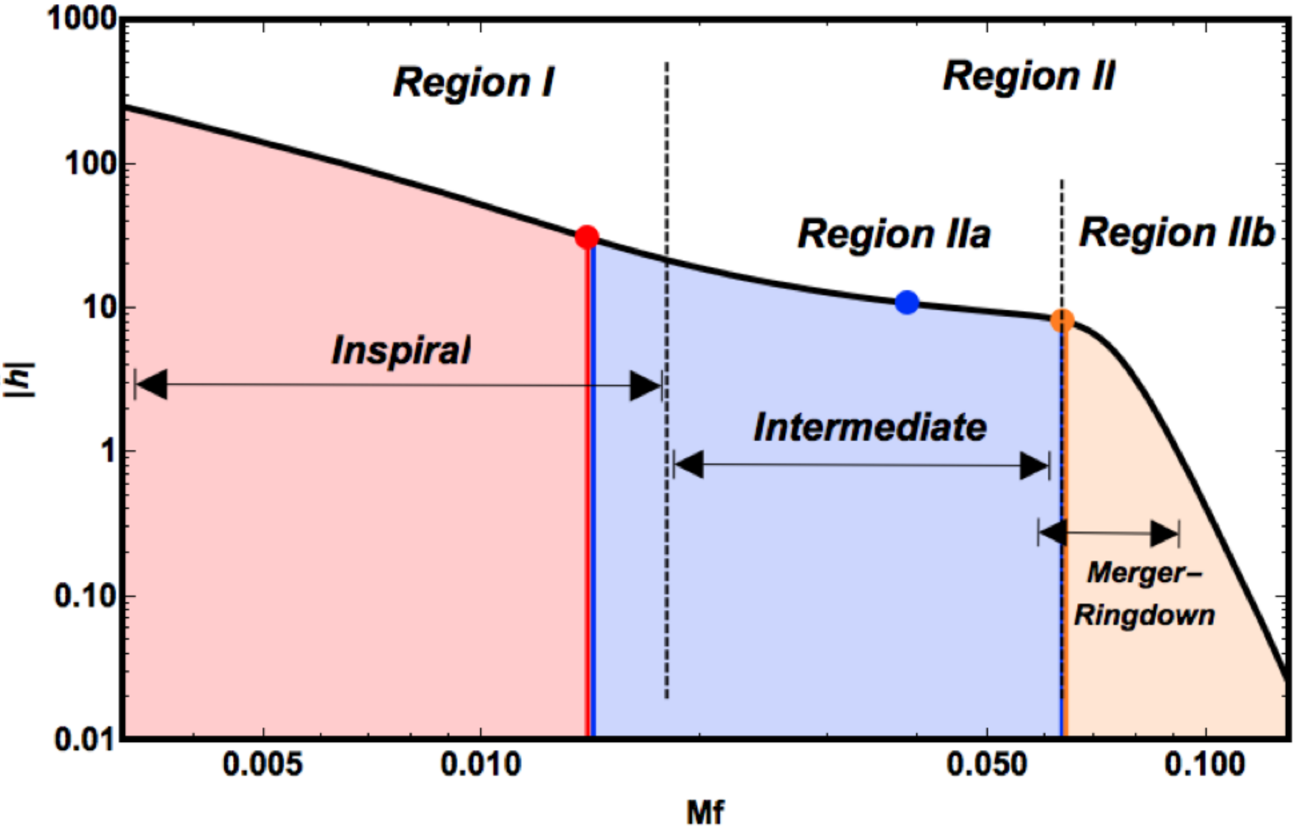
Computationally expensive

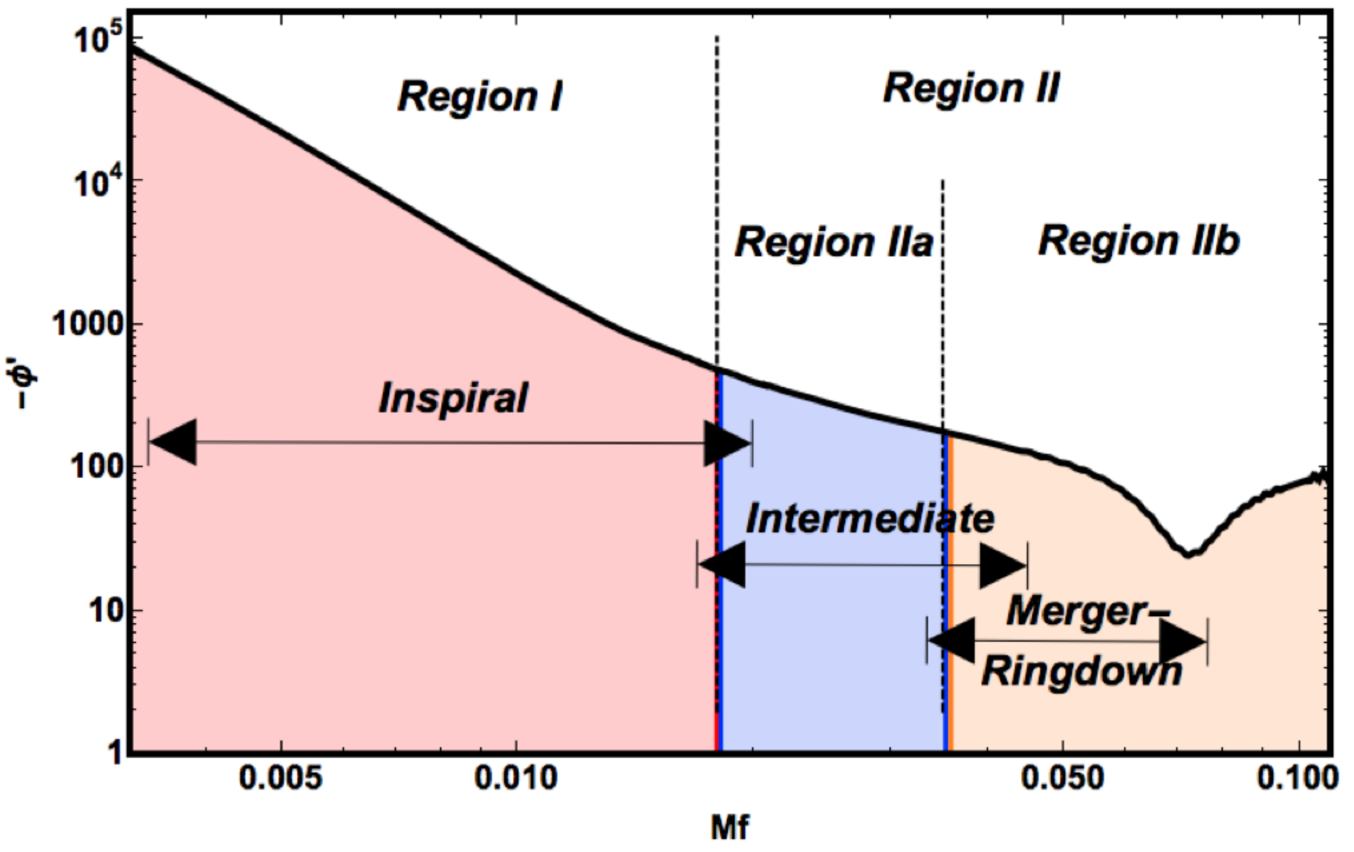
Slide courtesy of A. Buonanno

Phenomenological Waveforms

- Fast, frequency based model is a hybrid PN/EOB + NR (Ajith et al 2007, Pan et al 2007, Santamaria et al 2010, Khan et al 2015, Husa et al 2015)

$$\tilde{h}(f;\lambda_i) = \mathcal{A}(f;\lambda_i)\, e^{i\phi(f;\lambda_i)}$$





State of the Art: IMR

- Precessing spins (Babak, Taracchini & Buonanno 2016)
- Higher Harmonics (London et al 2017 with spins)
- Eccentric models (Huerta et al 2017 10 orbits before merger, no spin; Hinder, Kidder & Pfeiffer 2017 no spin, PN + NR)
- Surrogate Models (Blackman et al 2017)

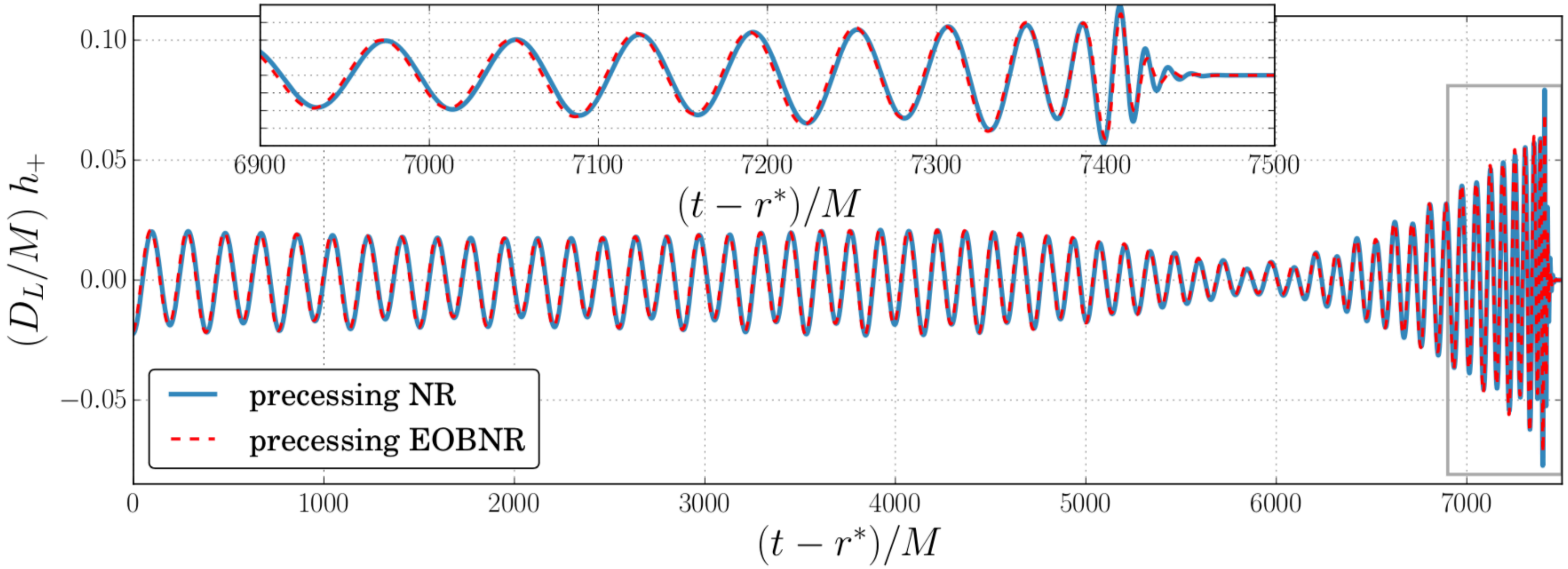


FIG. 5. **Comparison of NR and EOBNR + polarization** for a precessing BBH with $q = 5$, $\chi_1 = 0.5$, $\chi_2 = 0$, with spin 1 initially in the orbital plane (SXS:BBH:0058). The inclination is $\iota = \pi/3$. The NR (EOBNR) data are shown in blue (dashed red).

State of the Art: IMR

- Precessing spins (Babak, Taracchini & Buonanno 2016)
- Higher Harmonics (London et al 2017 with spins)
- Eccentric models (Huerta et al 2017 10 orbits before merger, no spin; Hinder, Kidder & Pfeiffer 2017 no spin, PN + NR)
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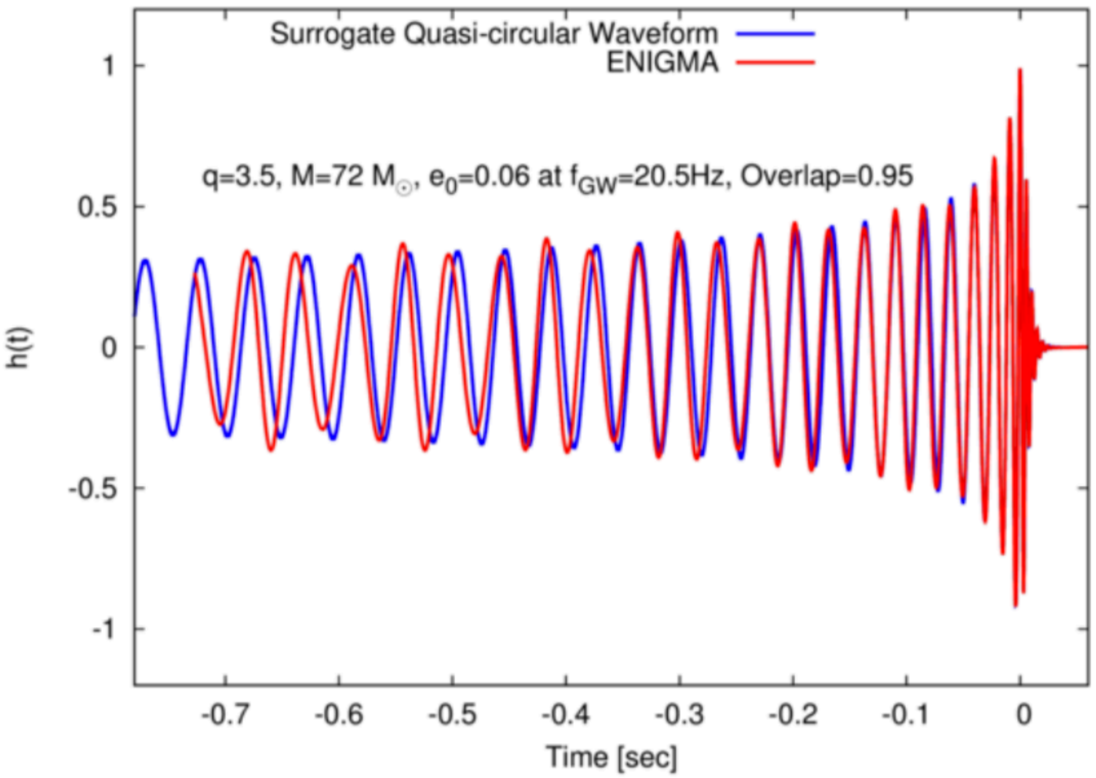
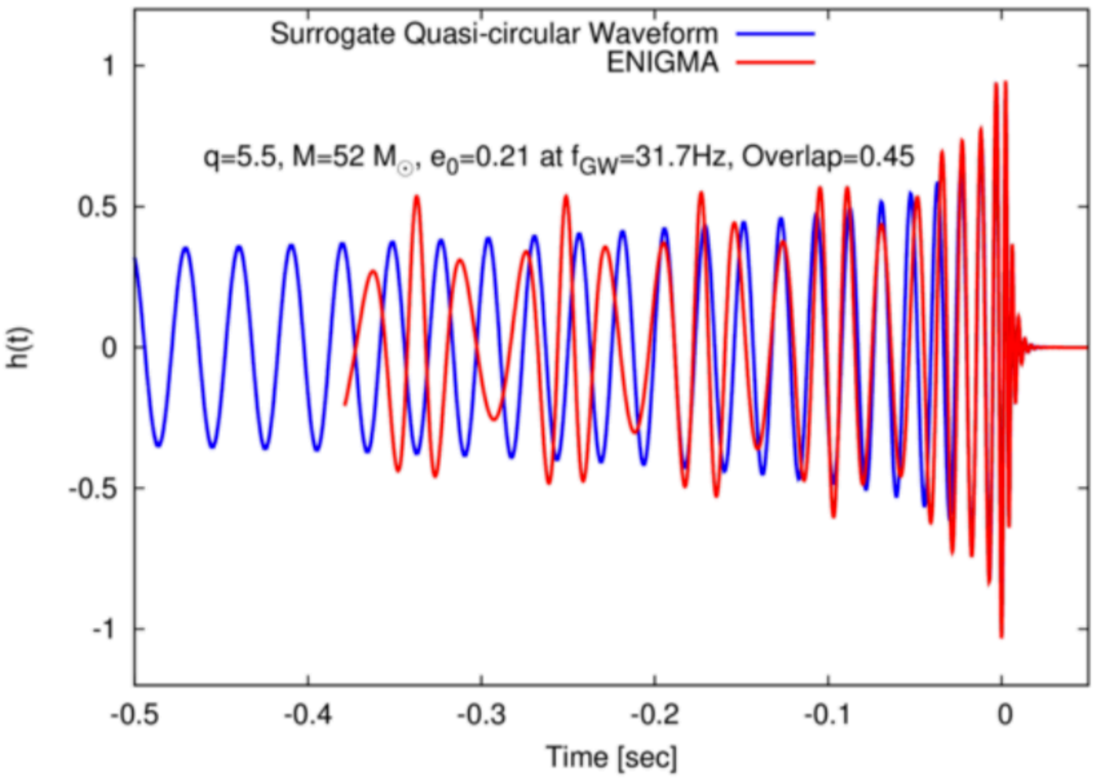
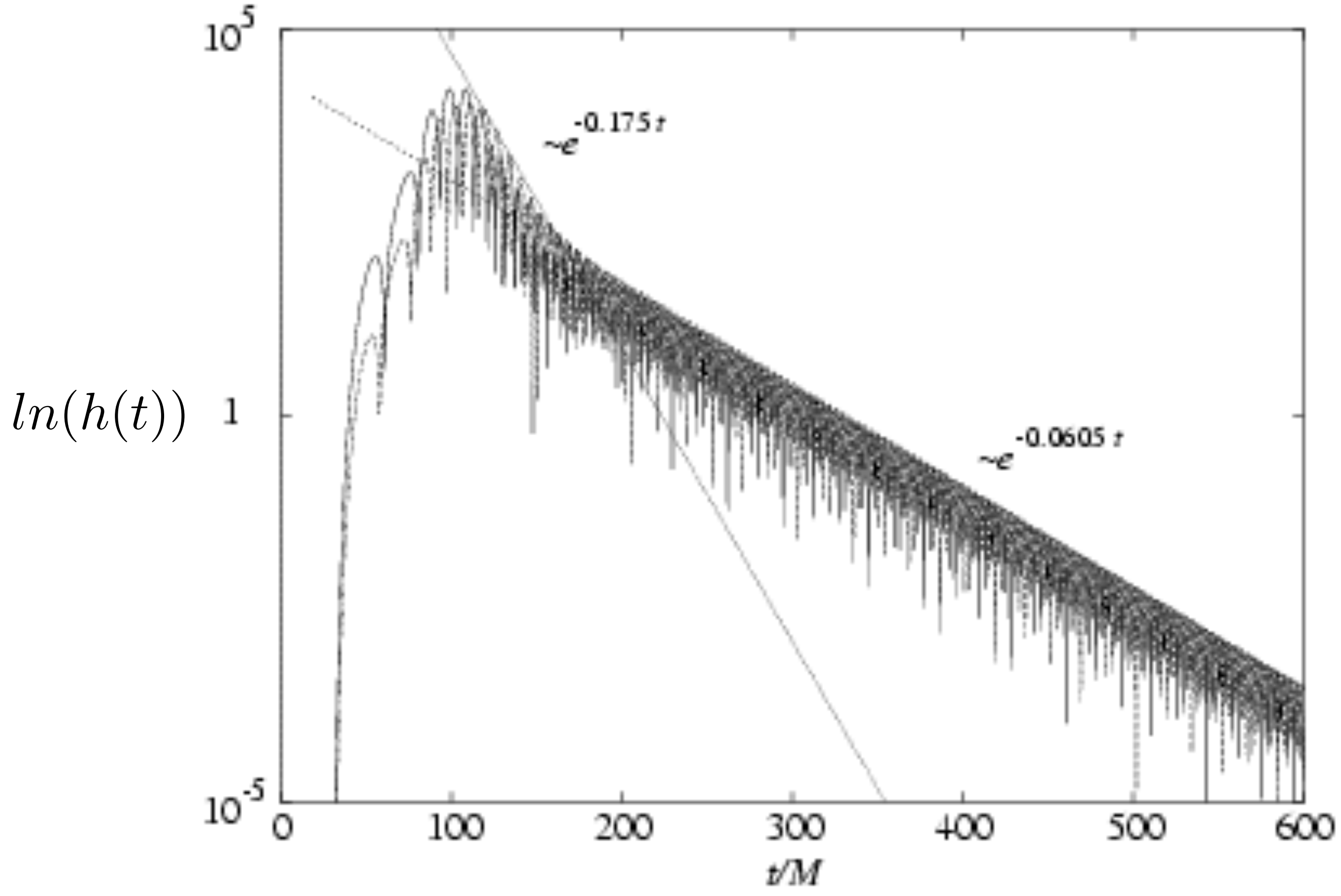




NR Provides a Map to Ringdown

15 parameters

2 parameters

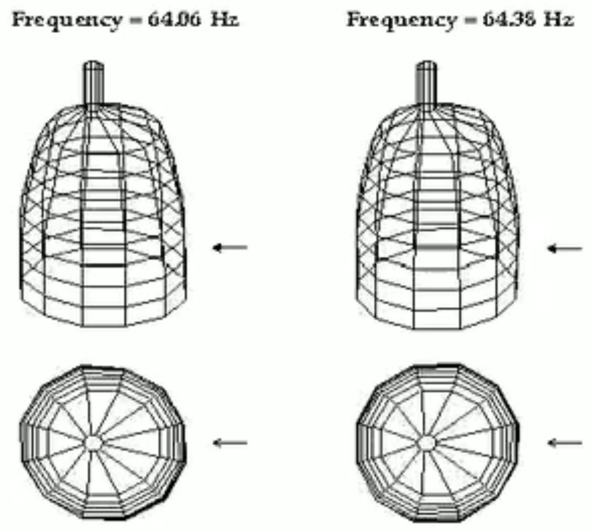


- Ringdown is completely describable by damped sinusoidal functions.

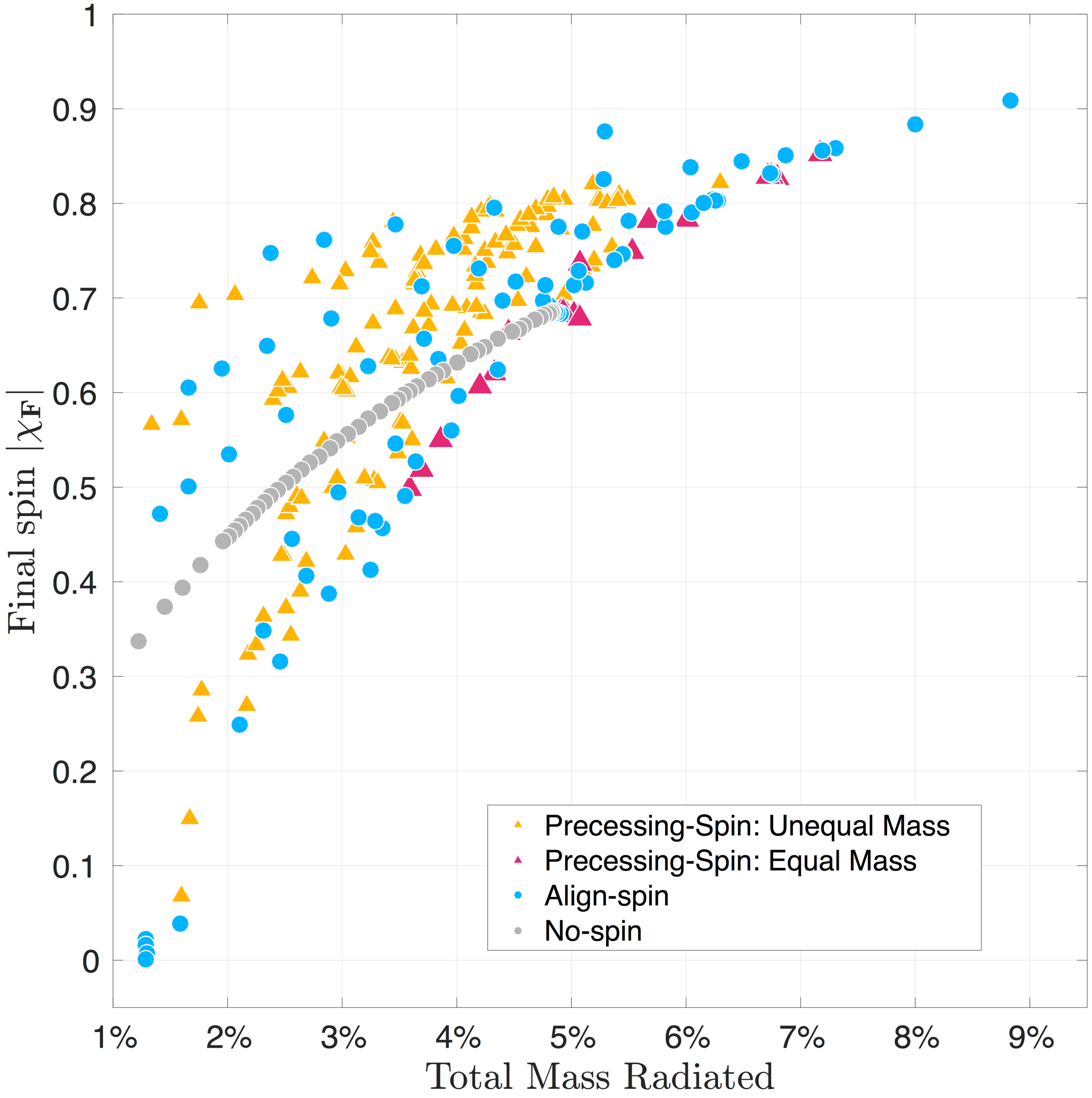
$$h(t) \propto e^{-t/\tau} sin(2\pi f t)$$

- The black hole “rings” in tones given by a set of unique complex frequencies.

$$f(M,a) \quad \tau(a)$$



Remnant Black Hole

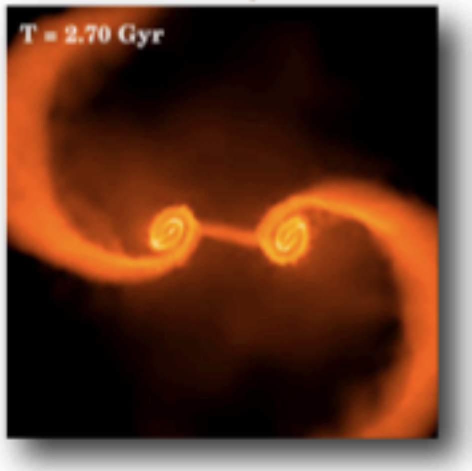


Input initial mass and spin values, formula predicts final mass, spin and recoil (Healy et al PRD 2014 & Barausse, et al APJ 2012)

Wet Binaries

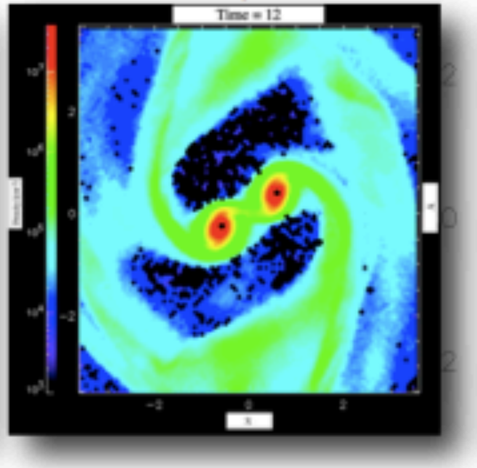
Massive BH Binaries and Astrophysical Environments

10s kpc



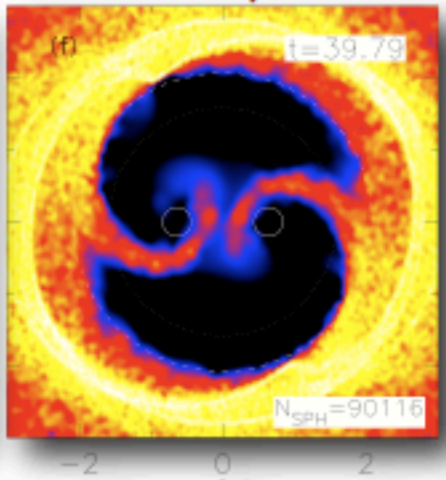
Mayer+ 07

10s pc



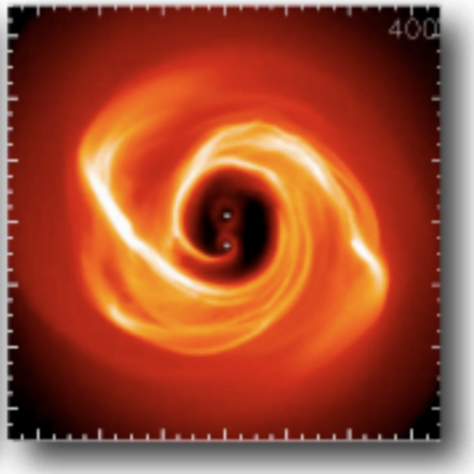
Escala+ 05

sub-pc



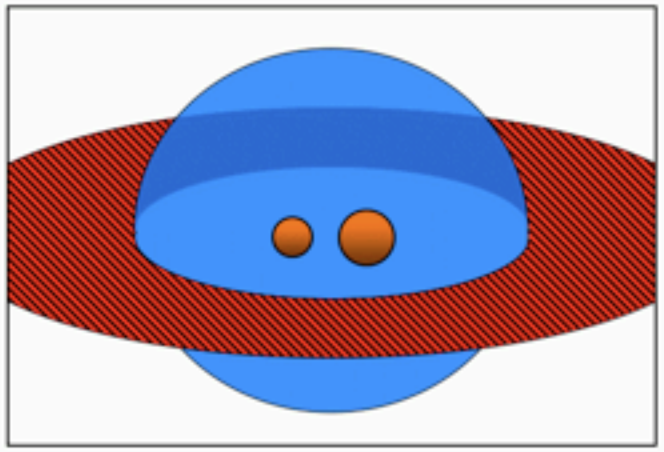
Hayasaki+ 07

sub-pc

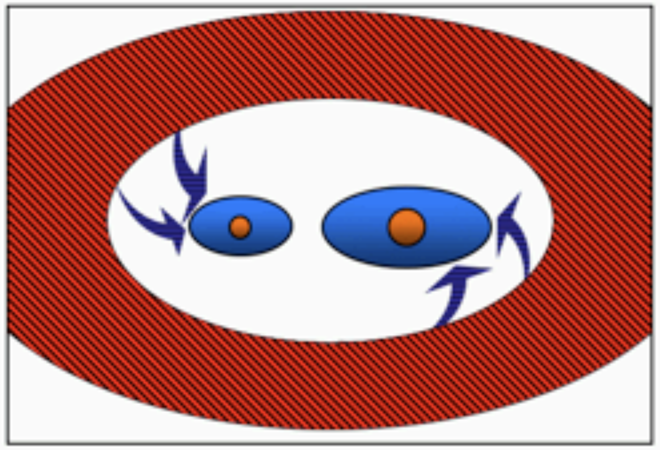


Cuadra+ 09

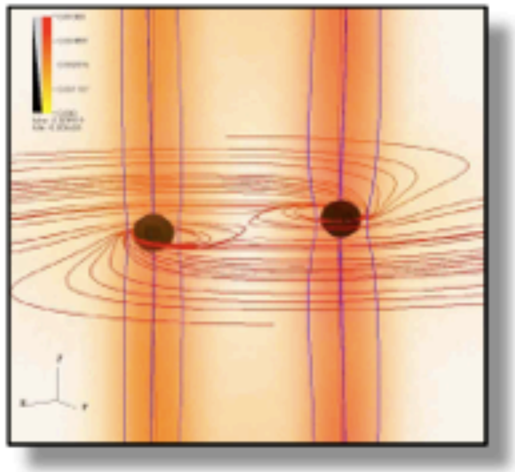
- What is the environment in the vicinity of massive BBH?
- Is there a smoking gun of GR+EM signatures



Radiatively Inefficient Hot Gas Cloud



Circumbinary disk



Plasma dominated by magnetic fields

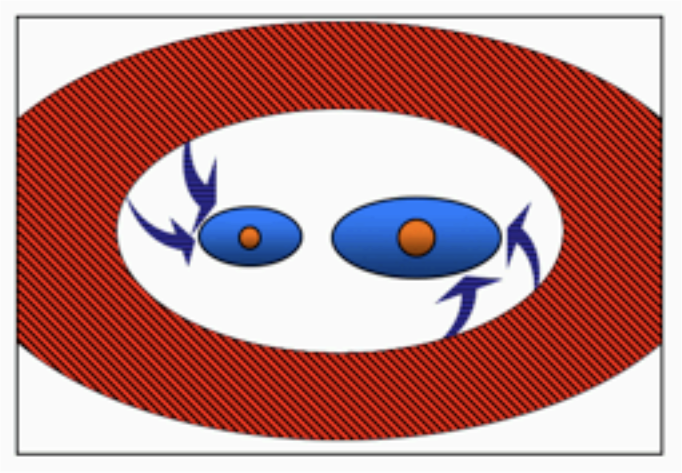
Tremendous computational modeling grand challenge!
10^5 pc ↔ 10^{-5} pc

Three Phases

1) Predecoupling phase disk viscous timescale (t_{vis}) is shorter than $t_{GW,}$ disk relaxes to quasi-equillibrim state BBH slowly inspires

2) post decoupling phase, t_{vis}>t_{GW}, binary decouples from disk before disk relies

3) post-merger (afterglow) disk fills the hollow left behind and accretion ramps up on BH



Wet Binaries

Analytic and semi-analytic models focus on the geometrically thin, optically thick disk (Haiman, Kocsis & Menou 2010, Shaprio 2010 & 2013, Liu & Shapiro 2010, Kocsis, Haiman& Loeb 2012)

Inner cavity of lowered density near the binary, were revealed in hydrodynamic studies in Newtonian gravity in 2&3D (Artymowicz & Lubow 1994, Cuadra et al 2008, Roedig et al 2011, Roesig et al 2012, MacFayden & Milosavljevic 2008, D'Orazio, Haiman & MAcFayden 2012, Darris et al 2014, MHD in 3D Shi et al 2012 and PN (Noble et al 2012, Zilho et al 2014)

Infalling clouds onto and the subsequent disk formation (Dunhill et al 2014)

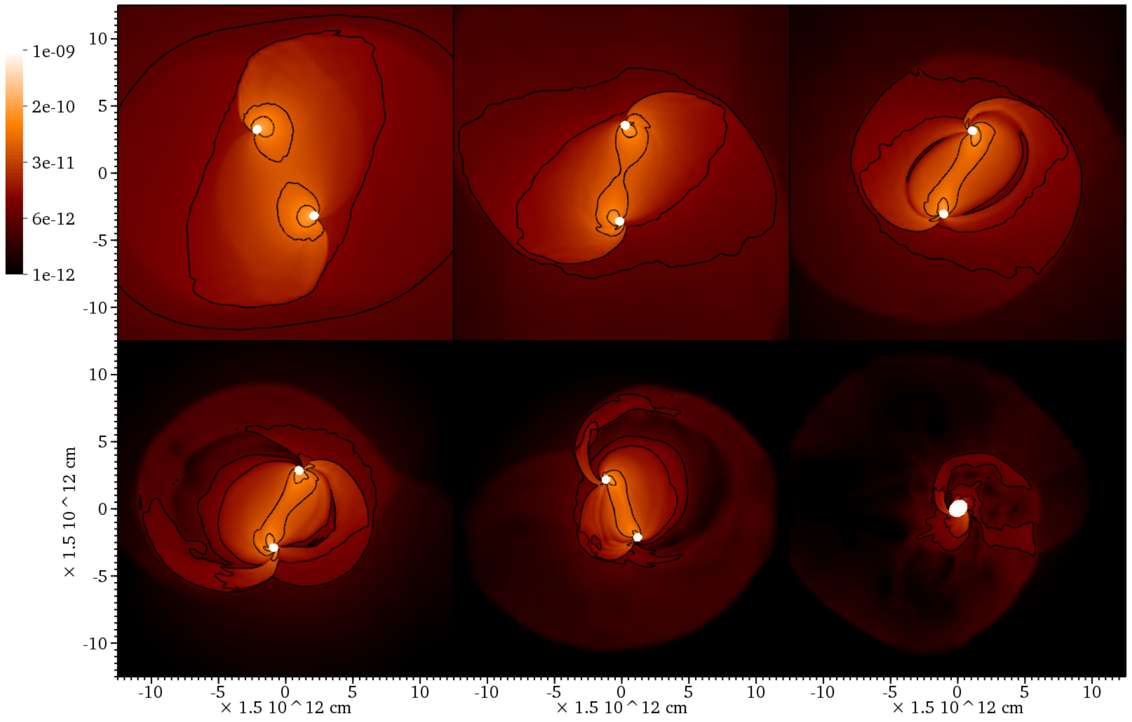
EM fields in force-free electrodynamics in GR, without modeling the disk (Mosta et al 2010, Neilsen et al 2011, Palenzuela et al 2010, Palenzuela, Lehner & Liebling 2010, Alic et al 2012)

MHD Circumbinary Disks (Giacomazzo et al 2012, Noble et al 2012)

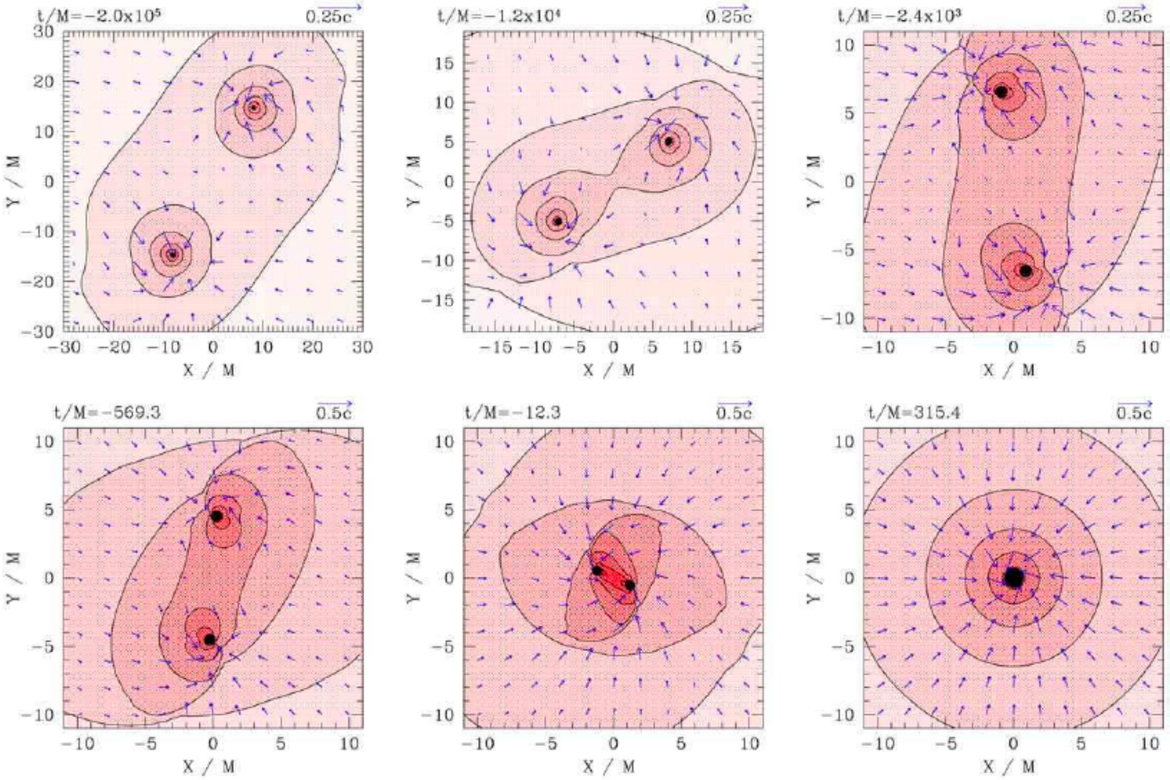
Maxwell Fields & Force-Free (Neilsen, et al 2012, Palenzuela, Bona, Lehner, Reula 2011, Palenzuela, et al 2010 , Alic et al 2012)

GR evolutions of geometrically thick disks (Bode et al 2010, Bogdanovic et al 2011, Bode et al 2012, Farris, Liu & Shapiro 2011) and with MHD (Farris et al 2012, Gold et al 20

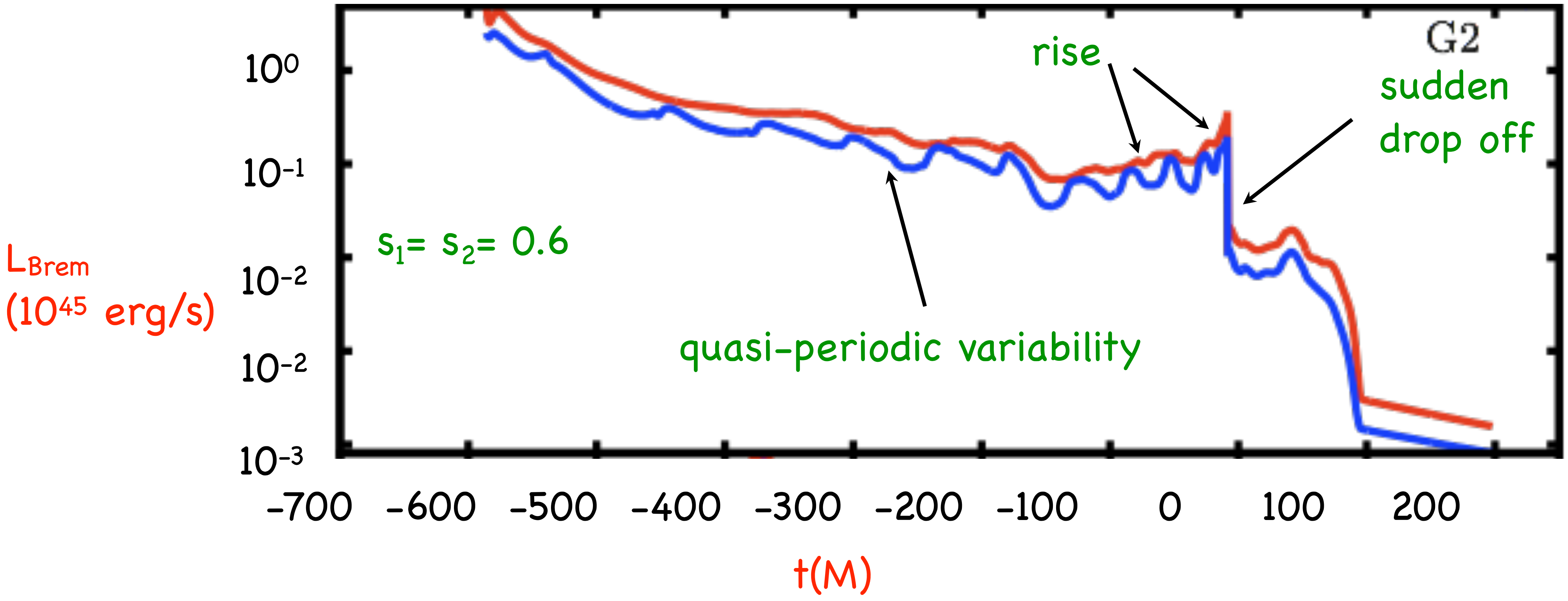
SMBH Mergers in Hot Gas



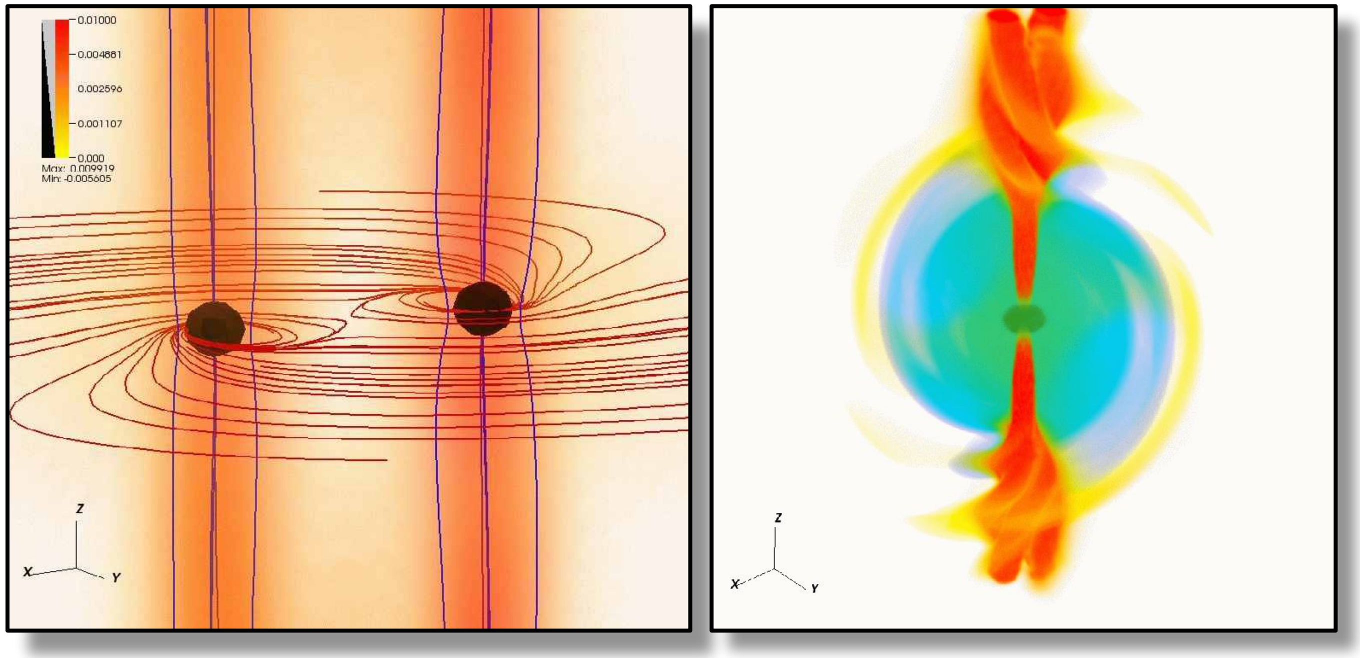
Relativistic Mergers of Supermassive Black Holes and their Electromagnetic Signatures
Bode, Bogdanovic, Haas, Laguna, Shoemaker (2010)

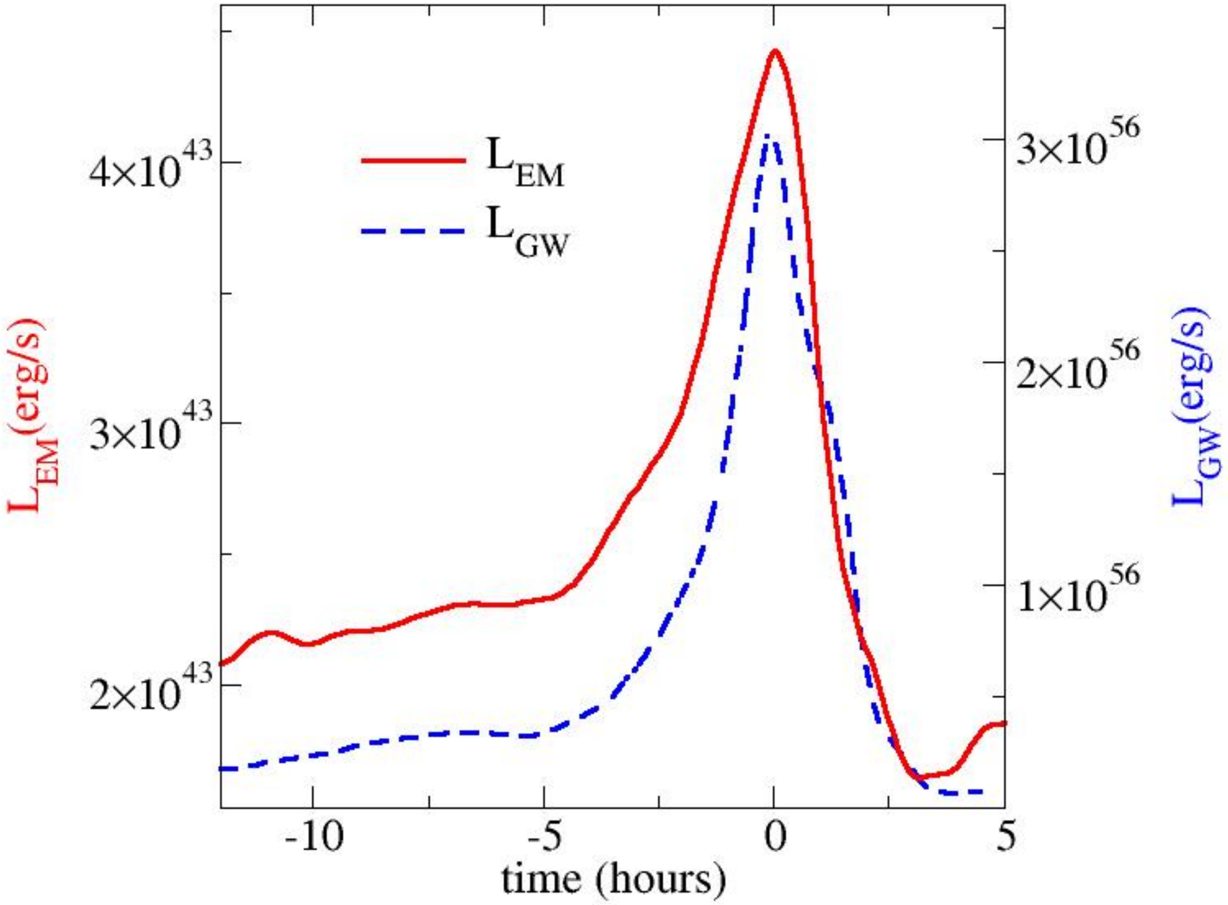


Binary Black Hole Mergers in Gaseous Environments: "Binary Bondi" and "Binary Bondi-Hoyle-Lyttleton" Accretion
Farris, Liu, Shapiro (2010)



SMBH Mergers Surrounded by EM Fields





(Palenzuela, Lehner Liebling 09a, 09b, 10;
Mösta+ 09)

- Unlikely that this EM emission can be detected directly.
- The EM emission could be observable indirectly from its effects on the BH accretion rate.

Transient signals” distinguishing single SMBH from SMBHB

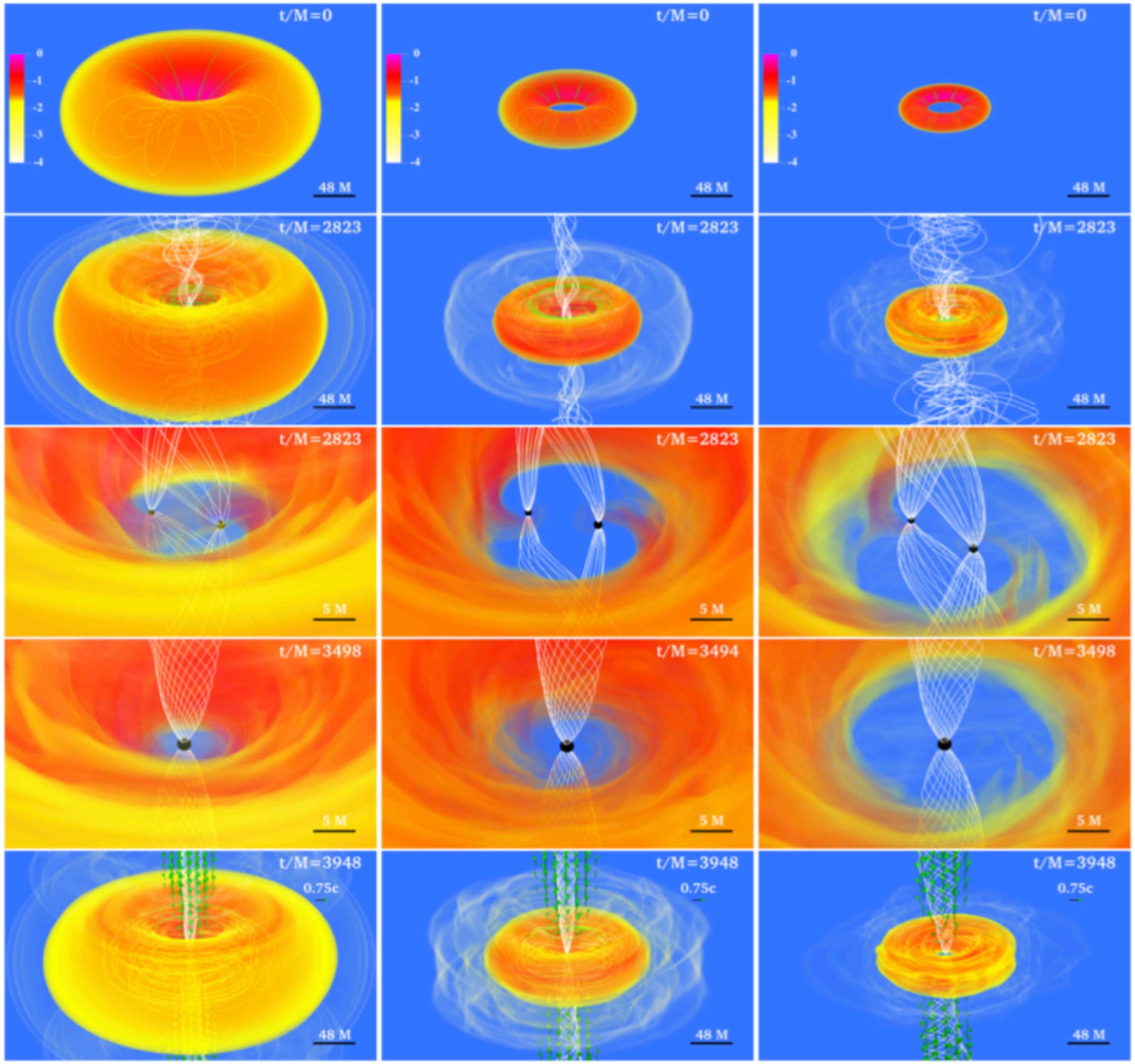


Fig 3 Khan et al 2018: volume rendering of rest-mass density normalized to its initial maximum value for 3 disk configurations. Green represents velocity and white magnetic fields.

State of the Art: Wet Binaries

- In the absence of information regarding the environment surrounding the binary, our best option is to explore a range of scenarios and look for characteristic features (flares, variability).
- More work is needed to explore more astrophysical plausible configurations (MHD, cooling, radiation) but progress is significant!
- Shapiro et al found little decrease in nearly all luminosity diagnostics after decoupling, indicating that such sources may be bright.
- Aftermath EM signatures are more prominent than precursor EM signals. Generally, the dependence of EM signatures on mass ratio is stronger after merger than before merger or in the predecoupling epoch.
- A robust acceleration and boost in magnetic energy density of the outflowing material was observed (Shapiro et al) as a one- time EM signature for merging SMBH binaries

Hopes and Directions

- “How, when and where go the first massive black holes form, grow and assemble, and what is the connection with galaxy formation?

- What is the nature of gravity near the horizons of black holes and on cosmological scales?”

- To satisfy baseline, must have Waveforms of a certain accuracy over the relevant parameters

- Cautionary tale of LIGO: the surprises, even mild ones.

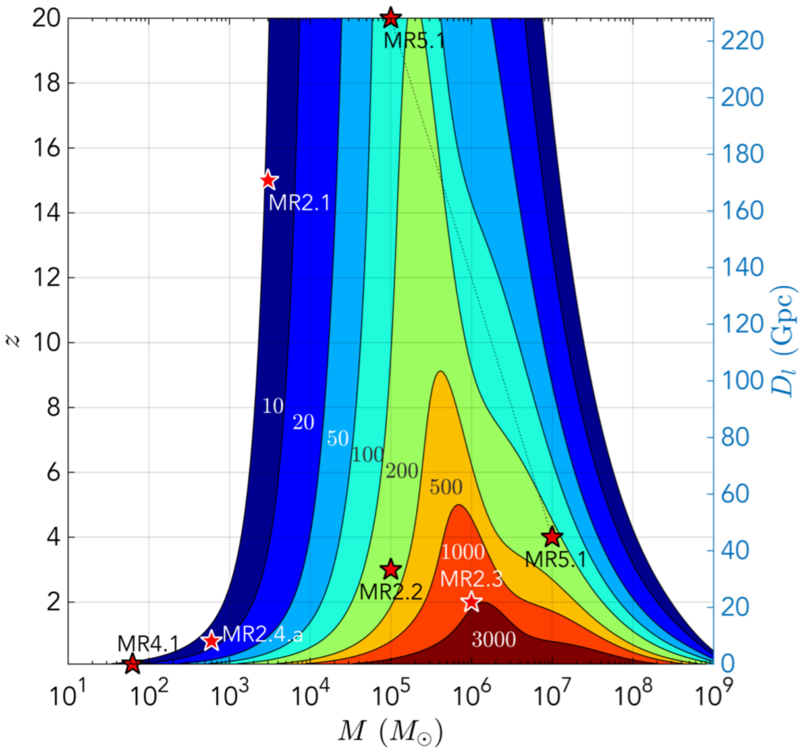
    - We aren’t that ready for post merger of BBHs

    - We do not have good enough models with eccentricity, precession and higher modes

- Alternate Theories of Gravity

My Hope for Workshop

Amplitude, phase (and eccentricity) bounds on waveforms based on threshold boundaries for each SO/SI



To prove the existence of IMBH:

- detect total 10^4-10^6M BH
- lightest BH 10^2-10^4M
- at z<3
- with 10% precision on component masses & thus SNR 20